Identifying Unknown Dynamic Forces Applied on Two Dimensional Frames

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Abstract—A time domain approach is used in this paper to identify unknown dynamic forces applied on two dimensional frames using the measured dynamic structural responses for a sub-structure in the two dimensional frame. In this paper a sub-structure finite element model with short length of measurement from only three or four accelerometers is required, and an iterative least-square algorithm is used to identify the unknown dynamic force applied on the structure. Validity of the method is demonstrated with numerical examples using noise-free and noise-contaminated structural responses. Both harmonic and impulsive forces are studied. The results show that the proposed approach can identify unknown dynamic forces within very limited iterations with high accuracy and shows its robustness even noise- polluted dynamic response measurements are utilized.

Keywords—Dynamic Force Identification, Dynamic Responses, Sub-structure and Time Domain.

I. INTRODUCTION

ACCURATE identification of dynamic forces can be very important to the structural design process. In addition, in order to identify the locations of the structural elements that suffered structural damage and to determine the amount and importance of the defects on the overall structural behavior; system identification techniques are usually used that require the information about the dynamic forces applied.

The system identification techniques that have been used in the last three decades [1] have three components; input excitation, the system to be identified, and the output response information. The input excitation is the force that excites the system. The system is a mathematical model of the structure. The output is the response of a structural system due to the input excitation, reflecting the current state of the structure. Knowing the input excitation and the output response information, the system (third component) can be identified. Unfortunately, in most cases [2] and [3], it is impossible to insert force gauges into the force transfer path to measure those dynamic forces directly. Therefore, in order to get better damage detection it is required to identify the dynamic forces applied to the structures.

There are many methods available in the literature for force identification; [4] proposed a polynomial to approximate the impact force history. The coefficients in the polynomial are directly used as unknown parameters. The relation between these unknown parameters and the strain responses at the specified positions is formulated through the finite element

method and the mode superposition method. After obtaining the impact force history, the impact position is identified by comparing the numerical strains and experimental ones directly.

Reference [5] proposed an extension of the Inverse Structural Filter (ISF) force reconstruction algorithm that utilizes data from multiple time steps simultaneously to improve the accuracy and robustness of the ISF. The ISF algorithm uses a discrete time system model of a structure and the measured response to estimate the forces causing the response.

Reference [6] proposed a genetic algorithm (GA)-based approach for impact load identification, which can identify the impact location and reconstruct the impact force history simultaneously. In this study, impact load is represented by a set of parameters, thus the impact load identification problem in both space (impact location) and time (impact force history) domains is transformed to a parameter identification problem. A forward model characterizes the dynamic response of the structure subject to a known impact force is incorporated in the identification procedure. By minimizing the difference between the analytical responses given by the forward model and the measured ones, GA adaptively identify the impact location and force history with its global search capability.

Reference [7] proposed an iterative approach for both structural parameters and dynamic loading identification, referred to as weighted adaptive iterative least- squares estimation with incomplete measured excitations (WAILSE-IME). The accuracy, convergence, and robustness of the proposed approach was demonstrated via numerical simulation on a six-story shear building model with noise-free and different levels of noise-polluted structural dynamic response measurements.

In this paper, a time domain approach based on the least-square method is used to identify the dynamic forces applied on two dimensional steel frames using a sub-structure finite element model. A short length of measurement from only three or four accelerometers is required for the identification process. The location of the input force is assumed to be known in the identification. Validity of the method is demonstrated with numerical examples using noise-free and noise-contaminated structural responses.

II. MODELING SUB-STRUCTURE REQUIRED FOR IDENTIFYING FORCE APPLIED ON 2D FRAMES

The sub-structure required for force identification should be selected in a way that the response measurements are available at all Degrees of Freedom (DOF) of the sub-structure and the

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location of the dynamic excitation force is assumed to be known and included in the sub-structure. The selection of the sub-structure will start by determining the node where the unknown dynamic force is applied. Then it is required to determine the nodes and elements that are attached to that node. Accordingly, the governing dynamic equation for the sub-structure can be written as:

$$\mathbf{K}_s\, \mathbf{x}_s(t) + \mathbf{C}_s\, \dot{\mathbf{x}}_s(t) + \mathbf{M}_s\, \ddot{\mathbf{x}}_s(t) = \mathbf{f}(t) \tag{1}$$

where Ks, Cs and Ms are the global stiffness, damping and mass matrices for the sub-structure, respectively, and $\mathbf{X}_s$, $\dot{\mathbf{X}}_s$, $\ddot{\mathbf{X}}_s$ are vectors containing the dynamic responses in terms of displacement, velocity and acceleration at time t for the sub-structure, respectively, and f (t) is the unknown dynamic force vector applied on the structure.

The global stiffness matrix for the sub-structure (Ks) can be assembled by using the method of superposition, the direct stiffness method, for the local stiffness matrices of all the elements in the sub-structure. The local stiffness matrix for two-dimensional beam element of uniform cross section is given by:

$$\bar{K}^i_{6\times6} = \frac{E_i I_i}{L_i}\begin{bmatrix} A_i/I_i & 0 & 0 & -A_i/I_i & 0 & 0 \\ 0 & 12/L_i^2 & 6/L_i & 0 & -12/L_i^2 & 6/L_i \\ 0 & 6/L_i & 4 & 0 & -6/L_i & 2 \\ -A_i/I_i & 0 & 0 & A_i/I_i & 0 & 0 \\ 0 & -12/L_i^2 & -6/L_i & 0 & 12/L_i^2 & -6/L_i \\ 0 & 6/L_i & 2 & 0 & -6/L_i & 4 \end{bmatrix} \tag{2}$$

where Ei, Ii, and Li are the Young's modulus, moment of inertia, and length of the ith element, respectively.

The damping matrix Cs is assumed to be Rayleigh-type damping and can be represented as:

$$\mathbf{C}_s = \alpha\,\mathbf{M}_s + \beta\,\mathbf{K}_s \tag{3}$$

where α is the mass-proportional damping coefficient and β is the stiffness-proportional damping coefficient.

The global consistent mass matrix for the sub-structure (Ms) can be assembled by using the method of superposition for the local mass matrices of all the elements in the sub-structure. The local consistent mass matrix for two-dimensional beam element of uniform cross section is given by:

$$\bar{M}^i_{6\times6} = \frac{\bar{m}_i L_i}{420}\begin{bmatrix} 140 & & & & & \\ 0 & 156 & & & \text{Sym.} & \\ 0 & 22L_i & 4L_i^2 & & & \\ 70 & 0 & 0 & 140 & & \\ 0 & 54 & 13L_i & 0 & 156 & \\ 0 & -13L_i & -3L_i^2 & 0 & -22L_i & 4L_i^2 \end{bmatrix} \tag{4}$$

where Li is the element length and m_i is the mass per unit length.

Accordingly, (1) can be rewritten in a matrix form as:

$$[\mathbf{A}] * \{\mathbf{B}\} = \{\mathbf{G}\} \tag{5}$$

where [A] is a matrix of size (3 x n) × Ls; n is the total number of sample time points, Ls is the total number of elements and damping coefficients in the sub-structure and can be expressed as:

$$[\mathbf{A}] = [\, \mathbf{Q}^1 \mathbf{x}_s(t)\ \ \mathbf{Q}^2 \mathbf{x}_s(t) \ldots \mathbf{Q}^{nes} \mathbf{x}_s(t)\ \mathbf{Q}^1 \dot{\mathbf{x}}_s(t)\ \ \mathbf{Q}^2 \dot{\mathbf{x}}_s(t)\ ]$$
$$\ldots \mathbf{Q}^{nes} \dot{\mathbf{x}}_s(t)\ \mathbf{M}_s \dot{\mathbf{x}}_s(t)] \tag{6}$$

where Q is the 6x6 matrix (2) excluding (EI/L) for each element in the sub-structure and nes is the total number of elements in the sub-structure.

{B} vector in (5) is a vector of size Ls x 1 and can be shown to be:

$$\{\mathbf{B}\} = \begin{Bmatrix} k_1 \\ k_2 \\ \ldots \\ k_{nes} \\ \beta\,k_1 \\ \beta\,k_2 \\ \ldots \\ \beta\,k_{nes} \\ \alpha \end{Bmatrix} \tag{7}$$

{G} vector in (5) is a vector of size (3 x n) x 1:

$$\{\mathbf{G}\} = \begin{Bmatrix} f_1(t) - \mathbf{M}_s\,\ddot{x}_s(t) \\ f_2(t) - \mathbf{M}_s\,\ddot{x}_s(t) \\ \ldots \\ f_{TD}(t) - \mathbf{M}_s\,\ddot{x}_s(t) \end{Bmatrix} \tag{8}$$

where f is the unknown dynamic force need to be identified and TD is the total number of DOF in the sub-structure.

The stiffness of each element (EI/L) in the two dimensional frame is assumed unknown and will be identified with the unknown dynamic force. A least-squares-based procedure proposed by [8] is used in this paper for the solution of unknown dynamic force f (t) by starting an iteration process by assuming that the unknown dynamic force to be zero at all n time sample points. It is observed through the numerical examples shown below that the method is not sensitive to this initial assumption, or the type and form of excitation.

Using the least-squares-based procedure proposed by [8]; the solution of unknown system parameters {B} and unknown dynamic force f(t) are evaluated using the following expression:

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$$\{\mathbf{B}\} = \left[\left[\mathbf{A}^{\mathbf{T}}\right]\left[\mathbf{A}\right]\right]^{-1}\left[\mathbf{A}^{\mathbf{T}}\right]\left[\mathbf{G}\right] \qquad (9)$$

The algorithm will iterate until a convergence in the unknown dynamic force with a predetermined tolerance set to be 10^{-6}. Accordingly, the unknown dynamic force is determined with a reasonable accuracy.

III. NUMERICAL EXAMPLES

A four story two bay plane steel frame (shown in Fig. 1) is used in this section to validate the effectiveness of the method in identifying the unknown dynamic forces. The frame consists of 20 members; 12 columns and 8 beams. The height of the columns in each floor is 4.0m and each bay width is 10.0m. W18x71 steel section is used for all the members. Assuming the bases are fixed; the structure is represented by 36 Degrees of Freedom (DOFs); 3 DOFs at each node.

Two cases representing two types of dynamic forces are adopted in this example:

Case 1: A harmonic force $\mathbf{f(t)}$ = 10 sin (20π t) is applied on node 1 of the two dimensional frame as shown in Fig. 1.

Case 2: An impact force of 10 kN at 0 sec and 0 kN at 0.05 sec is applied on node 1 of the two dimensional frame as shown in Fig. 1.
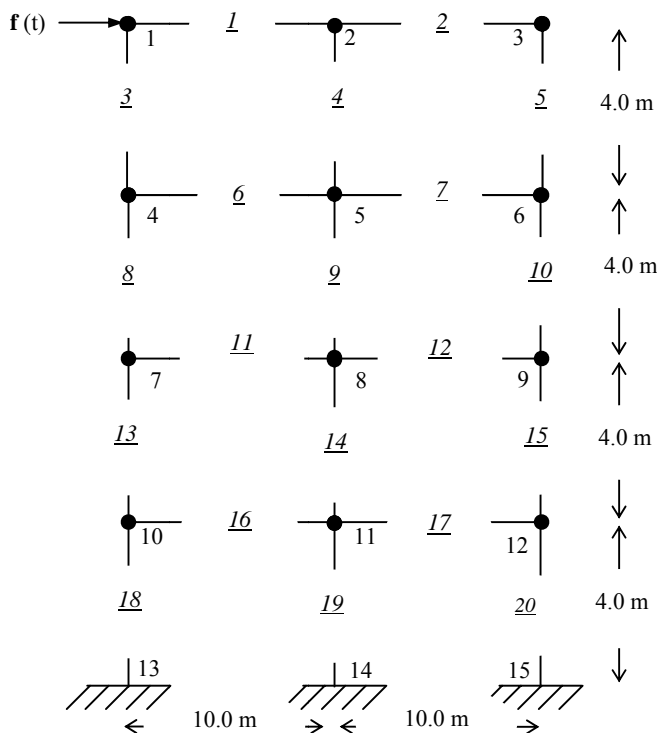
Fig. 1 Two dimensional steel frame used in the numerical examples

Based on the basic modeling and formulation of the least-square method, the selection of the sub-structure is started by determining the node where the unknown dynamic force is applied which is node 1 in this example. Then determine the nodes and elements that are attached to node 1, which are nodes 2 and 4 and elements 1 and 3. Fig. 2 shows the sub-

structure needed. It consists of 3 nodes and 2 elements only. Accordingly, three accelerometers are needed to be placed at nodes 1, 2 and 4 to measure the dynamic responses.

A. Case 1: Identifying Unknown Harmonic Force

The two dimensional steel frame is modeled using finite element software package SAP 2000 [9]. The harmonic force is applied on node 1. The theoretical dynamic responses including the acceleration, velocity and displacement of all 36 DOFs were obtained. After the theoretical responses are evaluated, the information on the harmonic force is completely ignored, and the nodal responses of the subs-structure, i.e. 9 DOFs are only used in the algorithm. The location of the input harmonic force is assumed to be known in the identification and the stiffness of each element (EI/L) in the sub-structure is assumed to be unknown as mentioned earlier. The responses used in the algorithm are with short length of measurement. In this case the responses are from 0.05 sec to 0.40 sec at a time interval of 0.01 sec only. The responses are assumed to be noise free. However, from experimental point of view, noise in the response measurements cannot be avoided. To address the issue of noise in the dynamic responses, a numerically generated noise with intensity of 10% of the root mean square (RMS) values of the responses observed at all DOFs are added to the theoretical responses. Accordingly, the unknown harmonic force is identified by using both noise free and noise included dynamic responses.
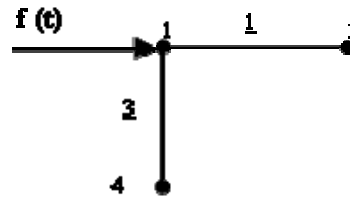

Fig. 2 Sub-structure needed for identifying the unknown dynamic force used in the numerical examples
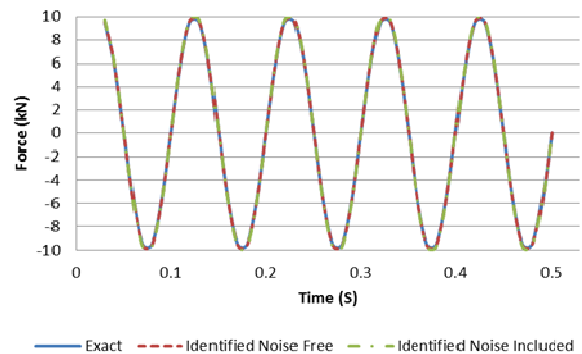

Fig. 3 Force identification at Node 1 for Case 1

Fig. 3 shows the results of the harmonic force identification for noise free and noise included cases compared with the exact force. It is obvious that the algorithm and the sub-structure identified the unknown harmonic force very well in both cases. The maximum error in force identification in the

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noise free case was less than 1% and this number was more for the noise included dynamic responses but less than 4%.

B. Case 2: Identifying Unknown Impact Force

The two dimensional steel frame is modeled again using finite element software package SAP 2000 [9]. The responses used in the algorithm are with short length of measurement. In this case the responses are from 0.01 sec to 0.05 sec at a time interval of 0.001 sec, yielding 41 time points only. The unknown impact force is identified by using both noise free and noise included dynamic responses. To address the issue of noise in the dynamic responses, a numerically generated noise with intensity of 10% of the root mean square (RMS) values of the responses observed at all DOFs are added to the theoretical responses.

Fig. 4 shows the results of impact force identification for noise free and noise included cases compared with the exact force. It is obvious that the algorithm and the sub-structure identified the unknown impact force very well in both cases. The maximum error in force identification in the noise free case was less than 0.8% and this number was more for the noise included dynamic responses but less than 3%.
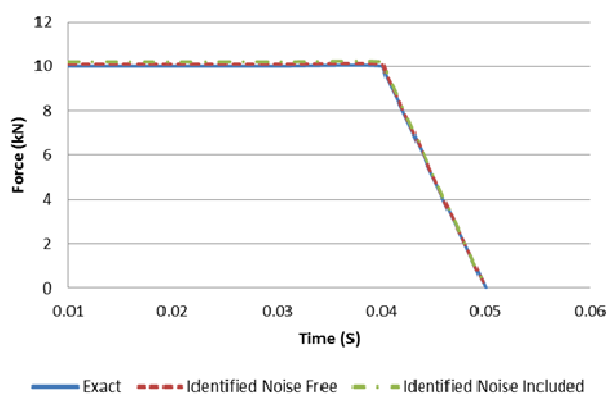

Fig. 4 Force identification at Node 1 for Case 2

IV. CONCLUSIONS

A time domain approach based on the structural responses is presented for identifying dynamic excitation forces applied on two dimensional steel frames. A sub-structure finite element model with short length of measurement from only two or three accelerometers was required for the iterative least-square algorithm to identify the unknown dynamic force applied on the structure.

The results showed that:

1) The maximum error in harmonic force identification in the noise free case was less than 1% and this number was more for the noise included dynamic responses but less than 4%.

2) The maximum error in impact force identification in the noise free case was less than 0.8% and this number was more for the noise included dynamic responses but less than 3%.

Accordingly, the method identified the unknown dynamic force applied on two dimensional frames accurately for both harmonic and impulsive forces. Also, the results showed that the approach can identify unknown excitations within very limited iterations with high accuracy and showed its robustness even noise-polluted dynamic response measurements were utilized.

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