

# Improved Stability Criteria for Neural Networks with Two Additive Time-Varying Delays

Miaomiao Yang, Shouming Zhong

**Abstract**—This paper studies the problem of stability criteria for neural networks with two additive time-varying delays. A new Lyapunov-Krasovskii function is constructed and some new delay dependent stability criterias are derived in the terms of linear matrix inequalities(LMI), zero equalities and reciprocally convex approach. The several stability criterion proposed in this paper is simpler and effective. Finally, numerical examples are provided to demonstrate the feasibility and effectiveness of our results.

**Keywords**—Stability, Neural networks, Linear Matrix Inequalities (LMI), Lyapunov function, Time-varying delays.

## I. INTRODUCTION

RECURRENT years neural networks have been studied extensively and have been widely applied within various engineering fields such as associative memories, neuro-biology, population dynamics, and computing technology [1-5]. Existing stability criteria can be classified into two categories, that is, delay-independent ones and delay-dependent ones. It is well known that delay-independent ones are usually more conservative than the delay-dependent ones, so much attention has been paid in recent years to the study of delay-dependent stability conditions [6-8]. It should be pointed out that the stability results mentioned are based on systems with one single delay in the state.

In this paper, we consider the stabilization of the system described by

$$\dot{x}(t) = Ax(t) + Bx(t - \tau(t)) \quad (1)$$

where  $\tau(t)$  is a time delay in the state  $x(t)$ ,  $A \in \mathbb{R}^{n \times n}$ , and  $B \in \mathbb{R}^{n \times n}$  are known real constant matrices,  $0 \leq \tau(t) \leq \tau$ , and  $\dot{\tau}(t) \leq u$ . In recent years, there are many people propose the model with multiple additive time-delays, the model as following:

$$\dot{x}(t) = Ax(t) + Bx(t - \sum_{i=1}^n \tau_i(t))$$

The paper in [5, 9] analysis the stability of system with two additive time-varying delay components, which is

$$\dot{x}(t) = Ax(t) + Bx(t - \tau_1(t) - \tau_2(t))$$

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Miaomiao Yang and Shouming Zhong are with the School of Mathematics Science, University Electronic Science and Technology of China, Chengdu 611731, PR China.

Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu 611731, PR China.

Email address: miaomiaoyang1989@163.com

The stability of the system (2) was studied in [5], and a delay-dependent stability criterion was obtained. An improved stability criterion was derived in [9] by construct a Lyapunov functional to employ the information of the marginally delayed state  $x(t - \tau)$ , where  $u = u_1 + u_2$ . However, another marginally delayed state  $x(t - \tau_1)$  was not considered, do not make full use of the information about  $\tau(t)$ ,  $\tau_1(t)$ ,  $\tau_2(t)$ , which would be inevitably conservative to some extent. What is more, the purpose of reducing conservatism is still limited due to the existence of multiple coefficients the number of the LMIs decision variables, from a theoretical point of view, still remains challenging.

In this paper, we first consider delay-dependent stability for the system (2) by constructing a new Lyapunov functional which employs information of the marginally delays state  $x(t - \tau_1)$  as well as  $x(t - \tau)$ . By construction a new Lyapunov Krasovskii functional, obtain the identical maximum allowable delay bounds, we derived a new and less conservative delay dependent stability condition for a system with two additive delay components. Finally a numerical examples given to illustrate the effectiveness of the proposed methods.

**Notation:** Throughout this paper, the superscripts  $' - 1'$  and  $'T'$  stands for inverse and transpose of matrix, respectively;  $\mathbb{R}^n$  denotes an  $n$ -dimensional Euclidean space;  $\mathbb{R}^{m \times n}$  is the set of all  $m \times n$  real matrices;  $P > 0$  means that the matrix  $P$  is symmetric positive definite,  $diag(\cdot, \cdot, \cdot)$  denotes a block diagonal matrix. In block symmetric matrix or long matrix expression, we use  $(*)$  to represent a term that is induced by symmetry,  $I$  is an appropriately dimensional identity matrix.

## II. PROBLEM STATEMENT.

Consider the following neural networks system with two additive time-varying delays:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t - \tau_1(t) - \tau_2(t)) \\ x(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \quad (2)$$

where  $\tau_1(t)$ ,  $\tau_2(t)$  is a time delay in the state  $x(t)$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times n}$  are known real system constant matrices of appropriate dimensions corresponding to non-delayed and delayed.  $\phi(t)$  is the initial condition on the segment  $[-\tau, 0]$ .

$$\begin{aligned} 0 \leq \tau_1(t) \leq \tau_1, 0 \leq \tau_2(t) \leq \tau_2, \dot{\tau}_1(t) \leq u_1, \dot{\tau}_2(t) \leq u_2 \\ \tau(t) = \tau_1(t) + \tau_2(t), \dot{\tau}(t) = u \\ \tau = \tau_1 + \tau_2, u = u_1 + u_2 \end{aligned} \quad (3)$$

where  $\tau_1, \tau_2, u_1, u_2$ , are constants.

**Lemma 1.**[10].For any positive constant matrix  $Z = Z^T > 0$ ,  $Z \in \mathfrak{R}^{n \times n}$ , scalars  $h_1 > h_2 > 0$  such that the following integrations are well defined, then

$$\begin{aligned}
 & -(h_2 - h_1) \int_{h_2}^{h_1} x^T(s)Zx(s)ds \\
 & \leq - \int_{h_2}^{h_1} x^T(s)dsZ \int_{h_2}^{h_1} x(s)ds \\
 & -\frac{1}{2}(h_2^2 - h_1^2) \int_{h_2}^{h_1} \int_{t+\theta}^t x^T(s)Zx(s)dsd\theta \\
 & \leq - \int_{h_2}^{h_1} \int_{t+\theta}^t x^T(s)dsZ \int_{h_2}^{h_1} \int_{t+\theta}^t x(s)ds
 \end{aligned} \tag{4}$$

**Lemma 2.**[11].For any constant matrix  $X \in \mathfrak{R}^{n \times n}$ ,  $Y \in \mathfrak{R}^{n \times n}$ ,  $\Omega = \begin{bmatrix} R & X^T \\ X & R \end{bmatrix}$ , scalars  $0 \leq \tau_0 \leq \tau(t) \leq \tau_M$ , and vector function  $\dot{x} : [-\tau_M, -\tau_0] \rightarrow \mathfrak{R}^n$ , such that the following integrations are well defined, then

$$\begin{aligned}
 & - \int_{t-\tau_M}^{t-\tau_0} \dot{x}^T(s)R\dot{x}(s)ds \leq - \begin{bmatrix} x^T(t-\tau_M) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t-\tau_0) \end{bmatrix}^T \\
 & \times \Omega \begin{bmatrix} x^T(t-\tau_M) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t-\tau_0) \end{bmatrix}
 \end{aligned} \tag{5}$$

### III. MAIN RESULTS

**Theorem 1.**For given scalars  $0 \leq \tau < \infty$ ,  $0 \leq \tau_1 < \infty$ , and  $0 \leq \tau_2 < \infty$ ,  $u > 0$ ,  $u_1 > 0$ ,  $u_2 > 0$ , then the system(2) is asymptotically stable with delays  $\tau(t)$ ,  $\tau_1(t)$ ,  $\tau_2(t)$ , if exist positive-definite matrices  $Q_i$  ( $i = 1, 2, \dots, 6$ ),  $\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$ ,  $R_i$  ( $i = 1, 2, 3, 4, 5$ ) and  $P$ , for any matrices  $X_1, X_2, X_3$ , with appropriate dimension such that the following LMIs hold:

$$E = \begin{bmatrix} e_{11} & e_{12} & X_1 & e_{14} & X_5 & e_{16} & X_3 & e_{18} & e_{19} \\ * & e_{22} & e_{23} & 0 & 0 & 0 & 0 & e_{28} & e_{29} \\ * & * & e_{33} & 0 & 0 & 0 & 0 & e_{38} & e_{39} \\ * & * & * & e_{44} & e_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & e_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & e_{66} & e_{67} & 0 & 0 \\ * & * & * & * & * & * & e_{77} & 0 & 0 \\ * & * & * & * & * & * & * & e_{88} & 0 \\ * & * & * & * & * & * & * & * & e_{99} \end{bmatrix}$$

$$\begin{aligned}
 e_{11} &= PA + AP^T + \sum_{i=1}^6 Q_i + \tau^2 A^T R_1 A + \tau_1^2 A^T R_2 A \\
 &+ \tau_2^2 A^T R_3 A - R_1 - R_2 - R_3 + \tau^2 R_4 + \frac{\tau^4}{4} A^T R_5 A \\
 e_{12} &= PB + \tau^2 A^T R_1 B + \tau_1^2 A^T R_2 B + \tau_2^2 A^T R_3 B + R_1 \\
 &+ \frac{\tau^4}{4} A^T R_5 B - X_1 \\
 e_{14} &= R_2 - X_2, e_{16} = R_3 - X_3 \\
 e_{18} &= P_{11} - P_{12} - P_{12} A, e_{19} = P_{12} - P_{22} - P_{22} A \\
 e_{22} &= -(1-u)Q_1 + \tau^2 B^T R_1 B + \tau_1^2 B^T R_2 B + 2X_1 \\
 &+ \tau_2^2 B^T R_3 B + \frac{\tau^4}{4} B^T R_5 B - 2R_1
 \end{aligned}$$

$$\begin{aligned}
 e_{23} &= R_1 - X_1, e_{28} = P_{12} B, e_{29} = P_{22} B \\
 e_{33} &= -R_1 - Q_2, e_{38} = P_{12} - P_{11}, e_{39} = P_{22} - P_{12} \\
 e_{44} &= -(1-u_1)Q_3 - 2R_2 + 2X_2, e_{45} = R_2 - X_2 \\
 e_{55} &= -Q_4 - R_2, e_{67} = R_3 - X_3 \\
 e_{66} &= -(1-u_2)Q_5 - 2R_3 + 2X_3 \\
 e_{77} &= -Q_6 - R_3, e_{88} = -R_4, e_{99} = -R_5
 \end{aligned}$$

*Proof:* Construct a Lyapunov function as follows:

$$V(x_t) = \sum_{i=1}^6 V_i(x_t)$$

where

$$V_1(x_t) = x^T(t)Px(t)$$

$$\begin{aligned}
 V_2(x_t) &= \int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds + \int_{t-\tau}^t x^T(s)Q_2x(s)ds \\
 &+ \int_{t-\tau_1(t)}^t x^T(s)Q_3x(s)ds + \int_{t-\tau_1}^t x^T(s)Q_4x(s)ds \\
 &+ \int_{t-\tau_2(t)}^t x^T(s)Q_5x(s)ds + \int_{t-\tau_2}^t x^T(s)Q_6x(s)ds
 \end{aligned}$$

$$\begin{aligned}
 V_3(x_t) &= \tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
 &+ \tau_1 \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\
 &+ \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s)R_3\dot{x}(s)dsd\theta
 \end{aligned}$$

$$V_4(x_t) = \tau \int_{-\tau}^0 \int_{t+\theta}^t x^T(s)R_4x(s)dsd\theta$$

$$V_5(x_t) = \frac{\tau^2}{2} \int_{-\tau}^0 \int_{t+\theta}^t \int_{t+\lambda}^t \dot{x}^T(s)R_5\dot{x}(s)dsd\theta d\lambda$$

$$\begin{aligned}
 V_6(x_t) &= \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\
 &\times \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}
 \end{aligned}$$

The time derivative of  $V(x_t)$  along the trajectory of system (2) is given by

$$\dot{V}(x_t) = \sum_{i=1}^6 \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t)P\dot{x}(t) \tag{6}$$

$$\begin{aligned}
 \dot{V}_2(x_t) &= x^T(t) \sum_{i=1}^6 Q_i x(t) - x^T(t-\tau)Q_2x(t-\tau) \\
 &- x^T(t-\tau_2)Q_6x(t-\tau_2) - x^T(t-\tau_1)Q_4x(t-\tau_1) \\
 &- (1-u)x^T(t-\tau(t))Q_1x(t-\tau(t))
 \end{aligned}$$

$$\begin{aligned}
 &-(1-u_2)x^T(t-\tau_2(t))Q_5x(t-\tau_2(t)) \\
 &-(1-u_1)x^T(t-\tau_1(t))Q_3x(t-\tau_1(t))
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_3(x_t) &\leq \dot{x}^T(t)[\tau^2R_1 + \tau_1^2R_2 + \tau_2^2R_3]\dot{x}(t) \\
 &\quad - \tau \int_{t-\tau}^t \dot{x}^T(s)R_1\dot{x}(s)ds \\
 &\quad - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\
 &\quad - \tau_2 \int_{t-\tau_2}^t \dot{x}^T(s)R_3\dot{x}(s)ds
 \end{aligned}$$

Based on the lemma 1 ,we have

$$\begin{aligned}
 &-\tau \int_{t-\tau}^t \dot{x}^T(s)R_1\dot{x}(s)ds \leq - \begin{bmatrix} x^T(t-\tau) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t) \end{bmatrix}^T \\
 &\quad \times \begin{bmatrix} R_1 & X_1^T \\ X_1 & R_1 \end{bmatrix} \begin{bmatrix} x^T(t-\tau) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &-\tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_2\dot{x}(s)ds \leq - \begin{bmatrix} x^T(t-\tau_1) - x^T(t-\tau_1(t)) \\ x^T(t-\tau_1(t)) - x^T(t) \end{bmatrix}^T \\
 &\quad \times \begin{bmatrix} R_2 & X_2^T \\ X_2 & R_2 \end{bmatrix} \begin{bmatrix} x^T(t-\tau_1) - x^T(t-\tau_1(t)) \\ x^T(t-\tau_1(t)) - x^T(t) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &-\tau_2 \int_{t-\tau_2}^t \dot{x}^T(s)R_3\dot{x}(s)ds \leq - \begin{bmatrix} x^T(t-\tau_2) - x^T(t-\tau_2(t)) \\ x^T(t-\tau_2(t)) - x^T(t) \end{bmatrix}^T \\
 &\quad \times \begin{bmatrix} R_3 & X_3^T \\ X_3 & R_3 \end{bmatrix} \begin{bmatrix} x^T(t-\tau_2) - x^T(t-\tau_2(t)) \\ x^T(t-\tau_2(t)) - x^T(t) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_4(x_t) &= \tau^2x^T(t)R_4x(t) - \tau \int_{t-\tau}^t x^T(s)R_4x(s)ds \\
 &\leq \tau^2x^T(t)R_4x(t) - \int_{t-\tau}^t x^T(s)dsR_4 \int_{t-\tau}^t x(s)ds
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_5(x_t) &\leq \frac{\tau^4}{4}\dot{x}^T(t)R_5\dot{x}(t) - \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta\right)^T R_5 \\
 &\quad \times \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta\right)
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_6(x_t) &= 2 \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\
 &\quad \begin{bmatrix} x(t) - x(t-\tau) \\ \tau\dot{x}(t) - x(t) + x(t-\tau) \end{bmatrix}
 \end{aligned}$$

Then,from (6)-(14),we can obtain

$$\dot{V}(x_t) \leq g^T(t)Eg(t)$$

(7) where,

$$\begin{aligned}
 g^T(t) &= [x(t), x(t-\tau(t)), x(t-\tau), x(t-\tau_1(t)), x(t-\tau_1), \\
 &\quad x(t-\tau_2(t)), x(t-\tau_2), \int_{t-\tau}^t x^T(s)ds, \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)ds]
 \end{aligned}$$

(8) **Corollary 1.**For given scalars  $0 \leq \tau < \infty, 0 \leq \tau_1 < \infty,$  and  $0 \leq \tau_2 < \infty,$  then the system(2) is asymptotically stable with delays  $\tau(t), \tau_1(t), \tau_2(t)$  ,if there exist positive-definite matrices  $P, Q_2, Q_4, Q_6, R_i (i = 1, 2, 3, 4, 5),$   $\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix},$  for any matrices  $X_1, X_2, X_3,$  with appropriate dimension such that the following LMIs hold:

$$(9) \quad E = \begin{bmatrix} e_{11} & e_{12} & X_1 & e_{14} & X_5 & e_{16} & X_3 & e_{18} & e_{19} \\ * & e_{22} & e_{23} & 0 & 0 & 0 & 0 & e_{28} & e_{29} \\ * & * & e_{33} & 0 & 0 & 0 & 0 & e_{38} & e_{39} \\ * & * & * & e_{44} & e_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & e_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & e_{66} & e_{67} & 0 & 0 \\ * & * & * & * & * & * & e_{77} & 0 & 0 \\ * & * & * & * & * & * & * & e_{88} & 0 \\ * & * & * & * & * & * & * & * & e_{99} \end{bmatrix}$$

$$\begin{aligned}
 (10) \quad e_{11} &= PA + AP^T + \sum_{i=1}^6 Q_i + \tau^2 A^T R_1 A + \tau_1^2 A^T R_2 A \\
 &\quad + \tau_2^2 A^T R_3 A - R_1 - R_2 - R_3 + \tau^2 R_4 + \frac{\tau^4}{4} A^T R_5 A \\
 e_{12} &= PB + \tau^2 A^T R_1 B + \tau_1^2 A^T R_2 B + \tau_2^2 A^T R_3 B + R_1 \\
 &\quad + \frac{\tau^4}{4} A^T R_5 B - X_1 \\
 e_{14} &= R_2 - X_2, e_{16} = R_3 - X_3 \\
 e_{18} &= P_{11} - P_{12} - P_{12} A, e_{19} = P_{12} - P_{22} - P_{22} A \\
 e_{22} &= \tau^2 B^T R_1 B + \tau_1^2 B^T R_2 B + 2X_1 + \tau_2^2 B^T R_3 B \\
 &\quad + \frac{\tau^4}{4} B^T R_5 B - 2R_1 \\
 e_{23} &= R_1 - X_1, e_{28} = P_{12} B, e_{29} = P_{22} B \\
 e_{33} &= -R_1 - Q_2, e_{38} = P_{12} - P_{11}, e_{39} = P_{22} - P_{12} \\
 e_{44} &= -2R_2 + 2X_2, e_{45} = R_2 - X_2 \\
 e_{55} &= -Q_4 - R_2, e_{67} = R_3 - X_3 \\
 e_{66} &= -2R_3 + 2X_3 \\
 e_{77} &= -Q_6 - R_3, e_{88} = -R_4, e_{99} = -R_5
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad e_{14} &= R_2 - X_2, e_{16} = R_3 - X_3 \\
 e_{18} &= P_{11} - P_{12} - P_{12} A, e_{19} = P_{12} - P_{22} - P_{22} A \\
 e_{22} &= \tau^2 B^T R_1 B + \tau_1^2 B^T R_2 B + 2X_1 + \tau_2^2 B^T R_3 B \\
 &\quad + \frac{\tau^4}{4} B^T R_5 B - 2R_1 \\
 e_{23} &= R_1 - X_1, e_{28} = P_{12} B, e_{29} = P_{22} B \\
 e_{33} &= -R_1 - Q_2, e_{38} = P_{12} - P_{11}, e_{39} = P_{22} - P_{12} \\
 e_{44} &= -2R_2 + 2X_2, e_{45} = R_2 - X_2 \\
 e_{55} &= -Q_4 - R_2, e_{67} = R_3 - X_3 \\
 e_{66} &= -2R_3 + 2X_3 \\
 e_{77} &= -Q_6 - R_3, e_{88} = -R_4, e_{99} = -R_5
 \end{aligned}$$

(13) *Proof:* Choosing  $Q_1 = 0, Q_3 = 0$  and  $Q_5 = 0$  in Theorem 1, one can easily obtain this result. ■

#### IV. NUMERICAL EXAMPLES

(14) In this section, we provide the simulation of examples to illustrate the effectiveness of our method.

**Example 1.** Considering the system (2) with the following parameters:

$$(15) \quad A = \begin{bmatrix} -2 & 0 \\ 0 & -9 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

and we suppose that  $\dot{\tau}_1(t) \leq 0.1, \dot{\tau}_2(t) \leq 0.8$ .  
First, the maximum delay bounds  $\tau_2$  are shown under different  $\tau_1$  are list in Table I.  
Then, the maximum delay bounds  $\tau_1$  are shown under different  $\tau_2$  are list in Table II.  
The maximum delay bounds  $\tau_2$  are shown under different  $\tau_1$  about corollary 1 are list in Table III.

TABLE I  
ALLOWABLE UPPER BOUND OF  $\tau_2$  WITH VARIOUS  $\tau_1$

Method	$\tau_1 = 1.0$	$\tau_1 = 1.1$	$\tau_1 = 1.2$	$\tau_1 = 1.5$
[12]	0.180	0.080	-	-
[13]	0.378	0.278	0.178	-
[14]	0.415	0.376	0.340	0.248
[15]	0.512	0.457	0.406	0.283
[16]	0.519	0.486	0.453	0.378
this works	0.810	0.710	0.610	0.310

TABLE II  
ALLOWABLE UPPER BOUND OF  $\tau_1$  WITH VARIOUS  $\tau_2$

Method	$\tau_2 = 0.3$	$\tau_2 = 0.4$	$\tau_2 = 0.5$
[12]	0.880	0.780	0.680
[13]	1.078	0.978	0.878
[14]	1.324	1.039	0.806
[15]	1.453	1.214	1.021
this works	1.510	1.410	1.310

TABLE III  
ALLOWABLE UPPER BOUND OF  $\tau_2$  WITH VARIOUS  $\tau_1$

Method	$\tau_1 = 0.3$	$\tau_1 = 0.5$	$\tau_1 = 1.0$
Corollary	1.088	0.888	0.388

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**Miaomiao Yang** was born in Anhui Province, China, in 1989. She received the B.S. degree from Huaibei Normal University in 2012. She currently pursuing the M.S.degree from University of Electronic Science and Technology of China. Her research interests include stability of neural networks, switch and delay dynamic systems.

**Shouming Zhong** was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.