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New Stability Analysis for Neural Networks with Time-Varying Delays

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Abstract—This paper studies the problem of asymptotically stability for neural networks with time-varying delays.By establishing a suitable Lyapunov-Krasovskii function and several novel sufficient conditions are obtained to guarantee the asymptotically stability of the considered system. Finally,two numerical examples are given to illustrate the effectiveness of the proposed main results.

Keywords—Neural networks, Lyapunov-Krasovskii, Time-varying delays, Linear matrix inequality.

I. Introduction

RECURRENT neural networks including Hopfield neural networks (HNNs) and cellular neural networks (CNNs) have been studied extensively over the recent decades [1]-[10] and have been widely applied within various engineering fields such as neuro-biology, population dynamics and computing technology.Up to now,various stability conditions have been obtained. But because of the hight speed of information processing,there inevitably exist time-varying delays in neural networks. Therefore, the problem of stability of recurrent neural networks with time-varying delay is importance in both theory and practice.

The problem of global asymptotically stability analysis for delay neural networks has been studied by many investigators in the past years. Through employing different Lyapunov Krasovskii functionals and LMI technique stability criteria were obtained. The following works have studied the global asymptotically stability for delayed neural networks . In [5], some sufficient conditions are obtained for existence and global asymptotically stability of constructing a new Lyapunov functional and using free-weighting matrix method,some more less conservative criteria were obtained.In [11], By introducing triple-integral terms and convex optimization approach,the results obtained were improved further than [5].

Motivated by these observations, it is of great importance to further investigate the stabilization problem of delayed neural networks by making use of the delay interval of neurons. In this paper, our attention focuses on the asymptotically stabilization problem of a class of recurrent neural networks with time delay.By choosing a new Lyapunov functional which fractions delay interval and employing different free-weighting matrices in the upper bounds of integral terms to guarantee the stability of the delayed neural networks. It is shown that this obtained conditions have less conservatism. Finally, a numerical example is given to show the usefulness of the proposed criteria.

Notation: Throughout this paper, the superscripts $'-1'$ and $'T'$ stand for the inverse and transpose of a matrix,respectively; $\Re^n$ denotes an n-dimensional Euclidean space;$\Re^{m\times n}$ is the set of all $m\times n$ real matrices; $P>0$ means that the matrix P is symmetric positive definite, $diag(\dots)$ denotes a block diagonal matrix. In block symmetric matrix or long matrix expression, we use $(*)$ to represent a term that is induced by symmetry,I is an appropriately dimensional identity matrix.

II. Problem statement

Consider the following neural networks with time-varying delays:

$$\dot{z}(t)=-Cz(t)+Ag(z(t))+Bg(z(t-\tau(t)))+\mu \tag{1}$$

$$z(t)=\phi(t), t\in[-\tau_2,0] \tag{2}$$

where $z(t)=[z_1(t),z_2(t),\dots,z_n(t)]^T\in\Re^n$ is neuron vector $g(z(t)))=[g_1(z_1(t)),g_2(z_2(t)),\dots,g_n(z_n(t))]^T\in\Re^n$ denotes the neuron activation function, $A\in\Re^{n\times n}$, $B\in\Re^{n\times n}$ are the connection weight matrices and the delayed connection weight matrices,$C=diag\{c_1,c_2,\dots,c_n\}>0$. respectively, $\mu=[\mu_1,\mu_2,\dots,\mu_n]^T$ is constant input vector, $\tau(t)$ is a continuous time-varying function which satisfies.

$$\tau_1\le\tau(t)\le\tau_2, \dot{\tau}(t)\le\ u \tag{3}$$

where τ_1,τ_2 and u are constants.

The following assumption is made in this paper.

Assumption 1. The neuron activation functions $g_i(t)$ in (1) are bounded and satisfy

$$\gamma_i^-\le\frac{g_i(x)-g_i(y)}{x-y}\le\gamma_i^+, x,y\in\Re, x\ne y, i=1,2,\dots,n \tag{4}$$

Where $\gamma_i^-,\gamma_i^+(i=1,2,\dots,n)$ are positive constants.

Assumption 1 guarantees the existence of an equilibrium point of system(1) [13].Denote that $z^*=[z_1^*,z_2^*,\dots,z_n^*]^T$ is the equilibrium point. Using the transformation$x(\cdot)=z(\cdot)-z^*$ system (1) can be converted to the following error system:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau(t))) \tag{5}$$

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where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \Re^n$ is the neuron vector,$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in \Re^n$ denotes the neuron activation function.
Let $f_i(x(t)) = g_i(z_i(\cdot)) - g_i(z_i^*), i = 1, 2, \ldots, n$.we can get

$$\gamma_i^- \leq \frac{f_i(x_i(t))}{x_i(t)} \leq \gamma_i^+, f_i(0) = 0, i = 1, 2, \ldots, n \tag{6}$$

Lemma 1 [12]. For any constant positive matrix $Z \in \Re^{n\times n}$, $Z = Z^T > 0$, scalars $h_2 > h_1 > 0$ such that the following integrations are well defined, then

$$-(h_2 - h_1)\int_{h_2}^{h_1} x^T(s)Zx(s)ds \leq -\int_{h_2}^{h_1} x^T(s)dsZ\int_{h_2}^{h_1} x(s)ds \tag{7}$$

Lemma 2 [13].By (6) the following inequalities hold

$$0 \leq \int_0^{x_i(t)} [f_i(s) - \gamma_i^- s]ds \leq [f_i(x_i(t)) - \gamma_i^- x_i(t)]x_i(t) \tag{8}$$

$$0 \leq \int_0^{x_i(t)} [\gamma_i^+ s - f_i(s)]ds \leq [\gamma_i^- x_i(t) + f_i(x_i(t))]x_i(t) \tag{9}$$

III. Main results

In this section, we propose a new asymptotically criterion for the neural networks with time-varying delays system. Now, we have the following main results.

Theorem 1. For given scalars $\Gamma_1 = diag(\gamma_1^-, \gamma_2^-, \ldots, \gamma_n^-)$ $\Gamma_2 = diag(\gamma_1^+, \gamma_2^+, \cdot\cdot\cdot, \gamma_n^+)$,$u > 0$, the system (5) is globally asymptotically stability if there exist the symmetric positive definite matrices P, $Q_i(i = 1, 2, 3)$, $R_i(i = 1, 2, 3)$, M_1, M_2, N_1, N_2. positive diagonal matrices $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\Delta = diag(\delta_1, \delta_2, \ldots, \delta_n)$, and arbitrary matrices H_1, H_2, W_1,W_2, such that the following LMIs hold:

$$E = \begin{bmatrix} e_{11} & Q_1 & Q_2 & 0 & e_{15} & e_{16} & 0 \\ * & e_{22} & Q_3 & 0 & 0 & 0 & 0 \\ * & * & e_{33} & 0 & 0 & 0 & -H_2 \\ * & * & * & e_{44} & 0 & e_{46} & 0 \\ * & * & * & * & e_{55} & e_{56} & 0 \\ * & * & * & * & * & e_{66} & 0 \\ * & * & * & * & * & * & e_{77} \end{bmatrix} < 0 \tag{10}$$

$$e_{11} = -PC - CP - 2\Gamma_2\Lambda C + 2\Gamma_1\Delta C + R_1 + R_2 + R_3 + C^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]C - Q_1 - Q_2 + M_1 - 2\Gamma_1 W_1\Gamma_2$$

$$e_{15} = PA - C\Lambda - \Gamma_1\Lambda A + \Gamma_2\Delta A + \Delta C + W_1(\Gamma_1 + \Gamma_2) - C^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]A + H_1$$

$$e_{16} = PB - C^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]B - \Gamma_1\Lambda B + \Gamma_2\Delta B$$

$$e_{22} = -R_1 - Q_1 - Q_2, e_{33} = -R_2 - Q_3 - Q_2 - M_2$$

$$e_{44} = -(1 - u)(R_3 + M_1 + M_2) - 2\Gamma_1 W_2\Gamma_2$$

$$e_{46} = -(1 - u)(H_1 - H_2) + W_2(\Gamma_1 + \Gamma_2)$$

$$e_{55} = 2A\Lambda - 2\Delta A + A^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]A + N_1 - 2W_1$$

$$e_{56} = B\Lambda - \Delta B + A^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]B$$

$$e_{66} = B^T[\tau_1^2 Q_1 + \tau_2^2 Q_2 + (\tau_2 - \tau_1)^2 Q_3]B - (1 - u)(N_1 - N_2) - 2W_2$$

Proof: Construct a Lyapunov-Krasovskii function as the follows:

$$V(x_t) = \sum_{i=1}^{4} V_i(x_t)$$

where

$$V_1(x_t) = x^T(t)Px(t) + 2\sum_{i=1}^{n}[\int_0^{x_i(t)} \lambda_i(f_i(s) - \gamma_i^- s)ds + \int_0^{x_i(t)} \delta_i(\gamma_i^+ s - f_i(s))ds]$$

$$V_2(x_t) = \int_{t-\tau_1}^{t} x^T(s)R_1x(s)ds + \int_{t-\tau_2}^{t} x^T(s)R_2x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)R_3x(s)ds$$

$$V_3(x_t) = \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_1\dot{x}(s)dsd\theta + \tau_2\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_2\dot{x}(s)dsd\theta + (\tau_2 - \tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{x}^T(s)Q_3\dot{x}(s)dsd\theta$$

$$V_4(x_t) = \int_{t-\tau(t)}^{t} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^T \begin{bmatrix} M_1 & H_1 \\ * & N_1 \end{bmatrix} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix} ds + \int_{t-\tau_2}^{t-\tau(t)} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^T \begin{bmatrix} M_2 & H_2 \\ * & N_2 \end{bmatrix} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix} ds$$

The time derivative of $V(x_t)$ along the trajectory of system (5) is given by

$$\dot{V}(x_t) = \sum_{i=1}^{4} \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t)P\dot{x}(t) + 2[(f^T(x(t)) - x^T(t)\Gamma_1)\Lambda]\dot{x}(t) + 2[(x^T(t)\Gamma_2 - f^T(x(t)))\Delta]\dot{x}(t) \tag{11}$$

$$\begin{aligned}\dot{V}_2(x_t) &= x^T(t)(\sum_{i=0}^{3} R_i)x(t) - x^T(t-\tau_1)R_1x(t-\tau_1)\\ &- (1-\dot{\tau}(t))x^T(t-\tau(t))R_3x(t-\tau(t))\\ &- x^T(t-\tau_2)R_2x(t-\tau_2)\\ &\leq x^T(t)(\sum_{i=0}^{3} R_i)x(t) - x^T(t-\tau_1)R_1x(t-\tau_1)\\ &- (1-u)x^T(t-\tau(t))R_3x(t-\tau(t))\\ &- x^T(t-\tau_2)R_2x(t-\tau_2)\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_3(x_t) &= \dot{x}^T(t)[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]\dot{x}(t)\\ &- \tau_1\int_{t-\tau_1}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds\\ &- \tau_2\int_{t-\tau_2}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds\\ &- (\tau_2-\tau_1)\int_{t-\tau_1}^{t-\tau_2} \dot{x}^T(s)Q_3\dot{x}(s)ds\\ &\leq \dot{x}^T(t)[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]\dot{x}(t)\\ &- [x(t)-x(t-\tau_1)]^TQ_1[x(t)-x(t-\tau_1)]\\ &- [x(t)-x(t-\tau_2)]^TQ_2[x(t)-x(t-\tau_2)]\\ &- [x(t-\tau_2)-x(t-\tau_1)]^TQ_3[x(t-\tau_2)-x(t-\tau_1)]\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_4(x_t) &= \begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix}^T \begin{bmatrix} M_1 & H_1\\ * & N_1\end{bmatrix}\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix} - (1-\dot{\tau}(t))\\ &\times \begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix}^T \begin{bmatrix} M_1-M_2 & H_1-H_2\\ * & N_1-N_2\end{bmatrix}\\ &\times \begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix} - \begin{bmatrix} x(t-\tau_2)\\ f(x(t-\tau_2))\end{bmatrix}^T \begin{bmatrix} M_2 & H_2\\ * & N_2\end{bmatrix}\\ &\times \begin{bmatrix} x(t-\tau_2)\\ f(x(t-\tau_2))\end{bmatrix}\\ &\leq \begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix}^T \begin{bmatrix} M_1 & H_1\\ * & N_1\end{bmatrix}\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix} - (1-u)\\ &\times \begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix}^T \begin{bmatrix} M_1-M_2 & H_1-H_2\\ * & N_1-N_2\end{bmatrix}\\ &\times \begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix} - \begin{bmatrix} x(t-\tau_2)\\ f(x(t-\tau_2))\end{bmatrix}^T \begin{bmatrix} M_2 & H_2\\ * & N_2\end{bmatrix}\\ &\times \begin{bmatrix} x(t-\tau_2)\\ f(x(t-\tau_2))\end{bmatrix}\end{aligned} \tag{14}$$

From (6), there exist positive diagonal matrices W_1, W_2,such that the following inequalities hold:

$$\begin{aligned}&- 2f^T(x(t))W_1f(x(t)) + 2x^T(t)W_1(\Gamma_1+\Gamma_2)f(x(t))\\ &- 2x^T(t)\Gamma_1W_1\Gamma_2x(t) \geq 0\end{aligned} \tag{15}$$

$$\begin{aligned}&- 2f^T(x(t-\tau(t)))W_2f(x(t-\tau(t))) + 2x^T(t-\tau(t))W_2\\ &\times (\Gamma_1+\Gamma_2)f(x(t-\tau(t))) - 2x^T(t-\tau(t))\Gamma_1W_2\Gamma_2\\ &\times x(t-\tau(t)) \geq 0\end{aligned} \tag{16}$$

From $(11)-(18)$ we can get

$$V(x_t) \leq g^T(t)Eg(t) < 0 \tag{17}$$

where

$$\begin{aligned}g(t) =&[x(t), x(t-\tau_1), x(t-\tau_2), x(t-\tau(t)), f(x(t)),\\ &f(x(t-\tau(t))), f(x(t-\tau_2))]^T\end{aligned} \tag{18}$$

This means that the system (5) is asymptotically stable, which complete the proof. ■

Remark 1. Theorem 1 gives a stability criterion for system (5) with $\tau_1 \leq \tau(t) \leq \tau_2, \dot{\tau}(t) \leq u$, where u is a given constant. In many cases, u is unknown.Considering this case, there have the following corollary.

Corollary 1. For given scalars $\Gamma_1 = diag(\gamma_1^-, \gamma_2^-, \cdots, \gamma_n^-)$ $\Gamma_2 = diag(\gamma_1^+, \gamma_2^+, \ldots, \gamma_n^+)$, the system (5) is globally asymptotically stable if there exist symmetric positive definite matrices P, $Q_i(i=1,2,3)$, $R_i(i=1,2)$, M_1, M_2, N_1, N_2. arbitrary matrices H_1, H_2, W_1,W_2, positive diagonal matrices $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\Delta = diag(\delta_1, \delta_2, \ldots, \delta_n)$, such that the following LMIs hold:

$$E = \begin{bmatrix} e_{11} & Q_1 & Q_2 & 0 & e_{15} & e_{16} & 0\\ * & e_{22} & Q_3 & 0 & 0 & 0 & 0\\ * & * & e_{33} & 0 & 0 & 0 & -H_2\\ * & * & * & e_{44} & 0 & e_{46} & 0\\ * & * & * & * & e_{55} & e_{56} & 0\\ * & * & * & * & * & e_{66} & 0\\ * & * & * & * & * & * & e_{77}\end{bmatrix} < 0 \tag{19}$$

$$\begin{aligned}e_{11} &= -PC - CP - 2\Gamma_2\Lambda C + 2\Gamma_1\Delta C + R_1 + R_2 + R_3\\ &+ C^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]C - Q_1 - Q_2 + M_1\\ &- 2\Gamma_1W_1\Gamma_2\end{aligned}$$

$$\begin{aligned}e_{15} &= PA - C\Lambda - \Gamma_1\Lambda A + \Gamma_2\Delta A + \Delta C + W_1(\Gamma_1+\Gamma_2)\\ &- C^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]A + H_1\end{aligned}$$

$$\begin{aligned}e_{16} &= PB - C^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]B\\ &- \Gamma_1\Lambda B + \Gamma_2\Delta B\end{aligned}$$

$$e_{22} = -R_1 - Q_1 - Q_2, e_{33} = -R_2 - Q_3 - Q_2 - M_2$$

$$e_{44} = -2\Gamma_1W_2\Gamma_2, e_{46} = W_2(\Gamma_1+\Gamma_2)$$

$$\begin{aligned}e_{55} &= 2A\Lambda - 2\Delta A + A^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]A\\ &+ N_1 - 2W_1\end{aligned}$$

$$e_{56} = B\Lambda - \Delta B + A^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]B$$

$$e_{66} = B^T[\tau_1^2Q_1 + \tau_2^2Q_2 + (\tau_2-\tau_1)^2Q_3]B - 2W_2$$

Proof: Choosing $R_3 = 0, \begin{bmatrix} M_1-M_2 & H_1-H_2\\ * & N_1-N_2\end{bmatrix} = 0$ in Theorem 1,one can easily obtains this result. ■

TABLE I
ALLOWABLE UPPER BOUND OF τ_2 WITH VARIOUS u

u	0.6	0.8	0.9	1.2
[14]	2.9219	1.7428	1.3246	1.2165
[15]	2.9334	1.7557	1.3423	1.2323
[16]	2.9876	1.7750	1.3747	1.2612
this works	∞	∞	∞	∞

TABLE II
ALLOWABLE UPPER BOUND OF τ_2 WITH VARIOUS u

u	0.4	0.45	0.5	0.55
[17]	3.99	3.27	3.05	2.98
[18]	4.38	3.60	3.33	3.23
[19]	4.39	3.67	3.46	3.41
[20]	∞	∞	∞	∞
Theorem 1	∞	∞	∞	∞

IV. EXAMPLES

In this section,we provide the simulation of examples to illustrate the effectiveness of our method.

Example 1. Considering the system (5) with the following parameters:

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}$$

$\Gamma_1 = diag(0, 0)$, $\Gamma_2 = diag(0.4, 0.8)$

First,the maximum delay bounds τ_2 are shown under $\tau_1 = 0$ and different u are list in Table I.

Example 2. Considering the system (5) with the following parameters:

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, A = \begin{bmatrix} 0.053 & 0.0454 \\ 0.0987 & 0.275 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix}$$

$\Gamma_1 = diag(0, 0)$, $\Gamma_2 = diag(0.3, 0.8)$

Table II lists out the comparison results on the maximum delay bound allowed via the methods in recent paper and our new established criterion.

V. CONCLUSION

In this paper,a new stability analysis for neural networks with time-varying delay is proposed. A suitable Lyapunov functional has been proposed to derive some less conservative delay-dependent stability criteria by using the free-weighting matrices method and the convex combination theorem. Finally, two numerical examples have been given to illustrate the effectiveness of the proposed method.

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