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Nonoscillation Criteria for Nonlinear Delay Dynamic Systems on Time Scales

Xinli Zhang

Abstract—In this paper, we consider the nonlinear delay dynamic system

$$x^{\Delta}(t) = p(t)f_1(y(t)), \qquad y^{\Delta}(t) = -q(t)f_2(x(t-\tau)).$$

We obtain some necessary and sufficient conditions for the existence of nonoscillatory solutions with special asymptotic properties of the system. We generalize the known results in the literature. One example is given to illustrate the results.

Keywords—Dynamic system, oscillation, time scales, two-dimensional.

I. Introduction

In this paper we investigate the nonlinear delay dynamic system

$$\begin{cases} x^{\Delta}(t) = p(t)f_1(y(t)), \\ y^{\Delta}(t) = -q(t)f_2(x(t-\tau)). \end{cases} \tag{1}$$

where $p(t), q(t)$ are real rd-continuous nonnegative functions defined on $t \in [t_0,\infty)_T = [t_0,\infty)\bigcap T$, $p(t)$ is not identically zero on $t \in [t_0,\infty)_T$ such that $\int_{t_0}^{\infty} p(t)\Delta t = \infty$. Here, the time scale T is unbounded. We assume throughout that $f_i : R \to R, i = 1, 2$, are continuous functions with $uf_i(u) > 0$ for $u \neq 0, i = 1, 2$, and τ is a nonnegative constant.

By the solution of system (1), we mean a pair of non-trivial real-valued functions $(x(t), y(t))$ which has property $y \in C^1_{rd}([t_0,\infty)_T, R), x \in C^1_{rd}([t_0+\tau,\infty)_T, R)$ and satisfies system (1) for $t \in [t_0,\infty)_T$. Our attention is restricted to those solutions $(x(t), y(t))$ of system (1) which exist on some half-line $[t_x,\infty)_T$ and satisfy $\sup\{|x(t)| + |y(t)| : t \geq t_x\} > 0$ for any $t_x \geq t_0$. As usual, a continuous real-valued function defined on $[T_0,\infty)$ is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is said to be nonoscillatory. A solution $(x(t), y(t))$ of system (1) is called oscillatory if both $x(t)$ and $y(t)$ are oscillatory (i.e.,neither eventually positive nor eventually negative), and otherwise it will be called nonoscillatory. System (1) is called oscillatory if its solutions are oscillatory.

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D.thesis in 1988 in order to unify continuous and discrete analysis (see [1]). Not only can this theory of the so-called "dynamic equations" unify the theories of differential equations and difference equations, but also extend these classical cases to cases "in between", e.g., to the so-called q-difference equations and can be applied on other different types of time scales. Since Hilger formed the definition of derivatives and integral on time scales, several authors have expounded on various aspects of this new theory; see the survey paper by Bohner and Peterson [2] and references cited therein. A book on the subject of time scales (see [3]) summarizes and organizes much of time scale calculus. The reader is referred to Chapter 1 in [3] for the necessary time scale definitions and notations used throughout this paper.

In recent years, there has been an increasing interest in studying the oscillation and nonoscillatory of solutions of dynamic equations on time scales with attempts to harmonize the oscillation theory for the continuous and the discrete, to include them in one comprehensive theory, and to eliminate obscurity from both. We refer the readers to the paper [4-7] and the references cited there in. The system (1) reduces to some important second-order dynamic equations in the particular case, for example

$$x^{\Delta\Delta}(t) + p(t)f(x(t-\tau)) = 0, \tag{2}$$

$$[r(t)x^{\Delta}(t)]^{\Delta} + p(t)f(x(\tau(t))) = 0, \tag{3}$$

$$x^{\Delta\Delta}(t) + p(t)x^{\gamma}(t-\tau) = 0, \tag{4}$$

where $p(t)$ is rd-continuous on $[t_0,\infty)_T$. Some oscillation results for these equations have been presented in [8-10]. When $T = R$ and $\tau = 0$, system (1) becomes the two-dimensional differential system

$$\begin{cases} x'(t) = p(t)f_1(y(t)), \\ y'(t) = -q(t)f_2(x(t)), \end{cases} \tag{5}$$

whose oscillatory behavior has been investigated, see for example [11] and the references cited there in. When $T = N$ and $\tau = 0$, system (1) becomes the two-dimensional difference system

$$\begin{cases} x^{\Delta}(t) = p(t)f_1(y(t)), \\ y^{\Delta}(t) = -q(t)f_2(x(t)), \end{cases} \tag{6}$$

whose oscillatory behavior has been investigated, see for example [12] and the references cited there in.

On the other hand, recently, the theory of dynamic equations on time scales has become an important research field due to its tremendous potential for various applications. Since there are few works about nonoscillation of dynamic systems on time scales, motivated by [5,11-12], in the present paper we investigate nonoscillatory properties for the systems (1) on time scales. Our results not only unify the known results

Xinli Zhang is with College of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, 266061, China. (e-mail: zhangxinli1000@sina.com).
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of differential and difference systems, but also extend and improve the existing results of dynamic systems on time scales in the literature. In the next section, we present some useful lemmas. In Section III, by means of appropriate hypotheses on $f_i(u), i = 1, 2$ and fixed point theorem, we establish some new sufficient and necessary conditions for the existence of nonoscillatory solutions with special asymptotic properties for the system (1). Examples are given to illustrate the applicability of the obtained results.

II. Preparatory Lemmas

In this section, we will give some lemmas which are important in proving our main results. For convenience, we will employ the following notation:

$$A(s,t) = \int_s^t p(\tau)\Delta\tau, \qquad s, t \in [t_0, \infty)_T. \tag{7}$$

Lemma 1 If $(x(t), y(t))$ is a nonoscillatory solution of system (1), then the component $x(t)$ is also nonoscillatory.

Proof. Assume to the contrary that $x(t)$ is oscillatory but $y(t)$ is non-oscillatory. Without loss of generality, we let $y(t) > 0$ on $[t_0, \infty)_T$. In view of the first equation of system (1), we have $x^\Delta(t) \geq 0$ on $[t_0, \infty)_T$. Thus $x(t) > 0$ or $x(t) < 0$ for all large t, which leads to a contradiction. The case where $y(t)$ is eventually negative is similarly proved.

Lemma 2 Suppose $\lim_{t\to\infty} A(t_0, t) = \infty$ holds and $(x(t), y(t))$ is a non-oscillatory solution of system (1). Then there exist positive constants c_1, c_2, d_2 , nonnegative constant d_1 and $t_1 \geq t_0 + \tau$ such that

$$c_1 \leq x(t) \leq c_2 A(t_0 + \tau, t), \qquad d_1 \leq y(t) \leq d_2,$$

or

$$-c_2 A(t_0 + \tau, t) \leq x(t) \leq -c_1, \qquad -d_2 \leq y(t) \leq -d_1,$$

for $t \geq t_1$.

Proof. Without loss of generality, assume that $x(t) > 0$ for $t \geq t_0$. In view of the second equation of system (1), we have $y^\Delta(t) < 0$ on $[t_0 + \tau, \infty)_T$. Thus , there are two cases: $y(t) > 0$ and $y(t) < 0$ for $t \geq t_0 + \tau$. If $y(t) < 0$, then we have

$$x^\Delta(t) = p(t)f(y(t)) \leq p(t)f(y(t_0 + \tau)) < 0,$$

which yields, after integrating ,

$$\begin{aligned} x(t) &\leq x(t_0 + \tau) + \int_{t_0+\tau}^t p(s)f(y(t_0 + \tau))\Delta s \\ &= x(t_0 + \tau) + f(y(t_0 + \tau)) \int_{t_0+\tau}^t p(s)\Delta s. \end{aligned}$$

The left hand side tends to $-\infty$ in view of $\lim_{t\to\infty} A(t_0, t) = \infty$, which is a contradiction. Thus, $y(t) > 0, y^\Delta(t) < 0$ eventually, and $x^\Delta(t) > 0$ eventually by the first equation of system (1). Hence, $\lim_{t\to\infty} y(t)$ exists and $x(t) \geq c_1$ eventually for some positive constant c_1. Furthermore, the same reasoning just used also leads to

$$x(t) \leq x(t_0 + \tau) + f(y(t_0 + \tau)A(t_0 + \tau, t), \qquad t \geq t_0 + \tau.$$

Since $\lim_{t\to\infty} A(t_0, t) = \infty$, thus there is c_2 such that $x(t) \leq c_2 A(t_0 + \tau, t)$ for all large t. The proof is complete.

III. Main results

In this section, we generalize and improve some results of [8-12]. Some necessary and sufficient conditions are given for the system (1) to admit the existence of nonoscillatory solutions with special asymptotic properties.

Theorem 1 Suppose that $\lim_{t\to\infty} A(t_0, t) = \infty$ and $f_i, i = 1, 2$ are nondecreasing. Then system (1) has a nonoscillatory solution $(x(t), y(t))$ such that $\lim_{t\to\infty} x(t) = \alpha \neq 0$ and $\lim_{t\to\infty} y(t) = 0$ if and only if for some $c \neq 0$

$$\int_{t_0}^\infty p(t)f_1\left(\int_t^\infty q(s)f_2(c)\Delta s\right)\Delta t < \infty. \tag{8}$$

Proof. Suppose that $(x(t), y(t))$ is a nonoscillatory solution of system (1) such that $\lim_{t\to\infty} x(t) = \alpha \neq 0$ and $\lim_{t\to\infty} y(t) = 0$. Without loss of generality, we assume that $\alpha > 0$. Then there exist two positive constant c_1, c_2 and $t_1 \geq t_0$ such that $c_1 \leq x(t) \leq c_2$ for $t \geq t_1$. In view of the second equation of system (1), we have

$$y(s) - y(t) = -\int_t^s q(u)f_2(x(u - \tau))\Delta u.$$

Let $s \to \infty$ and noting that $\lim_{t\to\infty} y(t) = 0$, we have

$$y(t) = \int_t^\infty q(u)f_2(x(u - \tau))\Delta u,$$

for $t \geq t_1$. Thus, from the first equation of system (1), we see that

$$\begin{aligned} \infty &> \alpha - x(t_1 + \tau) \\ &= \int_{t_1+\tau}^\infty p(s)f_1(y(s))\Delta s \\ &= \int_{t_1+\tau}^\infty p(s)f_1\left(\int_s^\infty q(u)f_2(x(u - \tau))\Delta u\right)\Delta s \\ &\geq \int_{t_1+\tau}^\infty p(s)f_1\left(\int_s^\infty q(u)f_2(c_1)\Delta u\right)\Delta s. \end{aligned}$$

Conversely, suppose that (8) holds, we may assume that $c > 0$ and choose $t_1 \in [t_0, \infty)_T$ so large that

$$\int_{t_1}^\infty p(s)f_1\left(\int_s^\infty q(u)f_2(c)\Delta u\right)\Delta s \leq \frac{c}{2}. \tag{9}$$

Let $BC[t_0 + \tau, \infty)_T$ be the Banach space of all real-valued rd-continuous functions on $[t_0 + \tau, \infty)_T$ endowed with the norm $\|x(t)\| = sup_{t\in[t_0+\tau,\infty)_T}|x(t)| < \infty$. We defined a bounded convex, and closed subset Ω of $BC[t_0 + \tau, \infty)_T$ as

$$\Omega = \{x \in BC[t_0 + \tau, \infty)_T : \frac{c}{2} \leq x(t) \leq c\}. \tag{10}$$

Define an operator $\Gamma : \Omega \to BC[t_0 + \tau, \infty)_T$ as follows: $(\Gamma x)(t) =$

$$\begin{cases} c - \int_t^\infty p(s)f_1\left(\int_s^\infty q(u)f_2(x(u - \tau))\Delta u\right)\Delta s, \\ \qquad\qquad t \in [t_1 + \tau, \infty)_T \\ c - \int_{t_1+\tau}^\infty p(s)f_1\left(\int_s^\infty q(u)f_2(x(u - \tau))\Delta u\right)\Delta s, \\ \qquad\qquad t \in [t_0 + \tau, t_1 + \tau]_T. \end{cases} \tag{11}$$

Now we show that Γ satisfies the assumptions of Schauder's fix-point theorem (see [13, Corollary 6]).

(i) We will show that Γ maps Ω into Ω. In fact, for any $x \in \Omega$ and $t \in [t_1+\tau,\infty)_T$, in view of

$$\begin{aligned} c &\geq (\Gamma x)(t) \\ &= c - \int_t^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(x(u-\tau))\Delta u\right)\Delta s \\ &\geq c - \int_t^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s \\ &\geq c - \frac{c}{2} = \frac{c}{2}. \end{aligned}$$

Similarly, we can prove that $\frac{c}{2} \leq (\Gamma x)(t) \leq c$ for any $x \in \Omega$ and $t \in [t_0+\tau, t_1+\tau]_T$. Hence, $(\Gamma x)(t) \in \Omega$ for any $x \in \Omega$.

(ii) We prove that Γ is a completely continuous mapping. First, we consider the continuity of Γ. Let $x_n \in \Omega$ and $\|x_n - x\| \to 0$ as $n \to \infty$. Since Ω is closed, then $x \in \Omega$. Consequently, by the continuity of f_i, for any $t \in [t_0+\tau, t_1+\tau]_T$, we have

$$\begin{aligned} &\lim_{n\to\infty} |p(t) f_1\left(\int_t^\infty q(s) f_2(x_n(s-\tau))\Delta s\right) - \\ &-f_1\left(\int_t^\infty q(s) f_2(x(s-\tau))\Delta s\right)]| = 0. \end{aligned} \tag{12}$$

We also obtain that

$$\begin{aligned} &p(t)|f_1\left(\int_t^\infty q(s) f_2(x_n(s-\tau))\Delta s\right) \\ &-f_1\left(\int_t^\infty q(s) f_2(x(s-\tau))\Delta s\right)| \\ &\leq 2p(t) f_1\left(\int_t^\infty q(s) f_2(c)\Delta s\right). \end{aligned} \tag{13}$$

On the other hand, from (11) we have

$$\begin{aligned} &|(\Gamma x_n)(t) - (\Gamma x)(t)| \\ &\leq \int_{t_1+\tau}^\infty p(s)|f_1\left(\int_s^\infty q(u) f_2(x_n(u-\tau))\Delta u\right) \\ &-f_1\left(\int_s^\infty q(u) f_2(x(u-\tau))\Delta u\right)|\Delta s, \end{aligned} \tag{14}$$

for $t \in [t_0+\tau, t_1+\tau]_T$ and

$$\begin{aligned} &|(\Gamma x_n)(t) - (\Gamma x)(t)| \\ &\leq \int_t^\infty p(s)|f_1\left(\int_s^\infty q(u) f_2(x_n(u-\tau))\Delta u\right) \\ &-f_1\left(\int_s^\infty q(u) f_2(x(u-\tau))\Delta u\right)|\Delta s, \end{aligned} \tag{15}$$

for $t \in [t_1+\tau,\infty)_T$. Therefore, from (14) and (15), we have

$$\begin{aligned} &\|(\Gamma x_n)(t) - (\Gamma x)(t)\| \\ &\leq \int_{t_1+\tau}^\infty p(s)|f_1\left(\int_s^\infty q(u) f_2(x_n(u-\tau))\Delta u\right) \\ &-f_1\left(\int_s^\infty q(u) f_2(x(u-\tau))\Delta u\right)|\Delta s. \end{aligned} \tag{16}$$

Referring to Chapter 5 in [14], we see that the Lebesgue dominated convergence theorem holds for the integral on time scales. Then, from (12) (13) (16) yields $\lim_{n\to\infty} \|\Gamma x_n - \Gamma x\| = 0$, which implies that Γ is continuous on Ω.

Next, we show that $\Gamma\Omega$ is uniformly cauchy. In fact, for any $\epsilon > 0$, take $t_2 \in [t_1+\tau,\infty)_T$ and $t_2 > t_1$ such that

$$\int_{t_2}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s \leq \epsilon. \tag{17}$$

Then for any $x \in \Omega$ and $t, r \in [t_2,\infty)_T$, we have

$$\begin{aligned} |(\Gamma x)(t) - (\Gamma x)(r)| &\leq |\int_{t+\tau}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s| \\ &\quad +|\int_{r+\tau}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s| \\ &\leq 2\int_{t_2}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s \\ &\leq 2\epsilon. \end{aligned}$$

This means that $\Gamma\Omega$ is uniformly cauchy.

Finally, we prove that $\Gamma\Omega$ is equi-continuous on $[t_0+\tau, t_2]_T$ for any $t_2 \in [t_0+\tau,\infty)_T$. Without loss of generality, we set $t_2 > t_1$. For any $x \in \Omega$, we have $|(\Gamma x)(t) - (\Gamma x)(r)| \equiv 0$ for $t, r \in [t_0+\tau, t_1+\tau]_T$ and

$$\begin{aligned} |(\Gamma x)(t) - (\Gamma x)(r)| &\leq |\int_{t+\tau}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s \\ &\quad - \int_{r+\tau}^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s| \\ &\leq \int_{t+\tau}^{r+\tau} p(s) f_1\left(\int_s^\infty q(u) f_2(c)\Delta u\right)\Delta s. \end{aligned}$$

for $t, r \in [t_1+\tau, t_2]_T$ Now, we see that for any $\epsilon > 0$, there exists $\delta > 0$ such that when $t, r \in [t_0+\tau, t_2]_T$ with $|t-r| < \delta$, $|(\Gamma x)(t) - (\Gamma x)(r)| < \epsilon$ for any $x \in \Omega$. This means that $\Gamma\Omega$ is equicontinuous on $[t_0+\tau, t_2]_T$ for any $t_2 \in [t_0+\tau,\infty)_T$. By Arzela-Ascoli theorem (see [13, lemma4]), $\Gamma\Omega$ is relatively compact. From the above, we have proved that Γ is a completely continuous mapping.

By Schauder's fixed point theorem, there exists $x \in \Omega$ such that $\Gamma x = x$. Therefore, we have

$$\begin{aligned} x(t) &= (\Gamma x)(t) \\ &= c - \int_t^\infty p(s) f_1\left(\int_s^\infty q(u) f_2(x(u-\tau))\Delta u\right)\Delta s, \\ &\qquad t \in [t_1+\tau,\infty)_T. \end{aligned} \tag{18}$$

Set

$$y(t) = \int_t^\infty q(u) f_2(x(u-\tau))\Delta u, \qquad t \in [t_1+\tau,\infty)_T. \tag{19}$$

Then $\lim_{t\to\infty} y(t) = 0$ and $y^\Delta(t) = -q(t) f_2(x(t-\tau))$. On the other hand,

$$x(t) = c - \int_t^\infty p(s) f_1(y(s))\Delta s, \tag{20}$$

which implies $\lim_{t\to\infty} x(t) = c$ and $x^\Delta(t) = p(t) f_1(y(t-\tau))$. The proof is complete.

Theorem 2 Suppose that $\lim_{t\to\infty} A(t_0,t)=\infty$ and $f_i, i=1,2$ are nondecreasing. Then system (1) has a nonoscillatory solution $(x(t),y(t))$ such that $\lim_{t\to\infty} x(t)=\infty$ and $\lim_{t\to\infty} y(t)=\beta$ if and only if for some $c\neq 0$

$$\int_{t_0}^{\infty} |q(s)f_2(cA(s,t_0))|\,\Delta s<\infty. \tag{21}$$

Proof. Suppose that $(x(t),y(t))$ is a nonoscillatory solution of (1) such that $\lim_{t\to\infty} x(t)=\infty$ and $\lim_{t\to\infty} y(t)=\beta$. Without loss of generality, we assume that $\beta>0$. From lemma 2, there exists $t_1\in[t_0+\tau,\infty)_T$ and positive constant c_1,c_2,d_1,d_2 such that

$$c_1\le x(t)\le c_2A(t_0+\tau,t),\qquad d_1\le y(t)\le d_2,$$

for $t\in[t_1,\infty)_T$. According to the first equation in system (1), we have

$$\begin{aligned} x(s)&=x(t_1)+\int_{t_1}^{s} p(u)f_1(y(u))\Delta u\\ &\ge f_1(d_1)\int_{t_1}^{s} p(u)\Delta u\ge cA(s,t_1). \end{aligned}\tag{22}$$

It follows from the second equation in system (1) that

$$\begin{aligned} \infty>y(t_1+\tau)-\beta&=\int_{t_1+\tau}^{\infty} q(t)f_2(x(t-\tau))\Delta t\\ &\ge \int_{t_1+\tau}^{\infty} q(t)f_2(cA(t,t_1))\Delta t, \end{aligned}\tag{23}$$

which implies that (21) holds.

Conversely, pick large $t_1\ge t_0+\tau$ so large that

$$\int_{t_1}^{\infty} q(s)f_2(cA(s,t_0))\Delta s<d=\frac{f_1^{-1}(c)}{2}. \tag{24}$$

Let $BC[t_1,\infty)_T$ be the partially ordered Banach space of all real-valued and rd-continuous functions $x(t)$ with the norm $\| x \|=\sup_{t\in[t_1,\infty)_T}\frac{|x(t)|}{A(t_1,t)}$, and the usual pointwise ordering $\le$. Define

$$\Omega=\{x\in BC[t_1,\infty)_T: f_1(d)A(t_1,t)\le x(t)\le f_1(2d)A(t_1,t)\}.$$

It is easy to see that Ω is a bounded, convex and closed subset of $BC[t_1,\infty)_T$. Let us further define an operator $\Gamma:\ \Omega\to BC[t_1,\infty)_T$ as follows:

$$(\Gamma x)(t)=\int_{t_1}^{t} p(s)f_1\left(d+\int_{s}^{\infty} q(u)f(x(u-\tau))\Delta u\right)\Delta s,$$
$$t\in[t_1+\tau,\infty)_T. \tag{25}$$

It is easy to see that the mapping Γ is nondecreasing. On the other hand, Γ maps Ω into Ω. Indeed, if $x\in\Omega$, then

$$\begin{aligned} f_1(d)A(t_1,t)\le(\Gamma x)(t)&\le\int_{t_1}^{t} p(s)f_1\left(d+d)\Delta u\right)\Delta s\\ &\le f_1(2d)A(t_1,t). \end{aligned}$$

The mapping Γ satisfies the assumptions of Knaster's fixed-point theorem [15]. By Knaster's fixed-point theorem, we ensures that the existence of an $x\in\Omega$ such that $x=\Gamma x$, this is

$$x(t)=\int_{t_1}^{t} p(s)f_1\left(d+\int_{s}^{\infty} q(u)f_2(x(u-\tau))\Delta u\right)\Delta s,$$
$$t\in[t_1,\infty)_T.$$

Set

$$y(t)=d+\int_{t}^{\infty} q(u)f_2(x(u-\tau))\Delta u, t\in[t_1,\infty)_T.$$

Then $\lim_{t\to\infty} y(t)=d$ and $y^{\Delta}(t)=-q(t)f_2(x(t-\tau))$. On the other hand, we have

$$x(t)=\int_{t_1}^{t} p(s)f_1(y(s))\Delta s,$$

which implies that $\lim_{t\to\infty} x(t)=\infty$ and $x^{\Delta}(t)=p(t)f_1(y(t))$. The proof is complete.

Remark 1. Theorem 1 and2 improve the existing results of [11,12].

Example 1. Consider the system

$$x^{\Delta}(t)=y(t),\qquad y^{\Delta}(t)=-t^{v-u}(x(t-\tau))^r, \tag{26}$$

where $T=aN=\{an|n\in N\}$, $a,\ v,\ u,\ r>0$ and are constants.

Let

$$p(t)=1,\quad f_1(y)=y,\quad q(t)=t^{v-u},\quad f_2(x)=x^r.$$

It is easy to see that $f_i(x), i=1,2$ are nondecreasing and continuous with $uf_i(u)>0$ for $u\neq 0, i=1,2$.
For $u>v+2$, we have

$$\begin{aligned} &\int_{t_0}^{\infty} p(t)f_1\left(\int_{t}^{\infty} q(s)f_2(c)\Delta s\right)\Delta t\\ &= \int_{t_0}^{\infty}\left(\int_{t}^{\infty} s^{v-u}c^r\Delta s\right)\Delta t\\ &\le |c|^r\int_{t_0}^{\infty}\left(\int_{t}^{\infty} s^{v-u}\Delta s\right)\Delta t\\ &= |c|^r a^{v-u+2}\sum_{n=n_0}^{\infty}\sum_{k=n}^{\infty} k^{v-u}\\ &= |c|^r a^{v-u+2}\sum_{n=n_0}^{\infty} n^{v-u}<\infty. \end{aligned}$$

That is, (8) holds. By Theorem 1, system (26) has a nonoscillatory solution $(x(t),y(t))$ such that $\lim_{t\to\infty} x(t)=\alpha$ and $\lim_{t\to\infty} y(t)=0$.

On the other hand, For $u>v+r+1$, we obtain

$$\begin{aligned} \int_{a}^{\infty} |q(s)f_2(cA(s,a))|\,\Delta s &= \int_{a}^{\infty} s^{v-u}[c(s-a)]^r\Delta s\\ &\le |c|^r\int_{a}^{\infty} s^{v-u+r}\Delta s\\ &= |c|^r a^{v-u+r+1}\sum_{n=1}^{\infty} n^{v-u+r}\\ &< \infty. \end{aligned}$$

Hence, (21) holds. By Theorem 2, system (26) has a nonoscillatory solution $(x(t), y(t))$ such that $\lim_{t\to\infty} x(t) = \infty$ and $\lim_{t\to\infty} y(t) = \beta$.

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