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Synchronization of a Perturbed Satellite Attitude Motion

Sadaoui Djaouida

Abstract**—**In the paper, the predictive control method is proposed to control the synchronization of two perturbed satellites attitude motion. Based on delayed feedback control of continuous-time systems combines with the prediction-based method of discrete-time systems, this approach only needs a single controller to realize synchronization, which has considerable significance in reducing the cost and complexity for controller implementation.

Keywords**—**Predictive control, Synchronization, Satellite attitude.

I. INTRODUCTION

ATITUDE control of rigid bodies and of satellite in particular, is a thoroughly researched discipline. Nowadays, different techniques and methods have been proposed to achieve attitude control such as nonlinear control [1], sliding mode control (SMC) [2]-[4], adaptive control [5]… etc.

Attitude synchronization is required for modern space mission concepts involving multiple satellites flying information. Disturbances may however prevent the satellites from following their reference trajectories precisely. A synchronization control scheme copes with this by controlling the relative errors between the satellites attitudes. The goal is to find control torques that asymptotically drives the satellites attitudes towards the same orientation.

In this paper we propose a Leader/Follower feedback predictive synchronization scheme for control of the attitude of two satellites. The Leader/Follower architecture is a hierarchical structure where the Follower satellite is controlled so as to maintain a predefined relative position and attitude to the Leader. A reference projection is proposed, so that the follower satellite is commanded to follow a combination of its reference attitude and the measured or communicated leader satellite attitude.

Instead of designing reference trajectories for each satellite, the coordination of the two satellites is made directly in the control law. The attitude of the follower satellite should track the attitude of the leader satellite. The attitude of the leader system, on the other hand, should track any time-varying reference attitude, which is typically given mathematically [6].

When the leader satellite is subjected to disturbances, controlling the relative error directly may give smaller relative errors than traditional tracking control.

Djaouida Sadaoui is with the Hadj Lakhdar University Batna, Algeria (e-mail: sadaouidjaouida@yahoo.fr).

II. SYSTEM EQUATION

A. Kinematics

The kinematics of the satellite determines the attitude of the main body, and is derived by integration of the angular velocity. The corresponding rotation matrices are [7], [8].

$$\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta\\ 0 & \cos\varphi & -\sin\varphi\\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta\end{bmatrix}\begin{bmatrix}w_x\\ w_y\\ w_z\end{bmatrix} \tag{1}$$

φ,θ *and* ψ are three angles clockwise rotations about inertial axes I;J and K respectively.

B. Dynamics

A satellite can be regarded as an ideal rigid body. The dynamic model of the satellite is derived using a Newton-Euler formulation, where the angular momentum change related to applied torque. The satellite model is [8]-[12]

$$\begin{pmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\end{pmatrix}=\begin{pmatrix}\sigma_x\omega_y\omega_z\\ \sigma_y\omega_x\omega_z\\ \sigma_z\omega_x\omega_y\end{pmatrix}+\begin{pmatrix}M_x\\ M_y\\ M_z\end{pmatrix}\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix}+\begin{pmatrix}u_x\\ u_y\\ u_z\end{pmatrix} \tag{2}$$

where

$$\sigma_x=\frac{I_y-I_z}{I_x},\sigma_y=\frac{I_z-I_x}{I_y}\ and\ \sigma_z=\frac{I_x-I_y}{I_z}$$

where I_x, I_y *and* I_z are the principal moments of inertia; M_x, M_y *and* M_z are the perturbing torques; and u_x, u_y *and* u_z are the three controls torques.

III. PREDICTIVE CONTROLLER DESIGN

A. Definition

Consider the nonlinear system described by the dynamics as the Leader system

$$\dot{x}=A_1x+f_1(x) \tag{3}$$

where $x=(x_1,x_2,....,x_n)^T\in R^n$ denotes the system's n-dimensional state vector, A1 is a n x n matrix and $f_1:R^n\rightarrow R^n$ represent nonlinear part of the system dynamics. The controller $u(t)=(u_1(t),u_2(t),....,u_n(t)^T\in R^n)$ is added into the slave system, which is given by

$$\dot{y}=A_2y+f_2(y)+u(t) \tag{4}$$

where $y = (y_1, y_2, ..., y_n)^T \in R^n$ is the follower system's n-dimensional state vector, A_2 is an n x n matrix and $f_2 : R^n \to R^n$ represent nonlinear part of the follower system.

To synchronize the systems is to find a control signal $u(t) \in R^n$ that makes states of the follower system to evolve as the states of the Leader system.

To achieve the goal we define the synchronization errors dynamics as follows:

$$e = y - x = (e_1, e_2, ..., e_n)^T \in R^n$$

and subtracting the system (3) from the system (4), the error dynamics is determined by

$$\begin{aligned} \dot{e} &= A_2 y + f_2(y) - A_1 x - f_1(x) + u(t) \\ &= Ae + F(x, y) + u(t) \end{aligned} \tag{5}$$

where $e = y - x,\ F(x, y) = f_2(y) - f_1(x) + (A_2 - A_1)x$

The aim is to design the controller $u(t) \in R^n$ such that

$$\lim_{t \to \infty} \|e(t)\| = 0$$

According to the predictive control design procedure [13]-[15], the control input $u(t)$ is determined by the difference between the predicted states and the current states:

$$u(t) = K(e_p(t) - e(t)) \tag{6}$$

where K is a gain vector, $e_p(t)$ is the predicted future state of uncontrolled systems from the current state $e(t)$.

Using a one-step-ahead-prediction, the predictive control (6) becomes

$$u(t) = K(\dot{e}(t) - e(t)) \tag{7}$$

Therefore the error system (5) is then rewritten as

$$\dot{e} = Ae + F_1(x, y) + K(\dot{e}(t) - e(t)) \tag{8}$$

Near e_f, we can use the linear approximation for the uncontrolled system by

$$(\dot{e}(t) - e_f) = A(e(t) - e_f) \tag{9}$$

where $A \in R^{n\times n}$ is the Jacobian matrix evaluated at the fixed points e_f, which is defined as follow:

$$A = \left. \frac{\partial \dot{e}(t)}{\partial e(t)} \right|_{e_f} \tag{10}$$

Equation (9) is rewritten in the form

$$\delta \dot{e}(t) = A \delta e(t) \tag{11}$$

With

$$\delta e(t) = e(t) - e_f \tag{12}$$

The controlled system is linearized around e_f by

$$\begin{aligned} \delta \dot{e}(t) &= A\delta e(t) + K(\delta \dot{e}(t) - \delta e(t)) \\ &= A\delta e(t) + K(A\delta e(t) - \delta e(t)) \\ &= (A + K(A - I))\delta e(t) \end{aligned} \tag{13}$$

where $I \in R^{n\times n}$ is the identity matrix.

In order to apply the proposed predictive control strategy, we have to determine the gain vector K and the vicinity of the fixed point to adjust the next point so it falls on the fixed one. The feedback gain K is determined as follows: [14]

$$|A + K(A - I)| < I \tag{14}$$

and the vicinity of the fixed point is given by:

$$r(t) = |e(t) - e(t-1)| \tag{15}$$

The controlled system will be described by:

$$\dot{e}(t) = \begin{cases} Ae + F(x, y) + u(t) & if\ r(t) < \varepsilon \\ Ae + F(x, y) & otherwise \end{cases} \tag{16}$$

where ε is a positive small real number.

IV. Synchronization of Two Satellite Systems Using Predictive Control

Consider the following two identical satellites attitudes systems, where the Leader system and Follower system are denoted with x and y, respectively [16].

Leader system:

$$\begin{cases} \dot{x}_1 = \sigma_x x_2 x_3 + m_1 x_1 + m_2 x_2 + m_3 x_3 \\ \dot{x}_2 = \sigma_y x_1 x_3 + m_4 x_1 + m_5 x_2 + m_6 x_3 \\ \dot{x}_3 = \sigma_z x_1 x_2 + m_7 x_1 + m_8 x_2 + m_9 x_3 \end{cases} \tag{17}$$

Follower system:

$$\begin{cases} \dot{y}_1 = \sigma_x y_2 y_3 + m_1 y_1 + m_2 y_2 + m_3 y_3 + u_x \\ \dot{y}_2 = \sigma_y y_1 y_3 + m_4 y_2 + m_5 y_2 + m_6 y_3 + u_y \\ \dot{y}_3 = \sigma_z y_1 y_2 + m_7 y_1 + m_8 y_2 + m_9 y_3 + u_z \end{cases} \tag{18}$$

The error dynamics are determined as follows:

$$\begin{cases} \dot{e}_1 = \sigma_x (y_2 y_3 - x_2 x_3) + m1(y_1 - x_1) + m_2 (y_2 - x_2) + m_3 (y_3 - x_3) + u_1(t) \\ \dot{e}_2 = \sigma_y (y_1 y_3 - x_1 x_3) + m4(y_1 - x_1) + m_5 (y_2 - x_2) + m_6 (y_3 - x_3) + u_2(t) \\ \dot{e}_3 = \sigma_z (y_1 y_2 - x_1 x_2) + m_7 (y_1 - x_1) + m_8 (y_2 - x_2) + m_9 (y_3 - x_3) + u_3(t) \end{cases} \quad (19)$$

$$\begin{cases} \dot{e}_1 = m_1 e_1 + (\sigma_x y_3 + m_2) e_2 + (\sigma_x y_2 + m_3) e_3 - \sigma_x e_2 e_3 + u_1(t) \\ \dot{e}_2 = (\sigma_y y_3 + m_4) e_1 + m_5 e_2 + (\sigma_y y_1 + m_6) e_3 - \sigma_y e_1 e_3 + u_2(t) \\ \dot{e}_3 = (\sigma_z y_2 + m_7) e_1 + (\sigma_z y_1 + m_8) e_2 + m_9 e_3 - \sigma_z e_1 e_2 + u_3(t) \end{cases} \quad (20)$$

Therefore

$$A = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix}, F(x,y) = \begin{bmatrix} \sigma_x (y_2 y_3 - x_2 x_3) \\ \sigma_y (y_1 y_3 - x_1 x_3) \\ \sigma_z (y_1 y_2 - x_1 x_2) \end{bmatrix}$$

In order to control the system to the unstable equilibrium point $[0\ 0\ 0]^T$, we have to determine the correction which will be applied to the current state of the system. For this purpose, we determine the control input *u(t)* defined by (7).

$$u(t) = \begin{cases} 0 \\ K(((\sigma_y y_3 + m_4) e_1 + m_5 e_2 + (\sigma_y y_1 e_3 + m_6) - \sigma_y e_1 e_3) - e_2) \\ 0 \end{cases} \quad (21)$$

The controlled system is given by:

$$\begin{cases} \dot{e}_1 = m_1 e_1 + (\sigma_x y_3 + m_2) e_2 + (\sigma_x y_2 + m_3) e_3 - \sigma_x e_2 e_3 \\ \dot{e}_2 = (\sigma_y y_3 + m_4) e_1 + m_5 e_2 + (\sigma_y y_1 + m_6) e_3 - \sigma_y e_1 e_3 + K(((\sigma_y y_3 + m_4) e_1 + m_5 e_2 + (\sigma_y y_1 e_3 + m_6) - \sigma_y e_1 e_3) - e_2) \\ \dot{e}_3 = (\sigma_z y_2 + m_7) e_1 + (\sigma_z y_1 + m_8) e_2 + m_9 e_3 - \sigma_z e_1 e_2 \end{cases} \quad (22)$$

The controlled system is described by

$$u = \begin{cases} K(((\sigma_y y_3 + m_4) e_1 + m_5 e_2 + (\sigma_y y_1 e_3 + m_6) - \sigma_y e_1 e_3) - e_2) \\ \quad if \left| e_2(t) - e_2(t-1) \right| \langle \varepsilon \\ 0 \quad otherwise \end{cases} \quad (23)$$

V. Numerical Simulation Results

In the following simulations, the numerical values for the satellite as found in [17] have been used. $\sigma_x = 8, \sigma_y = -3$ and $\sigma_z = 3$.

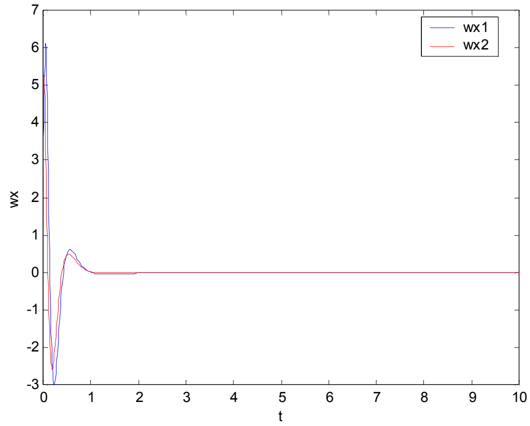
The perturbing torques:

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -8 & 8 & -0.6 \\ 0.5 & -1 & -0.21 \\ 0.7 & 3 & -3 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

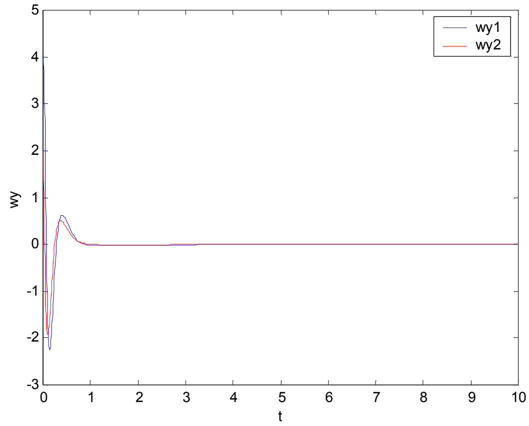
The initial states of the Leader and the Follower satellite are specified as:

$(x_1(0), x_2(0), x_3(0))^T=(3,4.1,2)^T$;
$(y_1(0),y_2(0),y_3(0))^T=(5,2,4)^T$.

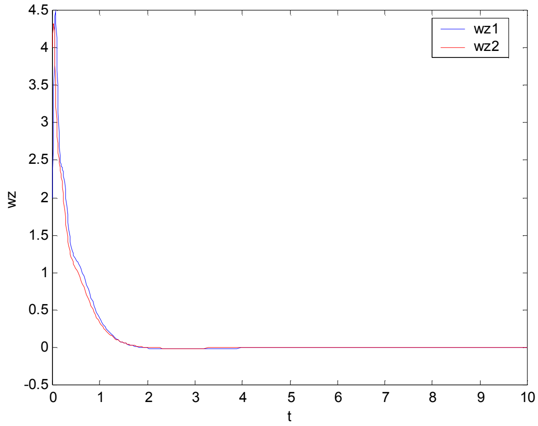
The simulation results are shown in Figs. 1 and 2 under the proposed predictive synchronization of two satellites via a single controller. Fig. 1 shows the time responses of state variables of the satellites attitude. The synchronization errors are shown in Fig. 2. From the simulation results, it shows that the attitude trajectory of the Follower track the attitude of the leader, on the other hand the attitude of the leader converge to the equilibrium point. The time responses of synchronization errors also converge to zero quickly. This means that, the proposed predictive control works well and the Leader and Follower satellites are indeed achieving synchronization.



(a)

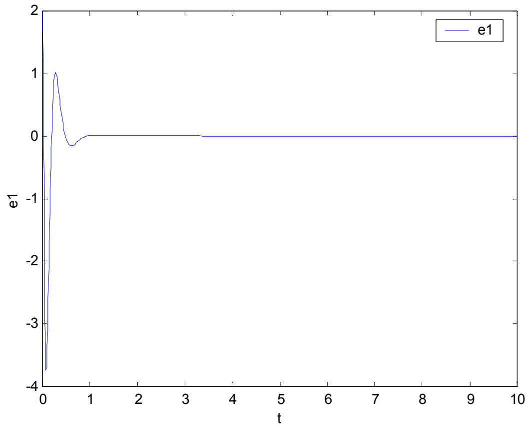


(b)

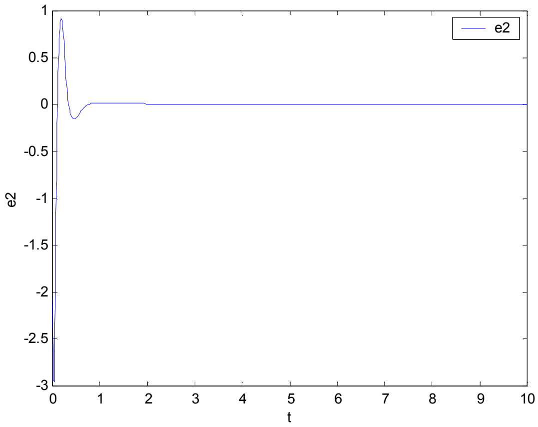


(c)

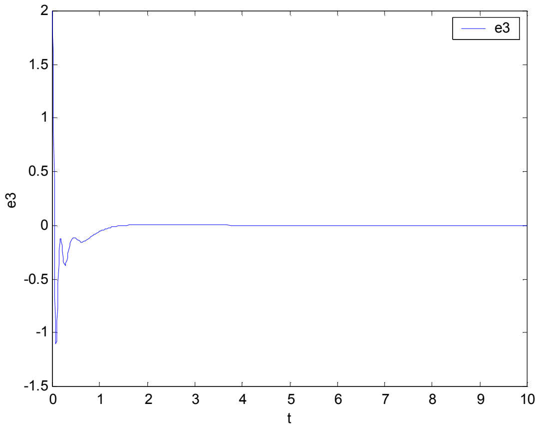
Fig. 1 State variables of the satellites Leader and Follower systems: (a) wx_1/wx_2, (b) wy_1/wy_2,wz_1/wz_2



(a)



(b)



(c)

Fig. 2 Synchronization errors of the satellites systems: (a) e_1, (b) e_2, (c) e3

VI. Conclusion

In this paper, a feedback predictive controller has been proposed to ensure the synchronization between the Leader and the Follower satellite attitude systems. A scheme for feedback predictive synchronization of the two satellites attitude systems was developed and the results were validated by simulation. It was shown that the perfect synchronization of the two systems was realized, and trajectory of error converges to zero.

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