Sensitivity Analysis of Principal Stresses in Concrete Slab of Rigid Pavement Made From Recycled Materials

Aleš Florian, Lenka Ševelová

Abstract—Complex sensitivity analysis of stresses in a concrete slab of the real type of rigid pavement made from recycled materials is performed. The computational model of the pavement is designed as a spatial (3D) model, based on a nonlinear variant of the finite element method that respects the structural nonlinearity, enables to model different arrangements of joints, and the entire model can be loaded by the thermal load. Interaction of adjacent slabs in joints and contact of the slab and the subsequent layer are modeled with the help of special contact elements. Four concrete slabs separated by transverse and longitudinal joints and the additional structural layers and soil to the depth of about 3m are modeled. The thickness of individual layers, physical and mechanical properties of materials, characteristics of joints, and the temperature of the upper and lower surface of slabs are supposed to be random variables. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used. For sensitivity analysis the sensitivity coefficient based on the Spearman rank correlation coefficient is utilized. As a result, the estimates of influence of random variability of individual input variables on the random variability of principal stresses $\sigma_1$ and $\sigma_3$ in 53 points on the upper and lower surface of the concrete slabs are obtained.

Keywords—Concrete, FEM, pavement, sensitivity, simulation.

I. INTRODUCTION

COMPLEX analysis of pavements is often very difficult for design practice. Rheological properties of materials, cracking, joints, contact of concrete slabs in joints, contact of slab and subsequent structural layer, temperature changes, non-homogeneity of pavement base, water regime in the subgrade, environmental changes, etc. influence serviceability of the structure in a decisive way. The problem is moreover complicated by the fact that the input data are generally random variables. Further source of uncertainties stems from vagueness of input data.

Taking into account specific properties of the particular type of structure, the combination of the proper analytical model with modern simulation techniques seems to be an effective tool for the solution of the problem [1]-[3]. The analysis of a pavement using these methods provides the designer with reliability limits of the structural response and enables the determination of possible critical development.

The results of the analysis also enable finding out which input variables require special attention due to their random variability dominantly influencing the structural behavior. This type of analysis is called sensitivity analysis.

The behavior of the older type of rigid pavement is analyzed. This type of pavement is made from plain concrete, no dowels are used, and joints are made during laying of concrete. Dimensions of individual concrete slabs are 7.5 x 3.75m, see Fig. 1. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and lower surface of the slab, and by the load of intensity 50 kN at a distance of 0.25m from the edge of slab - see point 26 in Fig. 1. Thus the total state of stress in the slab results from all three different sources of load acting together. Contrary to the traditional design, the base layer is supposed to be made from a recycled material instead of a natural one. It is made from recycled concrete of fractions 0–16mm.

The computational model is based on the nonlinear finite element method. Four concrete slabs, all other layers and longitudinal and transverse joints are modeled as 3D space. Joints, contact of slabs in joints, contact of slabs and subsequent layer, and the thermal loading are modeled in detail.

Total 17 basic random input variables describing layer thicknesses, mechanical properties of materials, characteristics of joints and temperature on both surfaces of concrete slabs are used in the study. They are described by the assumed cumulative distribution functions (generally three-parametric) and by the appropriate statistical parameters. The influence of uncertainties in input variables on the behavior of the pavement is respected in the analysis with help of numerical simulation techniques [4]. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used [5], [6].

The sensitivity analysis of principal stresses $\sigma_1$ and $\sigma_3$ in concrete slabs is performed to show possibilities of reliability methods in analysis of real pavement structures. The stresses are evaluated in 53 points on the upper and lower surface (totally 106 points) of the concrete slabs, see Fig. 1. Sign convention is chosen so the positive stresses are tensile, while the negative stresses are compressive. Principal stress $\sigma_1$ represents an extreme value of tensile stress in the given point of the structure, while $\sigma_3$ is an extreme value of compressive stress, which arises due to the spatial stress state. Although the calculation and measurement of deflections on the pavement

A. Florian is with the Faculty of Civil Engineering, Brno University of Technology, Czech Republic (phone: +420-541147378; e-mail: florian.a@fce.vutbr.cz).

L. Ševelová is with the Department of Landscape Formation and Protection, Mendel University of Brno, Czech Republic (e-mail: lenka.sevelova@mendelu.cz).
plays the most important role in today’s engineering practice, the calculation of stresses seems to be in fact much more important. Principal stress represents the extreme normal stress at a given point of the structure and thus it is the crucial characteristic which would be used in dimensioning process. If the principal stress exceeds the tensile strength of the material, a local tensile crack is created. If it exceeds the compressive strength of the material (only hypothetically for pavements), the material is locally crushed.

The modern simulation technique Updated Latin Hypercube Sampling [5], [6] with 20 simulations is used for statistical analysis. It is an improved variant of Latin Hypercube Sampling [4]. The method keeps the methodology of Latin Hypercube Sampling, but uses the improved strategy of generating input samples based on specially modified tables of random permutations of rank numbers. The modified tables consist of random permutations that are mutually statistically independent. Using of Updated Latin Hypercube Sampling generally results to the further increase of accuracy, quality and reliability of the results obtained from reliability analysis. The detailed description of Updated Latin Hypercube Sampling can be found in [5], [6].

To measure the relative influence of random variability of each input variable on the random variability of the output (principal stresses in 106 points in concrete slabs), the sensitivity coefficient based on the Spearman rank correlation coefficient is proposed, see [7]. It is not limited to the linear relationship like Pearson correlation coefficient. The sensitivity coefficient is defined as

$$ r_s = 1 - \frac{6 \sum d_i^2}{N(N-1)(N+1)} $$

where $r_s$ is the sensitivity coefficient among the k-th input variable and the output, $d_i$ is the difference between the rank numbers of the k-th input variable and the rank numbers of the output, and N is the number of simulation.

The sensitivity coefficient ranges within interval from −1 to +1. The higher is the coefficient (in absolute value), the higher is the sensitivity of the output to the appropriate input variable. The sign of the coefficient indicates positive or negative influence. Sensitivity coefficient lower than 0.30 (in absolute value) can be explained as practically no influence, higher than 0.30 as a low influence, higher than 0.50 as a moderate influence, higher than 0.70 as a high influence, and sensitivity
coefficient higher than 0.90 as a dominant influence. In the illustrative figures, the input variables with sensitivity coefficient higher than 0.30 are shown.

<table>
<thead>
<tr>
<th>No.</th>
<th>Layer</th>
<th>Input variable</th>
<th>mean</th>
<th>COV</th>
<th>skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>thickness</td>
<td>220</td>
<td>0.09</td>
<td>0.6</td>
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<td>X2</td>
<td>Young modulus</td>
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<td>0.0</td>
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<td>X3</td>
<td>Poisson’s coefficient</td>
<td>0.20</td>
<td>0.02</td>
<td>0.0</td>
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<tr>
<td>X4</td>
<td>thickness</td>
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<td>0.0</td>
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<tr>
<td>X5</td>
<td>Young modulus</td>
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<td>0.40</td>
<td>0.0</td>
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<tr>
<td>X6</td>
<td>Poisson’s coefficient</td>
<td>0.30</td>
<td>0.08</td>
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<tr>
<td>X7</td>
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<tr>
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<td>Poisson’s coefficient</td>
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<td>80</td>
<td>0.32</td>
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<td>X11</td>
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<td>0.075</td>
<td>0.9</td>
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<td>X12</td>
<td>width transversal</td>
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<td>0.15</td>
<td>0.0</td>
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<tr>
<td>X13</td>
<td>width longitudinal</td>
<td>1.5</td>
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<tr>
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<td>X15</td>
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<td>X16</td>
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<td>11</td>
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<tr>
<td>X17</td>
<td>lower surface</td>
<td>10</td>
<td>0.25</td>
<td>0.9</td>
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</tr>
</tbody>
</table>

* - truncation parameter

IV. INPUT RANDOM VARIABLES

Total of 17 variables are considered as random input variables, see Table I (units in MPa, mm, °C). Their statistical parameters are carefully evaluated taking into account the data obtained from the in-situ measurements, experimental tests, data from technological handbooks and scientific publications and the corresponding standards. Information about random properties of recycled concrete used in base layer is taken from [8]. To derive appropriate statistical parameters of input variables, the following procedure is utilized. At first, the limits are specified (minimum, maximum and mean value) in which input variables will occur with a high probability. Then, based on the assumption that values smaller than the minimum value and higher than the maximum value can occur only with a low probability, and choosing the appropriate cumulative distribution function (CDF) (normal, three-parametric lognormal, truncated normal), the other required statistical parameters are determined – coefficient of variation (COV) and skewness, see Table I. The normal and truncated normal CDF are used for symmetrically distributed variables, the three-parametric lognormal CDF for the other case. For simplicity, the mutual statistical independence of input variables is considered with the following exception - the temperatures of the upper and lower surface of the slabs are supposed to be fully statistically dependent.

The derived statistical parameters of input variables used in the presented study take into account uncertainties due to their random nature and also the uncertainties due to our incomplete knowledge of the structure, insufficient experimental research, modeling errors, vagueness of input data etc.

V. RESULTS

A. Principal Stress σ1

Sensitivity analysis of principal stress σ1 (maximal tensile stress) is performed in all points of the upper and lower surface of concrete slabs. Stresses on the both surfaces are generally affected by random variability of the input variables differently. Illustrative results are presented for some important points on the lower surface only, see Fig. 2.

Nominal values of stress obtained from deterministic analysis (based on mean values of input variables) on the lower surface have the character of tensile stress at all points, while on the upper surface there are both tensile and compressive stresses as well. In both cases, the nominal values are very close to the minimum values obtained from the statistical analysis taking into account uncertainties in input variables. Mean values of stress on both surfaces in all points have the character of tensile stress and are generally always greater than the nominal values. The nominal as well as mean values reach their maximum at points near the external load application (points 25 – 29), see Fig. 1.

Some points on the lower surface are influenced only by the minimum number of input variables. The stress at points near the external load application, the centers of slabs and the longitudinal edge of slabs (points 12-14, 20, 24-29, 32, 37, 38, 53, 56, 58, 63) are influenced only by two or three variables. The largest number of input variables - eight - has an influence in the outer corners of the loaded slab as well as the adjacent slab (points 11, 16, 22).

Most points on the lower surface are influenced by the temperature field acting on the upper (X16) and lower (X17) surface of the concrete slabs. Influence of these variables can be considered as dominant. Influence of the modulus of elasticity of joints material (X14), Poisson coefficient for sub-base layer (X9), and the width of transverse joints (X12) can be generally considered as moderate, in the case of the modulus of elasticity and the joint width at some points as high. Low influence show the modulus of elasticity of subgrade layer (X10); the thickness, the modulus of elasticity and Poisson coefficient of concrete slab (X1, X2, X3); the thickness, the modulus of elasticity and Poisson coefficient of base layer (X4, X5, X6); the thickness and the modulus of elasticity of sub-base layer (X7, X8); the coefficient of friction in joints (X15), and the width of the longitudinal joint (X13).
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calculated. The dominant influence shows the random 
varies in individual points in which sensitivity coefficients are 
σ3, 
sub-base layer (X8).

On the lower surface the influence of temperature field acting on the upper and lower 
surface of the concrete slabs.

VI. CONCLUSION

The presented sensitivity analysis of principal stresses in 
concrete slabs of rigid pavement shows that the principal 
stresses in the slabs are influenced by random variability of input variables in a different way. That is, random variability influences differently principal stress σ1 and principal stress σ3, differently principal stresses on the upper and lower surface of concrete slabs and differently its influence also varies in individual points in which sensitivity coefficients are calculated. The dominant influence shows the random 
variability of temperature field acting on the upper and lower surface of the concrete slabs.

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REFERENCES


