Kinematic Hardening Parameters Identification with Respect to Objective Function

Marina Franulovic, Robert Basan, Bozidar Krizan

***Abstract*—**Constitutive modeling of material behavior is becoming increasingly important in prediction of possible failures in highly loaded engineering components, and consequently, optimization of their design. In order to account for large number of phenomena that occur in the material during operation, such as kinematic hardening effect in low cycle fatigue behavior of steels, complex nonlinear material models are used ever more frequently, despite of the complexity of determination of their parameters. As a method for the determination of these parameters, genetic algorithm is good choice because of its capability to provide very good approximation of the solution in systems with large number of unknown variables. For the application of genetic algorithm to parameter identification, inverse analysis must be primarily defined. It is used as a tool to fine-tune calculated stress-strain values with experimental ones. In order to choose proper objective function for inverse analysis among already existent and newly developed functions, the research is performed to investigate its influence on material behavior modeling.

***Keywords*—**Genetic algorithm, kinematic hardening, material model, objective function.

I. Introduction

MATERIAL behavior modeling plays very important role in structural components design and their fatigue analysis. Material models differ in the range of material properties they can describe and proportionally, in complexity of their definition. Complex material models are characterized by numerous material parameters that have to be carefully identified to follow material behavior as accurately as possible. Due to the complexity of chosen Chaboche's material model [1], [2], it is necessary to use complex numerical procedures to identify material parameters. The usage of evolutionary algorithms is proposed because of their advantageous characteristics, mainly considering insensitivity to errors in measured data, reliability in achieving convergence to accurate results, improbability for convergence to local minima and it's robustness regarding the choice of objective function [3], [4]. Genetic algorithm is stochastic search method for obtaining good approximate solutions for complex problems [5]. It is based on mechanisms of natural evolution and genetic principles. The genetic algorithm creates a population of solutions and applies genetic operators, such as scaling, selection, mutation and crossover to evolve the solutions in order to find the best ones. The proper evolution of population is assured by selection of adequate genetic operators in order to achieve fast convergence to global optima. One of the main premises in genetic algorithm application for parameter identification is the choice of objective function for inverse problem solution. There are numerous published papers that suggest different objective functions for the problem solution. In order to evaluate these suggestions and the influence of objective function on simulation of material behavior by parameter identification with genetic algorithm usage, the most common ones are investigated [6]-[8], and also their modified versions that are proposed.

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II. Constitutive Material Model

The material model considered in this paper is based on continuum mechanics theory [1], [2], [9], [10]. Low-cycle fatigue material behaviour is described by means of models of kinematic and isotropic hardening according to Chaboche material model [1], [11]-[14]. The nonlinearity in kinematic hardening of the model makes it superior in relation to some simpler models [2], [15], but it also makes it very complicated and time–consuming to define. In order to account for material behaviour using Chaboche's material model, strain domain is observed through its elastic and plastic part. Elastic strain tensor corresponds to Hooke's law of linear elasticity, while the von Mises yield function for plasticity criteria description is given by

$$f = \sqrt{\frac{3}{2}\left(S_{ij} - X_{ij}\right)\left(S_{ij} - X_{ij}\right)} - R - \sigma_y = 0 \tag{1}$$

where S_{ij} is deviatoric stress tensor, X_{ij} is back stress tensor that defines the centre of the yield surface, R is the isotropic hardening variable and σ_y is initial yield stress. The flow rule [1] can be written as

$$d\varepsilon^{p} = d\lambda \frac{df}{d\sigma} \tag{2}$$

Non-linear kinematic hardening behavior is described by following three-decomposition rule of Armstrong-Frederick model [1], [11]

$$dX_{ij} = \frac{2}{3}\sum_{i=1}^{3}\left(C^{(i)} d\varepsilon_{ij}^{p} - \gamma^{(i)} X_{ij}^{(i)} dp\right) \tag{3}$$

where C and γ are the characteristic coefficients of the material. The integration of this equation leads to exponential expression suitable for identification of kinematic hardening material parameters

$$\frac{\Delta\sigma}{2} = R_\infty + \sum_{i=1}^{3}\left[X_\infty^{(i)} \tanh\left(\gamma^{i} \frac{\Delta\varepsilon^{p}}{2}\right)\right] \quad (4)$$

where $X_\infty^{(1)}, X_\infty^{(2)}, X_\infty^{(3)}, \gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}$ are kinematic hardening material parameters (kinematic hardening coefficients and rates of kinematic hardening), while R_∞ is the boundary of isotropic hardening. Considering high nonlinearity in (4), identification of six kinematic hardening parameters is made part of genetic algorithm procedure.

III. Material Parameters Identification

A. Genetic Algorithm for Parameter Identification

Based on proposed material model and it's mechanical principle, the parameters of kinematic hardening ($X_\infty^{(1)}, X_\infty^{(2)}, X_\infty^{(3)}, \gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}$) are obtained on the basis of material response in the fully reversed tensile – compressive cyclic tests, from the recorded cyclic stress – strain curves. The calculation procedure is automated by using genetic algorithm for material parameters identification and finite element method material behavior simulation.

The genetic algorithm based procedure consists of three main parts. The first part is system characterization, which means determination of parameters that can completely characterize the system. In the second part, forward modeling, mechanical principles and physical laws are defined to enable prediction of system behavior. The third part is backward or inverse modeling. Inverse analysis plays an important role in problems where the cause has to be defined from the results. It consists of defining the search methods of unknown sample characteristics by observing sample's response to a probing signal. Definition of objective function represents the solution of inverse problem. The mathematical structure of the model

$$\sigma = \hat{\sigma}(\varepsilon; a_i) \quad (5)$$

is defined by mapping function which defines the dependence among stress and strain values and the material parameter values a_i = [$X_\infty^{(1)}, X_\infty^{(2)}, X_\infty^{(3)}, \gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}$] that are considered within the chosen domain.

Parameter R_∞ is calculated as the difference between initial yield stress and yield stress in stable cycle and therefore isn't part of genetic algorithm calculation procedure.

B. Genetic Operators

The genetic algorithm creates a population of solutions and applies genetic operators, such as scaling, selection, mutation and crossover to evolve the solutions in order to find the best ones (Fig. 1).

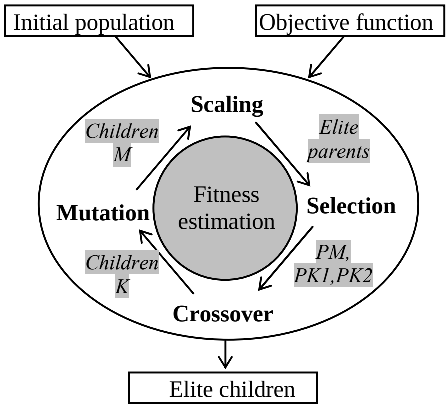

Fig. 1 Genetic algorithm procedure

The proper evolution of population is assured by choosing adequate genetic operators in order to achieve fast convergence to global optima [16]. Within selection procedure, 4-tournament method is used, while crossover is accomplished through intermediate recombination with 10% dispersion, characterized by children's' values

$$Child\_K = 0{,}9 \cdot PK_1 + Ratio\_K \cdot (1{,}1 \cdot PK_2 - 0{,}9 \cdot PK_1) \quad (6)$$

where *PK1* and *PK2* are parents values, achieved through selection procedure, while *Ratio_K* is random number between 0 and 1. In order to improve childrens' characteristics, corrections of their values are performed in case of unrealistic parameter values

$$Child\_K = PK_1 + Ratio\_K \cdot (PK_2 - PK_1) \quad (7)$$

$$Child\_K = 0{,}5 \cdot PK_1 + 0{,}5 \cdot PK_2 \quad (8)$$

Another improvement of recombination process is made by assuring impossibility of two equal parents' existence. In case of two identical parents' selection, one of them is mutated, using (9), with mutation domain that equals 0.25 instead of 0.1, as it is in original expression. All procedures of the proposed genetic algorithm have the same mutation routine. The possibility of mutation is set to 1, which means each variable is changing during mutation. Children's' values are calculated by

$$Child\_M = PM + Ratio\_M \cdot Change\_M \quad (9)$$

where *PM* is parent value, achieved through selection procedure, while *Change_M* is random number between 0 and 1. The mutation ratio is decreasing through generations, according to

$$Ratio\_M = 1 - \frac{Current\_generation}{Last\_generation} \quad (10)$$

IV. Objective Function

Scaling of population is based on the fitness values of the individuals, which is the solution of chosen objective function. In general

$$f = \sum_{i=1}^{n} \left[\frac{y_i^* - \hat{y}\left(x_i^*; parameters\right)}{y_i^*} \right]^2 \qquad (11)$$

where asterisk * refers to experimental value, while mark ^ refers to value calculated by using set of parameters. The best individuals have low fitness value and the possibility of their selection is high. In order to assure convergence of solutions to global optima, the "bad" individuals are also involved in evolution process, just with the lower possibility and expectancy of selection.

Since evolutionary algorithm for parameter identification is used, the solution of the problem is searched in the global domain. It is not necessary to localize solution domain in order to achieve more accurate data. The chosen objective functions used for comparison in this research are taken in the form published by some authors and also in modified form of each of them as shown in Table I.

TABLE I
Objective Functions for Genetic Algorithm

Source	Function	Equation
[6]	$f = \sum_{i=1}^{n} w_i[\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)]^2 \,; w_i = 1$	(12)
Modified [6]	$f = \sum_{i=1}^{n} w_i[\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)]^2 \,; w_i = 100$	(13)
[7]	$f = \sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2$	(14)
Modified [7]	$f = \sum_{i=1}^{n} \left\lvert\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right\rvert$	(15)
[8]	$f = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2}$	(16)
Modified [8]	$f = \sqrt[4]{\frac{1}{n}\sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2}$	(17)

V. Results Analyses

The procedure for determination of material parameters of the steel 42CrMo4 in normalized state with hardness of 296 HV is presented in this paper. The chemical composition of the material is given in Table II.

TABLE II
Chemical Composition of Tested Material (%)

Element	Percent
C	0,43
Si	0,26
Mn	0,65
P	0,015
S	0,021
Cr	1,07
Ni	0,19
Mo	0,16
Cu	0,16
Al	0,021
Sn	0,006

Detailed response of the material to the cyclic loading was recorded during own experiments and it serves as a basis for modeling of its behavior. Strain-controlled low-cycle fatigue testing [17] has been performed. Specimens (Fig. 2) used for the testing have solid circular cross section. The test is performed till total fracture of the specimen in two parts. The strain amplitude ε_a for cyclic testing is maintained at value 1.5%.

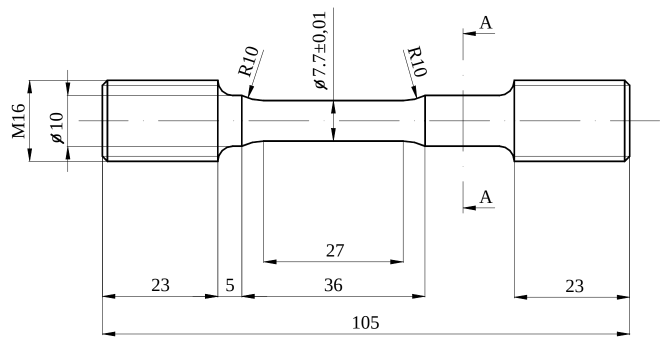

Fig. 2 Geometry of the specimen

Material parameters for modeling of material behavior of the steel 42CrMo4 in normalized state with hardness of 296 HV have been identified and are given in Table III. For this purpose, genetic algorithm procedure was performed with the applied objective functions (12) to (17).

TABLE III
Material Parameters for Presented Objective Functions

Equa tions	$X_\infty^{(1)}$ (N/mm^2)	$X_\infty^{(2)}$ (N/mm^2)	$X_\infty^{(2)}$ (N/mm^2)	$\gamma^{(1)}$ $(-)$	$\gamma^{(2)}$ $(-)$	$\gamma^{(3)}$ $(-)$
(12)	155	103	83	75	123	683
(13)	170	104	85	70	109	680
(14)	459	78	66	30	182	1089
(15)	157	93	72	114	95	825
(16)	140	54	112	139	1286	107
(17)	89	168	46	91	167	1554

The sets of material parameters for modeling material behavior of chosen steel show no similarity among themselves or notable tendency to any value. In order to understand this, all three components of Chaboche's model for kinematic hardening description are presented in Figs. 3 to 8, along with their total value (grey full line).

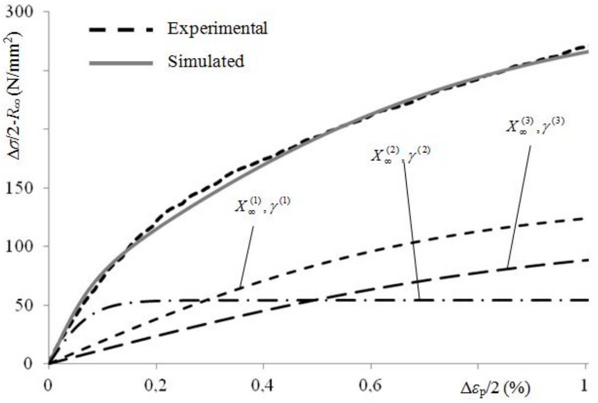

Fig. 3 Stress – strain material behavior calculated using (12)

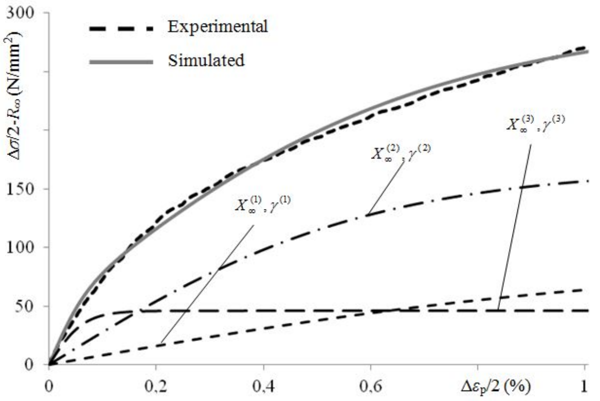

Fig. 4 Stress – strain material behavior calculated using (13)

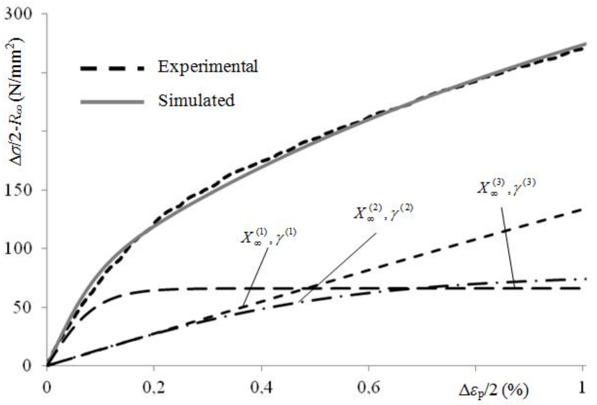

Fig. 5 Stress – strain material behavior calculated using (14)

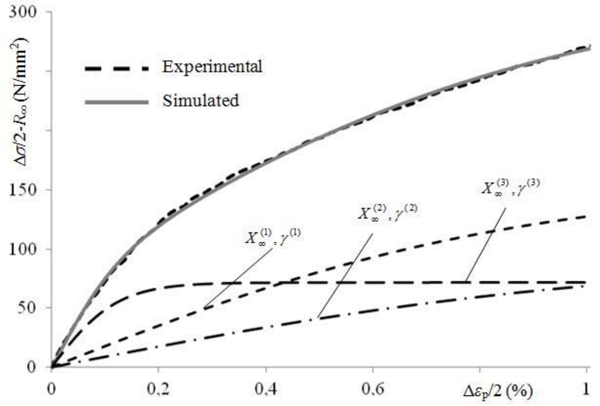

Fig. 6 Stress – strain material behavior calculated using (15)

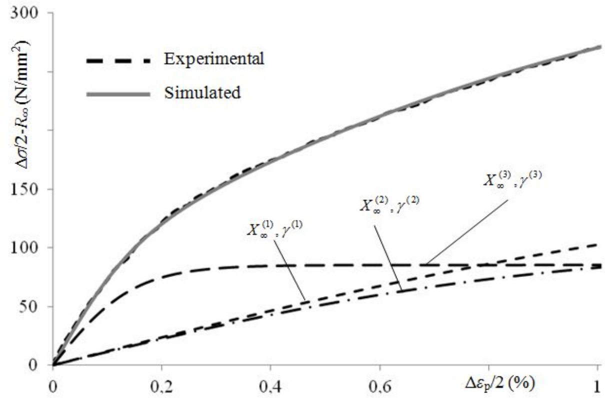

Fig. 7 Stress – strain material behavior calculated using (16)

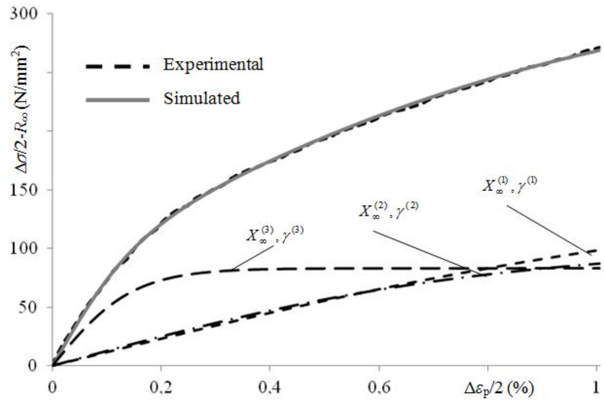

Fig. 8 Stress – strain material behavior calculated using (17)

Although parameter values as well as components of Chaboche's model (every component is in fact simple Armstrong-Frederick model) differ considerably among themselves, each group of parameters gives very good solution. Simulated kinematic hardening behavior of material follows real material behavior extremely well. The stress-plastic strain relationship in all simulations is completely acceptable for the material behavior simulation, as is shown in Fig. 9 (all curves coincide very closely with the experimental one).

The difference among experimental response of the material and simulated behavior is barely discernible. Deviations of simulated stress values from experimental ones are calculated. The biggest difference appears in identification process that has applied objective function (15), but even in this case, the calculated value differs from the experimental one for only 1.57%, which is negligible.

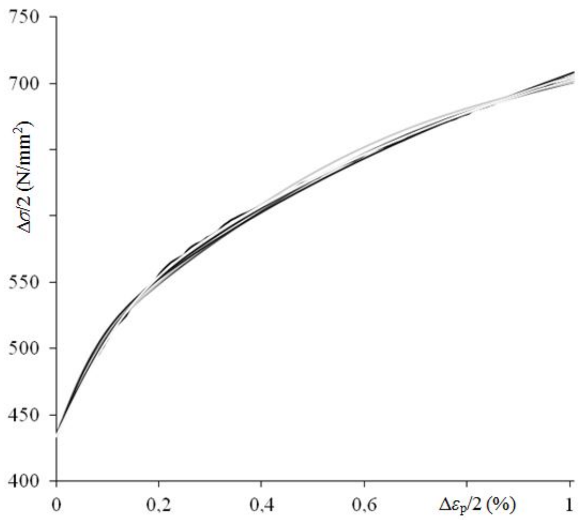

Fig. 9 Stress – strain material behavior

VI. Conclusion

Generally, when referring to the functional inverse problems for the parameter identification, appropriate objective function must be used in the most calculation procedures. The choice of the function depends on the numerical procedure in material behavior modeling that will be used. In genetic algorithm for parameter identification random applications were used to solve complex problem. In order to evaluate robustness of such calculation procedure, regarding the choice of objective function, the most commonly used functions and their modified versions were examined. The calculations showed extremely good compatibility in results and only very small deviations of simulated from real material's response. Therefore, it can be concluded that genetic algorithm in parameter identification for kinematic hardening behavior in low-cycle fatigue problems is robust enough to give reliable results without the need to consider the choice of the objective function for inverse problem. The probability of convergence to the accurate results is very high and there is no need for the improvement in the calculation procedure by using specifically oriented objective function.

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