On the Theory of Persecution

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Abstract—Classification of persecution movement laws is proposed. Modes of persecution in number of specific cases were researched. Modes of movement control using GLONASS/GPS are discussed.

Keywords—Controlled Dynamic Motion, Unmanned Aerial Vehicles, GPS.

I. INTRODUCTION

PRECISION of GPS/GLONASS positioning is getting increasing. This situation is favorable for using of satellite navigation systems in various areas. In our previous work, we showed a method for increasing of accuracy of positioning using GPS/GLONASS based on improving of mathematical calculations [1]. This approach allows us to manage vehicle not only control them. L.S. Pontryagin and co-workers developed an effective theory for managing of dynamic movement [2].

II. A MATHEMATICAL MODEL OF PERSECUTION

We discovered dynamic movements defined by following system of equations.

$$\frac{dx^i}{dt} = f^i(x^1, \ldots, x^n, u^1, \ldots, u^r, t), i = 1 \ldots n \tag{1}$$

$x^1, \ldots, x^n$ are the phase coordinates and velocities of the managed object, $u^1, \ldots, u^r$ –functions managing turn wheel, brakes and accelerators forming the closed limited set U in the r-dimensional space of object control, t is the time [3], [4].

Dynamic movement control problem is to select the motion control signals $u_i = u_i(t)$ for which (1) gives an unique solution $x_i = x_i(t)$, corresponding to the initial data $x^i(t_0) = (x_0) i, i = 1, \ldots, n$, which satisfy predefined condition of motion. The integral feature usually determines optimality condition of motion

$$J = \int_{t_1}^{t_2} f^{n+1}(x^1, \ldots, x^n, u^1, \ldots, u^r, t) dt \tag{2}$$

which has the property of transitivity for autonomous systems.

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In the case where the optimality criterion is a requirement of minimal motion time between two given points, or closed region of the phase space of generalized coordinates and velocities of the managed moving body, the problem of managing becomes a problem of performance according to [2].

Performance problems are simplest because optimality function in this case becomes equal to one. These ones are well known and there are number of solutions for specific classes of movement managing problems [2]. Partially continuous managing signals $u(t)$ located on the border of the limited set U during all the time are satisfy to solution of performance problems [2]. Therefore, to fastest movement from one point of trajectory to another driver should speed up at one point and have to maximum push on the brake at another point. In many practical situations, that a strategy is a good choice, but every driver knows the driving rule: accelerates smoothly and brake smoothly. It satisfies to another optimality condition of motion. Each optimality condition defines the own class of optimal controls and optimal trajectories. In the area of intersection of classes of managed movement trajectories simultaneously satisfies two or more of the conditions of optimality.

The simplest class of managed motion is a straight linear motion with constant velocity. Managing signals are constant and equal to zero [5].

We choice an optimal motion with minimal integral curvature as a generalized straight linear motion

$$J = \int_{t_1}^{t_1+\tau} \sigma(t) \cdot u(t) \cdot dt \tag{3}$$

or more common class – the motion with minimal torsion. Trajectories of performance with partially constant acceleration are given by quadratic splines. They cannot always approximate trajectory of persecution on a finite time interval, which have a smooth managing.

The problem of motion managing for a minimum curvature and torsion we call problem of persecution.

Communication between GLONASS/GPS receiver with satellite system is carried out at an interval Δt. However, persecution time may be both larger and smaller Δt.

Determining of the persecution time is a very important problem both in theory of managing and in theory of persecution. If the time between communication sessions is less than the persecution time $\Delta t < \tau$, then each session gives final trajectory persecution correction, time of persecution and managing signals [6].

Anyway, final time of persecution satisfies to own optimal law of persecution $x_i(t)$, usually different from the straight linear motion.

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

We proffer classification of the available laws of persecution based on geometric and algebraic approaches.

We can solve the generalized two point Cauchy problems for each class of trajectories and find control commands in each class of trajectories for the dynamic laws of persecution based on the selected class of the persecution.

Finally, the computer subsystem initially selects a class of trajectory then finds laws of dynamic control and solves the persecution problem based on the relative positions and velocities of interceptor and target.

In the simplest case, when the target does not deviate from the predetermined path, the command of persecution control is equal to zero. If the target is retarding linearly, the persecution command for increasing of linear tangent velocity is run. There is a finite set of situations that require more complex trajectories of persecution, which are analyzed below.

III. THE GEOMETRICAL APPROACH

A. Optimality Criteria and Equation of Persecution

Its known geometrical characteristics of trajectory of movement includes unit vectors for accompanying orthoreper:
$\frac{d\bar{r}}{ds} = \tau$ – unit vector of velocity tangent to the trajectory
$\frac{d^2\bar{r}}{ds^2} = \sigma \cdot n$, n – unit vector of normal to the trajectory, σ - curvature of the trajectory $\left[\frac{d\bar{r}}{ds}, X, \frac{d^2\bar{r}}{ds^2}\right] = \sigma \cdot b$, b – unit vector of binormal of the trajectory, s – length of the trajectory's arc, - a natural parameter; $\sigma2 = -\sigma^{-2} \cdot \left[\frac{dr}{ds} \times \frac{d^2r}{ds^2}\right] \cdot \left(\frac{d^3r}{ds^3}\right)$ – second curvature or twisting of the trajectory [3]. A straight line has zero curvature $\sigma = 0$ and as a result zero twisting $\sigma2 = 0$. They forms the first geometrical class of trajectories widely using in dynamic games. The next class of trajectories are the zero torsion trajectories with smallest curvature.

B. Planar Motion with Minimal Integral Curvature

Let us require (3) on the optimal trajectories of persecution should be smallest without using of dynamic equations of managed object (1). Using the time *t* as a parameter we have following:

$$\dddot{x}(t)\ddot{y}(t)\dot{z}(t) - \dddot{x}(t)\ddot{z}(t)\dot{y}(t) - \dot{x}(t)\dddot{y}(t)\dot{z}(t) +$$
$$\dot{x}(t)\dddot{z}(t)\dot{y}(t) + \dot{x}(t)\ddot{y}(t)\dot{z}(t) - \dot{x}(t)\ddot{z}(t)\ddot{y}(t) = 0 \quad (4)$$

which means the linear dependence of the velocity vectors, acceleration and acceleration of acceleration:

$$a1(t) \cdot \begin{bmatrix} \dddot{x}(t) \\ \dddot{y}(t) \\ \dddot{z}(t) \end{bmatrix} = a(t) \cdot \begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix} + b(t) \cdot \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} \quad (5)$$

Equation (5) represents a general type of linear homogeneous differential equation of the second order relatively components of interceptor's vector of velocity. This one has two fundamental solutions relatively vector of velocity leading to the plane motion. Indeed, the movement will be flat if the twisting of trajectory

$$\sigma2 = -\sigma^{-2} \cdot \left[\frac{dr}{ds} \times \frac{d^2r}{ds^2}\right] \cdot \left(\frac{d^3r}{ds^3}\right) \quad (6)$$

will be equal to zero in every point [3]. However, mixed multiplication of vectors in the right part of (6) becomes equal to zero due to linear dependency (5), as the determinant of a matrix with linearly dependent rows.

In the particular case of $a_1(t)=0$ the order of equation is decreasing, trajectories becomes straight linear and under $a(t)=0$ motion becomes inertial. Simplified variants of plane motion are discussed above. In the general case of plane motions we can consider $a(t)=1$. We can consider a plane passing through the radius-vector connecting interceptor at initial time of persecution and the target at final time of persecution and vector of velocity of interceptor at initial time of persecution as a plane of persecution. Plane of persecution may change when adjusting the persecution.

Arbitrary coefficients of equation connected to the control signals and have to be defined by methods of reversal problems of dynamics. Solutions founded of conjugated expression ((4) or (5)) with some control signals $a_1(t)$, $a(t)$, $b(t)$ we substitute into (1), from which we find the control signals $u_k(t)$ controlled dynamical system [7].

C. Movement with Minimal Integral Torsion

Planar motion of persecution is quite capable to solve the first problem of persecution and is not able to solve the second problem of persecution if the velocity vector at the time of completion of the persecution has a component orthogonal to the plane of the persecution.

In this case, a wider class of trajectories, such as converging spiral or helix - trajectory with torsion that goes out of the plane of persecution is needed.

To do this, we can choose the limitation in form of the generalization of (4) or the generalization of (5). We may require the minimal of functional

$$S = \int_{t_1}^{t_1+\tau} \left(\sigma2 + \lambda\left(\frac{d\bar{r}}{ds}\right)^2\right) ds \quad (7)$$

$\sigma2$ is a second curvature of trajectory defined by (4) [3].

Using the orthogonality properties of the unit vectors of the accompanying benchmark, we can cause the integral (8) to the form

$$S = \int (b(s)dn(s) + \lambda ds) \quad (8)$$

Variation of functional in (9) leads to the complex nonlinear equation interesting in some cases

$$\left(\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3}\right)\left(-\frac{1}{\sigma^2}\right) - \frac{d}{ds}\left(\left(\frac{d^3\bar{r}(s)}{ds^3} \times \frac{d\bar{r}(s)}{ds}\right)\left(-\frac{1}{\sigma(s)^2}\right)\right) + \frac{d^2}{ds^2}\left(\left(\frac{d\bar{r}}{ds} \times \frac{d^2\bar{r}}{ds^2}\right)\left(-\frac{1}{\sigma^2}\right)\right) - \frac{d}{ds}\left(\left(\frac{4}{\sigma^3} \cdot \frac{d^2\bar{r}}{ds^2}\right) \cdot \left(\frac{d\bar{r}}{ds} \times \frac{d^2\bar{r}}{ds^2}\right) \cdot \frac{d^3\bar{r}}{ds^3}\right) + 2\lambda\frac{d\bar{r}}{ds} = 0 \quad (9)$$

However, following a geometrical logic, as the conjugated dynamic equations we choose the generalized expression (6) of planar motion

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

$$\dddot{r}(t) = a(t)\ddot{r}(t) + b(t)\ddot{r}(t) + c(t)\dot{r}(t) \qquad (10)$$

which have three fundamental solutions. Two fundamental solutions give a planar curative motion and the third fundamental solution may give a torsion of trajectory [8].

IV. CLASSES OF PLANAR TRAJECTORIES OF PERSECUTION

A. Planar Circular Motion

The purpose of this and subsequent subsections is the solution of targeting problem at a particular class of trajectories. In case of constant speed planar motion the requirement of minimal integral curvature leads to the constant curvature round motion. This class of persecution is the second one after straight-line motion [9].

Algorithm is following. For a given initial velocity vector $\{V_x, V_y\}$ find the line perpendicular to the initial velocity vector and passing through the initial point $\{X_0, Y_0\}$. For given initial $\{X_0, Y_0\}$ and final $\{X_1, Y_1\}$ points of trajectory of persecution define coordinates of the center of curvature of the trajectory $\{x, y\}$ and the radius of curvature.

$$R = \sqrt{\frac{\frac{(X_0^2 - 2X_0X_1 + X_1^2 + Y_1^2 - 2Y_1Y_0 + Y_0^2)^2(V_y^2 + V_x^2)}{(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)^2}}{2}}$$

$$x = \frac{-V_yX_0^2 + V_yX_1^2 + Y_1^2V_y - 2Y_1V_yY_0 - 2Y_1V_xX_0 + Y_0^2V_y + 2Y_0V_xX_0}{2(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)}$$

$$y = -\frac{2Y_0V_yX_0 - 2Y_0V_yX_1 - Y_0^2V_x + V_xX_0^2 - 2V_xX_0X_1 + V_xX_1^2 + V_xY_1^2}{2(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)}$$

$$\varphi = \arctan\left(\frac{V_x}{V_y}\right) \qquad (11)$$

Formula (11) solves the Cauchy problem and we can definitely determine the law of prosecution:

$$x(t) = x + R\cos\left(\frac{ut}{R} + \varphi\right)$$
$$y(t) = y + R\cos\left(\frac{ut}{R} + \varphi\right)$$
$$z(t) = 0$$

The time of persecution $\tau = \frac{2R}{|V_0|}\arcsin\left(\frac{[X_xV_0]}{|X||V_0|}\right)$. Here X – vector from initial to final point of persecution, V_0 – vector of initial velocity.

- This class may occur circular trajectories with a constant centripetal acceleration, but with variable velocity and curvature when it is necessary to catch up with the target or close to it with a minimum speed.

One of variant is $R = K \cdot u^2$ with constant K.

The second variant is the twisting spiral.

Initial spiral's slope is defined by vector of velocity at the start of persecution. Indeed let us suppose

$$a(t) = -2\lambda, \ b(t) = -\lambda^2 - \omega^2$$

and find the solution

$$x(t) = a \cdot \exp(-\lambda t) \cdot \cos(\omega t + \lambda)$$
$$y(t) = a \cdot \exp(-\lambda t) \cdot \sin(\omega t + \lambda) \qquad (12)$$
$$z(t) = 0$$

Parameters of the low of persecution are defined from initial coordinates and velocities of the target and interceptor.

Note for simplicity we use a relativity principle and we suppose initial point is stationary at the origin. Parameters of the law of persecution are calculated via relative initial coordinates and velocities

$$a = \sqrt{x_0^2 + y_0^2}$$

$$\lambda = \frac{-vy_0y_0 - x_0vx_0}{x_0^2 + y_0^2}$$

$$\omega = -\frac{vx_0y_0 - x_0vy_0}{x_0^2 + y_0^2} \qquad (13)$$

$$\varphi = \arctan\left(\frac{y_0}{x_0}\right)$$

B. Planar Trajectories with Available Management

It has known in the problems of performance control signal take their boundary values from the area of management with an abrupt change [2]. Available control we understand as finite functions belong to the compact set of management U [2]. However, in contrast to the results of the theory of performance, our control signals are differentiable functions on the whole of set U.

Equation (6) with substitution $V(t) = \omega(t) \cdot Z(t)$ allows transition to equation

$$\ddot{Z}(t) = f(t) \cdot Z(t) \qquad (14)$$

where $f(t) = \left(b(t) + \frac{a(t)^2}{4} - \dot{a}(t)\right)$ and function $\omega(t) = e^{\frac{\int a(t)dt}{2}}$, which doesn't consist the first derivative. Among all kinds of functional dependencies the managing signal $a(t)$ in form

$$a(t) = \sum_{k=1}^{N} \frac{a_k}{t - t_k} \qquad (15)$$

is of particular interest, as it leads to expression

$$\omega(t) = \prod_{k=1}^{N}(t - t_k)^{\frac{a_k}{2}} \qquad (16)$$

which represent the limited function if all a_k are positive values. The numbers t_k are the moments of time of shifting retarding to accelerating when zeroing relative velocity occur. These moments in time are the moments of finishing of persecution τ. To complain this condition the coefficients of acceleration a_k should be positive and possibly integer values. Expression (14) has a solution:

$$Z(t) = C_1 \cdot e^{\int \xi(t)dt} \qquad (17)$$

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

where function ξ(t) is found as a solution of equation

$$\frac{d}{dt}\xi(t) + \xi(t)^2 = f(t) \tag{18}$$

Now we can limit the force of (14). Note the scale factor "force" $K^2 f(t)$ is transferred to the ξ(t) and t: $t_l = tK$, $\xi_1 = \frac{\xi}{K}$, that don't appear in (17).

C. The First Case

Let $f(t) = 0$. The cause by $b(t) + \frac{a(t)^2}{4} - \dot{a}(t) = 0$. For limb control signals and monotone subject that most closely approaching to Pontrjagin's "relay" type control, put

$$\frac{a(t)^2}{4} - \dot{a}(t) = \frac{n*(n+2)}{(t-\tau)2}, \qquad b(t) = -\frac{n*(n+2)}{(t-\tau)2}.$$

In this case, we find that

$$Z(t) = c1*(t+c2), \ V(t) = c1*(t-\tau)^n*(t+c2),$$

$$x_k(t) = \frac{(c1_k*(t-\tau)^{n+1}*((n+1)*t+\tau+c2_k)}{((n+1)*(n+2))} + x0_k$$

or by beginning coordinates and velocity

$$x_k(t) = \left((x1_k - x0_k)\left(1+(n+1)\frac{t}{\tau}\right) - V0_k t\right)(\frac{t}{\tau}-1)^{(n+1)} + x1_k \tag{19}$$

The time of persecution have evolved as

$$\tau = \frac{1}{ax}*\left(-(n+1)*V_x o + \sqrt{((n+1)V_x o)^2 + ax(n+2)(n+1)(x1-x0)}\right) \tag{20}$$

D. The Second Case

Let $b(t) = 0$
At that cause we have

$$V_k(t) = \frac{(-\tau+t)^n C_{1k}\sqrt{-\frac{\tau-t}{C_{2k}}}\left(\left(\left(\frac{-\tau+t}{C_{2k}}\right)^{\sqrt{(n+1)^2-\frac{3}{4}}}\right)^2 + 1\right)}{2\left(\frac{-\tau+t}{C_{2k}}\right)^{\sqrt{(n+1)^2-\frac{3}{4}}}}$$

Unlike the first case, the relative velocity does not vanish while reaching the time of persecution τ but it becomes the maximum physical and technical conditions for all positive integers even n, except for n = 0. When n = 0, we have uniformly accelerated motion with acceleration.

$$a_m = \frac{C_{1m}}{2C_{2m}} V_k(t) = \frac{C_{1k}\left(\left(\frac{-\tau+t}{C_{2k}}\right)+1\right)}{2}$$

$$V_k(\tau) = \frac{C_{1k}}{2}$$

$$V_k(0) = \frac{\left(\left(\frac{-\tau}{C_{2k}}\right)+1\right)}{2}, \text{ or}$$

$$V_k(t) = V_k(0) + a_k \cdot t$$

For arbitrary n> 0 motion, as the velocity increases rapidly in the final stage of persecution. This situation can be compared with the slow persecution before the targeting in a straight line and then shooting a laser beam. This one has conditional practical application.

$$x_k(t) = C_{1k}\left(\frac{\left(\frac{t-\tau}{C_{2k}}\right)^{n+\frac{3}{2}+\sqrt{(n+1)^2-\frac{1}{4}}}}{n+\frac{3}{2}+\sqrt{(n+1)^2-\frac{1}{4}}} + \frac{\left(\frac{t-\tau}{C_{2k}}\right)^{\frac{3}{2}+n-\sqrt{(n+1)^2-\frac{1}{4}}}}{\frac{3}{2}+n-\sqrt{(n+1)^2-\frac{1}{4}}}\right) + x_{0k}$$

E. The Third Case

The control function $b(t)$ such that the controlling force $f(t)$, defined on a finite time interval τ is given by Legendre polynomials

$$P_n(x) = \frac{1}{2^n \cdot n!}\cdot\frac{\partial^n}{\partial x^n}((x^2-1)^n), \text{ where } x = \frac{t}{\tau}.$$

Those complex movements can be useful for the persecution of managed aircraft when an effort against the persecution takes place. In simple cases may be useful more simple management techniques, which are defined by special cases of solutions of (6).

F. The Differentiable Case

Let us $b(t) = \dot{a}(t)$, in (3), then (6) reduces to

$$\dot{U}(t) = a(t)U(t) + \Omega,$$

where $U(t)$ is a vector of velocity and Ω are integration constants. Let us denote

$$u(t) = e^{-\int a(t)dt},$$

then general type solution of the equation becomes

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \frac{\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}\cdot\int u(t)dt + \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}}{u(t)} \tag{21}$$

Existing of two vectors reduces this solution to the plane case. Solution of (21) in the case of a constant $a(t)$, shows that vector of velocity shifts from one direction to another one exponentially under persecution. Without losing of significance, we can assign

$$a(t) = -\lambda,$$

$$\dot{x}(t) = \frac{\Omega_x}{\lambda} + C_x e^{-\lambda t}$$

$$\dot{y}(t) = \frac{\Omega_y}{\lambda} + C_y e^{-\lambda t}$$

$$\dot{z}(t) = 0$$

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

Coordinates and velocities of the initial and final points of persecution and time of persecution express parameters of the law of motion:

$$x(t) = V_{x1}t + \frac{V_{x1} - V_{x0}}{\lambda}e^{-\lambda t} + x_0$$

$$y(t) = V_{y1}t + \frac{V_{y1} - V_{y0}}{\lambda}e^{-\lambda t} + y_0$$

$$\lambda = -\frac{Vx_0Vy_1 - Vx_1Vy_0}{-Vx_1Y_1 + Vx_1Y_0 + X_1Vy_1 - X_0Vy_1}$$

$$x_0 = \frac{-Vx_1Vy_0X_0 + Vx_1^2Y_1 - Vx_1X_1Vy_1 + Vx_1X_0Vy_1 - Vx_1Y_1Vx_0 + Vx_1Y_0Vx_0 + X_1Vx_0Vy_1}{Vx_0Vy_1 - Vx_1Vy_0} \quad (22)$$

$$y_0 = \frac{Vy_0X_1Vy_1 - Vy_0X_0Vy_1 + Vy_1Vx_1Y_1 - Vy_1Vx_1Y_0 - X_1Vy_1^2 + X_0Vy_1^2 + Y_0Vx_0Vy_1 - Vy_0Vx_1Y_1}{Vx_0Vy_1 - Vx_1Vy_0}$$

Time of persecution is estimated by formula:

$$\tau = -\frac{Vy_0X_1 - Vy_0X_0 + Vx_1Y_1 - Vx_1Y_0 - X_1Vy_1 + X_0Vy_1 - Y_1Vx_0 + Y_0Vx_0}{Vx_0Vy_1 - Vx_1Vy_0}$$

We have the more general case of solution of (6). Let us transform (3) to the form

$$\frac{d}{dt}(\ddot{r}(t) - a(t)\dot{r}(t) + (\dot{a}(t) - b(t))r(t)) = r(t)\frac{d}{dt}(\dot{a}(t) - b(t))$$

In addition, require

$$\dot{a}(t) - b(t) = \alpha$$

should be constant. Then we have equation

$$\ddot{r}(t) - \alpha(t)\dot{r}(t) + \alpha r(t) = \beta,$$

and it's solution under

$$a(t) = \alpha \cdot t$$

$$r(t) = M\left(\frac{a-\alpha}{2a}, \frac{3}{2}, \frac{at^2}{2}\right)tC_2 + U\left(\frac{a-\alpha}{2a}, \frac{3}{2}, \frac{at^2}{2}\right)tC_1 + r_0 -$$

is a more general type of planar motion. There M and U are special functions.

V. PERSECUTION WITH TORSION

A. Screwed Spiral

The most important in the theory of the prosecution motion with torsion is a movement along a twisting of spiral.

$$x(t) = Rexp(-\mu\tau)\cos(\omega\tau) - R + x0, y(t) = Rexp(-\mu\tau)\sin(\omega\tau) + y0, z(t) = azt^2 + Vz0t + z0$$

It satisfies to (10) under finite control functions

$$a(t) = -\frac{2t\mu^3 + 3\mu^2 + 2t\omega^2\mu - \omega^2}{t\mu^2 + 2\mu + t\omega^2}$$

$$b(t) = -\frac{(\mu^4 + 2\mu^2\omega^2 + \omega^4)t}{t\mu^2 + 2\mu + t\omega^2} \quad (23)$$

$$c(t) = \frac{\mu^4 + 2\mu^2\omega^2 + \omega^4}{t\mu^2 + 2\mu + t\omega^2}$$

where the functions $a(t)$ and $b(t)$ tend to a finite limit, and $c(t)$ – tends to zero. Omitting cumbersome calculations show dependence of the rate of persecution time.
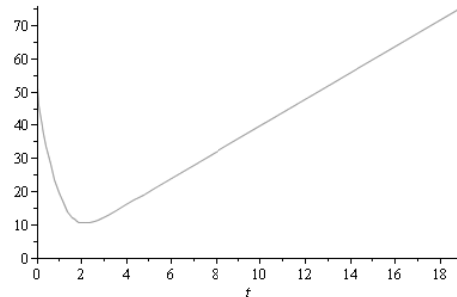

Fig. 1 Dependence of velocity of persecution on the time
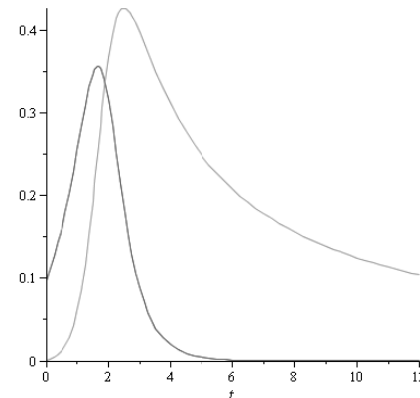

Fig. 2 Dependence of the torsion and curvature of the trajectory on the time of persecution, light line is torsion, dark line is curvature
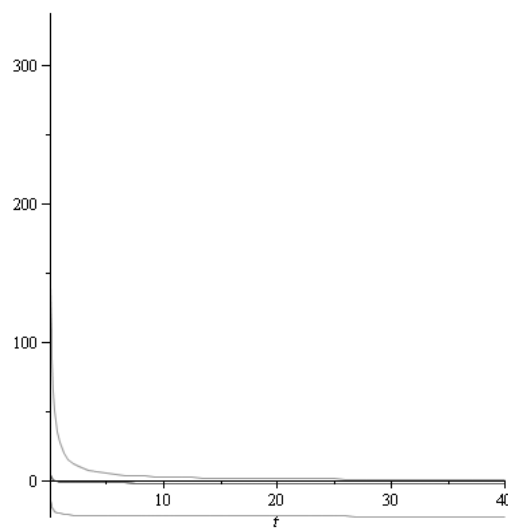

Fig. 3 Dependence of controlling forces on time

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VI. CONCLUSION

Stereotyped law of uniformly accelerated motion is using in the theory of performance. Complex trajectory is approximating with splines and reaching by triggering of control signals. The set of librarian trajectories is using in the theory of persecution where each class of trajectories has own analytical expression and constants as parameters of equation. The problem of targeting is solving for every class of equations as defining of time of persecution and numerical parameters of trajectories allowing moving from initial point of phase space to the final point. The law of motion with certain numerical parameters allows us find control signals as continuous functions of time.

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