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A Novel Solution Methodology for Transit Route Network Design Problem

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Abstract**—**Transit route Network Design Problem (TrNDP) is the most important component in Transit planning, in which the overall cost of the public transportation system highly depends on it. The main purpose of this study is to develop a novel solution methodology for the TrNDP, which goes beyond pervious traditional sophisticated approaches. The novelty of the solution methodology, adopted in this paper, stands on the deterministic operators which are tackled to construct bus routes. The deterministic manner of the TrNDP solution relies on using linear and integer mathematical formulations that can be solved exactly with their standard solvers. The solution methodology has been tested through Mandl's benchmark network problem. The test results showed that the methodology developed in this research is able to improve the given network solution in terms of number of constructed routes, direct transit service coverage, transfer directness and solution reliability. Although the set of routes resulted from the methodology would stand alone as a final efficient solution for TrNDP, it could be used as an initial solution for meta-heuristic procedures to approach global optimal. Based on the presented methodology, a more robust network optimization tool would be produced for public transportation planning purposes.

Keywords**—**Integer programming, Transit route design, Transportation, Urban planning.

I. INTRODUCTION

WITH increasing traffic on the roads, more mobility-related problems such as congestion, air pollution, noise pollution, and accidents are created. Public transportation is a very important means to reduce traffic congestions, to improve urban environmental conditions and consequently affects peoples' social lives. Therefore, the need for new public transportation infrastructures for serving new towns and/or improving existing transportation structures to cope with such increase is an urge [1].

Planning, designing and management of public transportation are the key issues for offering a competitive mode that can compete with the private transportation [2]. These transportation planning, designing and management issues are addressed in the Transit route Network Design Problem (TrNDP) [3].

The TrNDP aims to design a set of bus routes and manage their operation in an efficient manner for both users and operators. Different system functions and targets, required for each group of participants, have to be met through solution methodology. TrNDP, stated simply, relates to the determination of a set of routes defined over the street network to deal with demand trips [4].

TrNDP is sorted as one of the most difficult problems to be solved in the field of transportation. This might be due to its high degree of complexity. There are five main sources of complexity that often preclude finding a unique optimal solution for TrNDP [4]-[7].

1. Problem Formulation: Formulation complexity results from the difficulty of defining the decision variables and thus expressing the components of the objective functions.
2. Non-linearity and Non-convexity: Most TrNDP formulations exhibit non-linear decision variables and constraints. Non-convexity would be illustrated by the fact that a transit designer can deploy more resources in transit network (increasing operator's costs) and still obtain a higher total travel time (worse users' costs).
3. Combinatorial Complexity: This arises from the discrete nature of TrNDP. Discrete variables are always involved in route network design problem. Combinatorial optimization problem is a special case of integer problems. It makes the complexity of the problem grows exponentially with the size of transit network.
4. NP-hard: TrNDP optimization is classified as NP-hard, which refers to the problem for which the number of elementary numerical operations is not likely to be expressed by function of polynomial form.
5. Multi Objective Nature of TNDP: Many past approaches have recognized reducing users' costs or operator's costs as their solo objective. In practice, users and operators costs are conflicting objectives.

Over the last five decades, the TrNDP has been under study for many researchers, most likely because the problem is practically important, theoretically interesting, highly complicated, and multi-disciplinary as well. Each researcher considered different objective function, problem's Mathematical Programming representation and search algorithm (heuristics or Meta – heuristic) to be implemented in the solution methodology.

Heuristics is widely accepted by vast number of researchers as the best possible approach for TrNDP. Lampkin and Saalmans used skeleton routes as initial solution for TrNDP. These skeleton routes were expanded by inserting other network nodes under specified conditions [8]. Baaj and Mahmassani implemented the concept of inserting node using Lisp language. They used different strategies for inserting nodes to skeleton routes in attempt of trading off between user

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cost and operator cost [5], [9], [10].Shih and Mahmassani's extended Baaj and Mahmassani work introducing Transit Centre (TC) notion [11]. Mauttonw & Urquhart used Pair Insertion Algorithm (PIA) to improve Baaj and Mahmassani work. PIA simply uses pair node insertion instead of single node insertion per time of insertion [12].

Recently, Meta-heuristics have been considered as one of the most practical approaches to TrNDP solution, because these methods are generally designed for combinatorial optimization problems. They implement efficient mechanisms to reach optimal or near optimal solutions. Examples of Meta-heuristics approaches are Genetic Algorithm (GA), Simulated Annealing (SA), and Swarm Intelligence (SI). Pattnnik et al. generated a set of feasible bus routes through heuristic procedure, then GA was called to select optimal (or near – optimal) bus routes network. Their objective was to minimize the total system cost for users and operators [13]. Zhao defined a neighbor search space around selected routes, called master paths, and then implemented SA for the selection of final solution [14]. Generally speaking, SI is a relatively new approach to TrNDP solving that takes inspiration from the social behaviors of insects in selecting their routes between nest and food. Tracking the behavior of ants in finding paths according to the density of their pheromone is found in [15]. Bee behavior is also tackled in [16].

To the best of our knowledge, there is a rare work on deterministic techniques for TrNDP. This may back to complexity sources mentioned before, besides the well-known deterministic algorithm Travel Salesman Problem (TSP) is devoted to the case of one to many transit demand problem "unlike our case many to many".

In this study, we took the challenge to propose a simple deterministic solution methodology for TrNDP. We decomposed the problem into three sub-problems. Each sub-problem is formed in a deterministic manner. First, an approximate transit network is obtained from the ordinary given street (road) network through a simple linear assignment model. Second, another linear model is used to generate Critical Demand (illustrated in Section II.D) on transit network links from the assigned network. Third, Critical Demand is used in the final step of constructing transit routes to identify transit routes over the transit network through a mathematical formulation resembles the one used in TSP. The output of methodology should reflect the objectives of design and inviolate planning constraints.

The structure of this paper is as follows. Section II gives basic transit concepts to help the reader to get a good grasp of the remainder of this paper. Section III describes the route design stage objectives and constraints. Section IV gives the solution methodology. Section V Mandl benchmark transit problem is used to evaluate the proposed methodology. Section VI is the conclusion.

The nominations used in the paper are presented as follows:

d_{i-j}	the transit demand from (i) to (j) expressed as trips per unit time
t^{r}_{i-j}	minimum in vehicle travel time between (i) and (j) for passengers' demand (d_{i-j}) using route (r), r $\in$ R
t^{s}_{i-j}	travel time between node (i) and node (j) through the shortest path
R	R=(r_1, r_2,……,r_n) a set of bus routes
C_a	the fixed cost on link (a) (time or length)
X_a	flow on link (a) trips per unit time
q^{k}_{i-j}	flow on path k connecting node (i) and (j)
$\delta^{a,k}_{i-j}$	dummy variable, 1 if flow q^{k}_{i-j} pass through link a, 0 otherwise
A	the set of network arcs (links)
θ_{i-j}	model decision variable equal to 1 if directed arc i-j is selected in bus route (r) under construction, 0 otherwise
X_{i-j}	flow value on link i-j
$d(R)$	Transit route network directness indicator
D_o	demand covered directly by the set of routes R
D_{o1}	Demand covered with one transfer by the set of routes R
D_{tot}	total passengers demand within service area to be covered by bus service
TD	network total demand
i, j	origin and destination nodes $\in N$
K	set of available paths between (i) and (j) k= 1,2,……,k_n
S	of all network links combinations that include at least three nodes and perform closed circular route, S = [s_1, s_2, s_3,……s_n]
CD_{i-j}	Critical Demand on directed link i-j
N	network set of nodes
t_{i-j}	arc travel time or length
T^r	bus route (r) time (or length)
T^r_{max}	maximum allowable bus route travel time (or length)

II. Basic Transit Concepts

A. Transit Network Infrastructure Representation

The term "network" in transportation planning is used to describe a structure of streets and intersections (nodes) to be used in Mathematical Programming. Street network and nodes are considered the infrastructure for TrNDP. Two types of graphs are presented in Fig. 1. Undirected graph which arcs are bi-directional is depicted in Fig. 1 (a). A directed graph is a graph in which the arcs have specified directions by arrow heads as shown in Fig. 1 (b). A directed graph representation was adapted in the proposed solution methodology. For a given urban network, it can be defined as G = (N, A), where (N) is the set of nodes (│N│ = n). The set of nodes is connected by the set of arcs (links) (│A│= a).

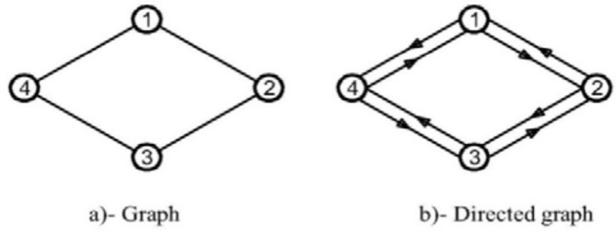

Fig. 1 Graphs and directed graphs

B. Street (Road) Network versus Transit Network

Essential speaking, planners should differ between two terms, namely; street network and transit network. Road network refers to all existing streets and their intersected nodes presented in the studied network. Transit network, as mentioned before, aims to identify sets of bus routes over the road network achieving some transit objectives under some constraints. These sets of connected bus routes constitute a transit network.

C. Route Directness (d(R))

It should be noted, a particular transit network will differ from the original street network from which it is derived, provided some links present in the street network are absent from the transit network. As a consequence, shortest path distances for travelers between the various node pairs will need to be recalculated for each new route set that is evaluated, using a distance or time matrix specific to that new transit route network.

Route directness (d(R)) is an indicator to measure bus route deviation from the shortest path among main transit nodes pairs since; d(R) = 1 indicates that all bus users would take the shortest path along their travel between origin and destination. The value of d(R) which exceeds one, it would indicate the delay caused by the set of bus routes to all users.At the network level, we would evaluate the overall network directness d(R) according (1) taking into account the weight of each transit route demand.

$$d(R) = \frac{\sum_{i \in N} \sum_{j \in N} d_{i-j} \times \frac{t^r_{i-j}}{t^s_{i-j}}}{TD} \quad (1)$$

D. Critical Demand (CD)

In this section, we would define a new notion of Critical Demand (CD) for demand load profile of the transit route. Fig. 2 depicts two possible load profiles of the transit route encompasses five link segments and six terminals (A to F). Case (a) shows that the sustainable demand (CD) on all segments is (28 passenger/hr) which is the lower demand of this transit route. Case (b) reveals the importance of CD in conserving route sustainability. If CD decreased to approach zero in this segment, the route would be split into two routes. The value of the CD is an indicator of all route segments rider-ship.

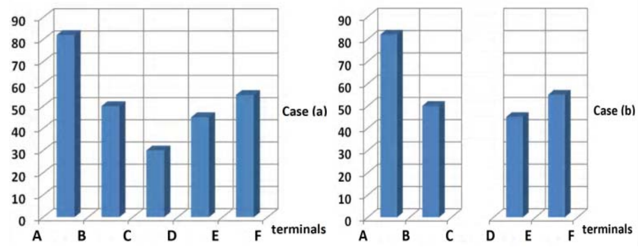

Fig. 2 Transit route demand load profile

In this paper, we would concern of calculating CD for all transit network segments which build all transit network routes. It is considered the criterion for selecting these segments in the constructing route process. For a certain transit route, CD is considered as fingerprint, which is conserved along all segments. We would formulate a linear mathematical model to obtain a CD for transit load profile given in case (a). It may appear too simple to need a formulation, but its usefulness would appear in transit network level, when transit routes aren't constructed yet.

Maximize

$$\sum_i \sum_j CD_{i-j} \quad (2)$$

$$\text{s.t.} \sum_i CD_{i-j}(in) - \sum_m CD_{j-m}(out) = 0 \;\; \forall j \in N \quad (3)$$

$$-X_{i-j} \le CD_{i-j} \le X_{i-j} \;\; \forall i, j \in N \quad (4)$$

Solution;

maximize $CD_{A-B} + CD_{B-C} + CD_{C-D} + CD_{D-E} + CD_{E-F}$

s.t. $CD_{A-B} = CD_{B-C}$; $CD_{B-C} = CD_{C-D}$; $CD_{C-D} = CD_{D-E}$; $CD_{D-E} = CD_{E-F}$

-80≤ CD_{A-B} ≤ 80; -58 ≤ CD_{B-C} ≤ 58; -28≤ CD_{C-D} ≤ 28; -42 ≤ CD_{D-E} ≤ 42; -52≤ CD_{E-F} ≤ 52; this would result in CD = $CD_{A-B} = CD_{B-C} = CD_{C-D} = CD_{D-E} = CD_{E-F}$ = 28 passenger/hr

Note: A negative lower bound on links flow is permitted during the optimization process. This means; if CDi-j takes a negative value, it is an opposite flow direction on that link (i.e. the direction of flow is from (j) to (i)).

III. TRNDP Design Objectives and Constraints

Transit route design is multi-objective problem which constitutes a major obstacle in defining the global optimal solution. We could identify three major objectives to be minimized in TrNDP, namely operator cost, user cost and social cost. The operator cost is measured in terms of total travel time (length) of transit routes which reflects a total operation cost and required fleet size to maintain a certain frequency (level of service). User cost is measured by Transit network directness, "the more directed network, the less user cost". The social cost is minimized by maximizing direct demand coverage (Do) for the given transit network. Increasing Do is a motivation for an increase in transit rider-ship, and consequently a significant decrease in network congestion occurs.

The concerned objective function is staged in two levels, first, in assignment stage a compromise is made between operator and user cost. Passengers are assigned to shortest or kth-shortest paths (user cost), while minimizing the overall network cost (operator cost). Second, in route construction stage, we focused on minimizing social cost (maximizing direct demand coverage) which could be considered the primary concern in the design.

TrNDP constraints refer usually to resource availability and practical applicability. Transit route maximum length is a major constraint in route design stage. There is a constraint on maximum transit route length based on either heuristic guidelines, past experience or common practice accepted by transit planners. Longer bus routes may cause bus driver fatigue and consequently result in safety hazards. Maximum round trip shouldn't exceed 2 hours.

IV. Solution Methodology

The proposed solution methodology doesn't aim to reach the global optimality. It contributes to TrNDP by presenting a simple innovative solution methodology which is capable of producing efficient transit network in systematic deterministic methodology. The output network can stand as an efficient solution for TrNDP or be used as subroutine for other meta-heuristics procedures.

The challenge for conserving the deterministic manner of methodology appeared in route design stage. Three major problems hinder identifying transit routes over the transit network through TSP formulation; these problems are tackled in Section IV.C.

1. Hamilton Graph: TSP stipulates that the graph of network under study contains at least one Hamilton circuit (HC). HC is a path that visits each vertex exactly once, except the start and end are visited twice [17].
2. Sub-Tours: A key part of a TSP is to make sure the tour is continuous (the solution is one tour), that the arcs are linked from the base city all the way to every city visited. Pervious formulations of TSP quite often will get solutions containing detached tours between intermediate nodes and not connected to the base city.
3. One Circular Route: As known, TrNDP aims design set of open bus routes achieving design goals. TSP results in one circular route, so its usage in transit planning is limited to cases of CBD or railway station (Base city) would be connected to other demand zones (one to many) unlike our case of study (many to many). These problems are tackled in Section IV-C.

A. Transit Passenger Assignment

It could be defined as the query of passenger flows on transit network segments. Passenger assignment is a process of predicting passengers' behavior in selecting bus routes according to route time length and bus frequency for each bus route [18]. Transit passengers in many cases have to deal with overlapping bus routes with some routes sharing sections and common stops. This problem is sub-problem of transit passenger assignment, called common-lines problem. Various assumptions and studies are made in order to track passengers' behavior towards a given supply of transit service [19]-[21] Spiess developed the concept of clever passenger into passenger optimal strategies. Passengers would minimize the sum of waiting time and in-vehicle time in their boarding strategies. We would perform assignment step on the street network (instead of the transit network) to obtain an approximate solution for the transit network. Spiess formulation is mainly mixed integer non-linear programming which is difficult (impossible) to be solved exactly for real size networks. We would use only the first term of his model (passenger tendency to minimize in-vehicle time) assuming that bus frequencies are very high on all transit network segments. This assumption would be valid in that case in which the shortest transit line is turned to the attractive one. Assignment model would be easily written as follows:

Minimize

$$\sum_{a \in A} C_a \times X_a \tag{5}$$

$$\text{s.t.} \sum_{k \in K} q_{i-j}^{k} = d_{i-j} \ \forall i, j \in N \tag{6}$$

$$X_a = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \delta_{i-j}^{a,k} \times q_{i-j}^{k} \ \forall a \in A \tag{7}$$

$$X_a \leq L.F \times f_{\max} \times V_s \tag{8}$$

$$q_{i-j}^{k} \geq 0 \ \forall i, j, k \tag{9}$$

$$X_a \geq 0 \ \forall \mathbf{a} \tag{10}$$

K represents the set of shortest paths between node pair (i) and (j), it could be obtained by using Dijkstra's Algorithm and Yen's K-shortest Path [22], [23]. Assignment model would be considered the optimal (efficient) solution for TrNDP, since all trips are assigned to the shortest path or k^{th} shortest path. Shortest path set -for even small network- is a large set; therefore it is not a practical solution to consider all these paths as bus routes.

Our solution methodology depends on constructing high demand coverage routes from the resulted transit network (assigned network). For more illustration, a small instance of street network is given in Fig. 3 (a) with links length in km. and transit O/D matrix is given in Table I. By applying assignment model in (5)-(10), it would give the assigned transit network depicted in Fig. 3 (b).

TABLE I
O/D MATRIX REPRODUCED FROM [24]

OD	1	2	3	4	5	6
1	0	22	35	82	66	100
2	22	0	40	44	95	88
3	35	40	0	33	67	46
4	82	44	33	0	74	51
5	66	95	67	74	0	33
6	100	88	46	51	33	0

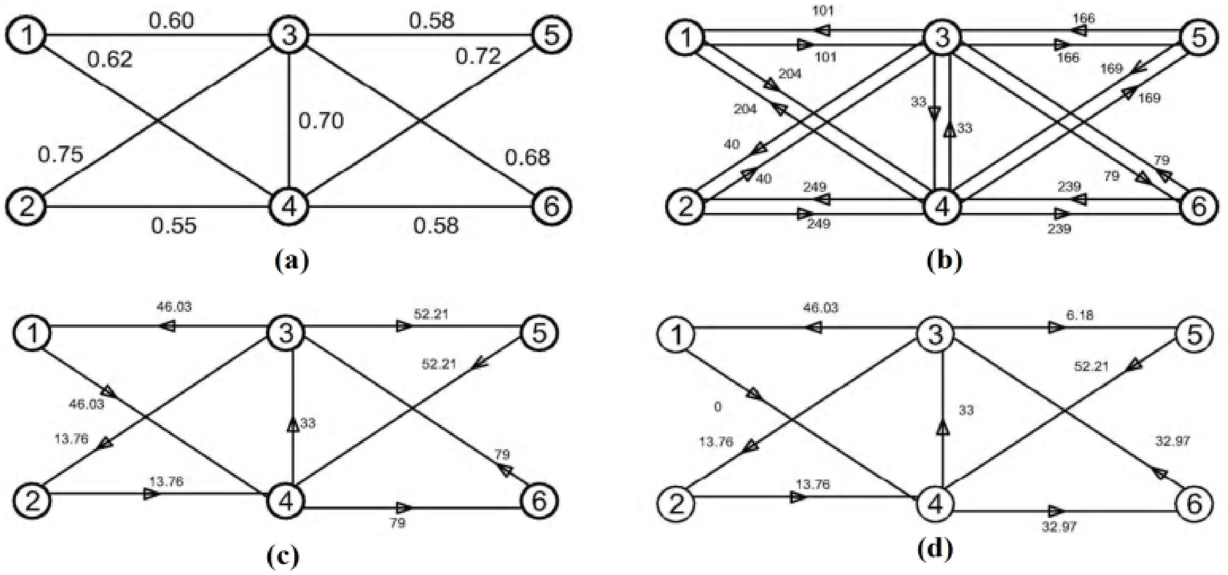

Fig. 3 An example network reproduced from [24]

B. Critical Demand Calculation

Assigned transit network in Fig. 3 (b) denotes aggregate demand load profile of all transit routes, while transit routes haven't been identified yet (sequence of nodes for each transit route). As mentioned before, the CD is considered an indicator for Transit route sustainability. We would exploit (2)-(4) to generate aggregate CD on the transit network links. In the real world, buses go in a directed path from its start terminal traversing other nodes in its route (path) till it reaches its end terminal. So, it is worth noting that the proposed formulation of CD helps in finding the best directional system of transit network, see Fig. 3 (c) (Note: All network links in this example are bi-direction, so there is an opposite system of directions with the same value of CD on links). This would aid at route construction stage. Transit route effectiveness would be defined by the summation of CD values over its links. Higher summation value is an indicator of higher route overall rider-ship.

C. Route Construction Procedure

To conserve the deterministic manner of proposed solution methodology, we would use a linear integer formulation for transit route construction, which can be solved exactly up to reasonable size with branch and bound algorithm. There are also efficient stochastic techniques for larger sizes. Integer programming algorithm presented in (11) resembles TSP formulation, except it aims to find the route with the highest CD summation (the most efficient transit route).

Maximize

$$\sum_{i\in N}\sum_{j\in N}\theta_{i-j}\times CD_{i-j} \quad (11)$$

$$s.t. \sum_{i}\sum_{j}\theta_{i-j}\times t_{i-j}\le T^{r}_{max} \quad (12)$$

$$\sum_{i}\theta_{i-j}=\sum_{m}\theta_{j-m} \quad \forall j\in N \quad (13)$$

$$\sum_{i}\theta_{i-j}+\sum_{m}\theta_{j-m}\le 2 \ \forall j\in N \quad (14)$$

$$\sum_{i}\theta_{i-f}+\sum_{m}\theta_{f-m}=2 \quad (15)$$

$$S=[s_1, s_2, s_3, \ldots\ldots s_n] \ s_n\subset N, |s_n|\ge 3 \quad (16)$$

$$\sum_{i\in s_n, j\in s_n}\theta_{i-j}\le |s_n|-1 \qquad \forall s_n\in S \quad (17)$$

Inequality (12) asserts that bus round trip travel time won't exceed its maximum allowable time or length, (13) provides a connected bus route [25].Inequality (14) assures that every node will be visited once at most. Equation (15) asserts that bus route under-construction must visit a node (*f*). Node (*f*) is a fictitious node connected to all network nodes with imaginary links, as depicted in Fig. 4. CD values on these imaginary links equal to zero. The only rule for this fictitious node *f* is to create open bus routes.

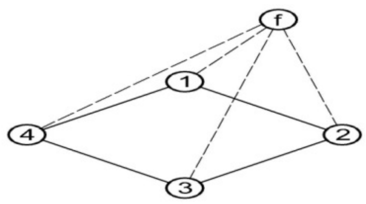

Fig.4 Transit network connected with a fictitious node (f)

Equation (16) defines S as the set of all network links combinations that include at least three nodes and perform closed circular route [26]. Equation (17) prevents the possible existence of any group of circular bus routes except the one passing the fictitious node (*f*). Note that S set is easily defined here since the network is directed (i.e. nodes combination with their links directions should coincide to perform circular route, see for instance route 2-4-5-3-2 Fig. 3 (c), this combination doesn't perform a route and thus doesn't require a constraint to be prevented.

Another way to define the set S may be as follows; 1- Apply (11)-(17) before adding the fictitious node to the network. 2- This would result in circular or combination of detached circular routes. 3- The resultant circular (or circulars) will be defined to the set S and be prevented by applying (22). 4- Apply again (11)-(17), this may result in another circular routes subset of S. 5-This process would be repeated till all S set are defined and there isn't any internal closed circular route can be formed.

Applying route construction model on network Fig. 3 (c) would result transit route (1-4-6-3-5) with ∑CD = 256.24. After first route construction, we would update CD for network links by subtracting the smallest value of the CD on the constructed bus route (CD = 46.03, link 1-4), from all route links. This value is the fingerprint of the route. We would obtain updated network in Fig. 3 (d). This subtraction is done to eliminate the possibility of selecting this route in the next route construction. Equations (11)-(17) are used to construct bus routes iteratively and this gives the operator flexibility to discard any unprofitable route from the solution of TrNDP.

By following the same procedure, we could construct the second bus route. This process can continue till a number of routes are constructed satisfying TrNDP objectives or no improving in route network design parameters is made. The transit route enumeration process gives the operator a possibility to choose the number of bus routes which satisfy his/her planning requirements and budgetary constraints. A summary of bus routes is given in Table II. Although the route number (2) doesn't increase network demand coverage, it improves route network directness (i.e. route number 2 approval or exclusion returns to the operator).

TABLE II
BUS ROUTES ENUMERATION

Route no.	[a]Route node sequence	[b]Route length in km	∑CD	CD	-0- transfer trips (%)	Total demand covered (%)
1	1-4-6-3-5	2.46	256.2	46.03	67	67
2	1-3-4-5	2.02	[c]131.2	33	67	67
3	2-4-6-3-5	2.17	85.88	6.18	97.48	100

[a]In this example nodes (1,2,5,6) are bus route start and end terminals [24]
[b]Route maximum length constraint equals 2.5 km [24]
[c]Summation of CD is calculated from updating network seeFig. 3 (c)

D. TrNDP Solution Methodology Structure

The proposed solution methodology for TrNDP can be concluded in these steps:

Step1. Construct a coded street network.
Step2. Identify travel time (or length) on links.
Step3. Assign nodes transit trips using model (5-10), reaching assigned Transit – network.
Step4. Assume arbitrary directed graph arrows, i.e. only one direction is chosen arbitrarily for each bi-directional link (N.B the algorithm adjusts the direction later in the process of reaching CD values on links).
Step5. Optimize the assigned Transit – network (2-4), reaching directed transit – network with CD values on links.
Step6. Construct first bus route (11)-(17), covering the highest CD summation, set r = 1.
Step7. Subtract the least value of CD on the constructed route from the route links along its path to get updated network with new CD values.
Step8. Compute the total demand satisfied by the set of *R*th routes (using -0- and -1- transfer), if that demand ≥ D_{tot}^{min} (the minimum total demand needed to be satisfied) terminate route construction process and output set R routes; otherwise, go to step 6.

V. COMPUTATIONAL CASE STUDY

A. Mandl's Benchmark Transit Network [27]

In order to demonstrate the effectiveness of the solution methodology proposed in this paper, a popular benchmark network is solved (Mandl's Swiss transit network). Mandl's Swiss transit network is the most popular transit network that has been utilized by many researchers as a benchmark network to compare their results with Mandl's solution results [4], [6], [9]-[12], [14], [27]-[30]. Mandl's transit network consists of 15 nodes connected by 21 links with a total demand of 15570 trips; see Fig. 5 [27].

B. Solution Procedure for Mandl's Transit Network [27]

The models presented in this paper are mainly linear and integer programming models. So, they can be solved with their standard solvers. We continued to construct bus routes for Mandl's networks [27] until no improvement in the network direct demand coverage.

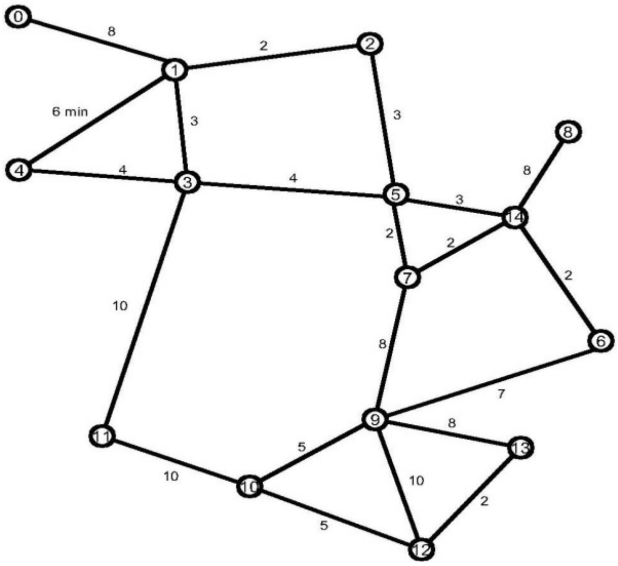

Fig. 5 Coded network with travel time on links in minutes [27]

C. Results and Discussions

The Transit route design resulted in six bus routes. A summary of the constructed routes is given in Table III.

TABLE III
CONSTRUCTED ROUTES SUMMARY

Route No.	Route node sequence	length	$\sum CD$	$\%\sum D_o$	%Total Demand satisfied
1	3-4-1-2-5-14-6-9-7	35 min	2955.36	50.16	50.16
2	6-14-5-7-9-12-10-11	40 min	2132.04	73.15	87.63
3	2-1-4-3-5-7-9-6-14	35 min	1501.72	73.15	87.63
4	8-14-5-7-9-12-13	33 min	982.36	78.87	91.52
5	0-1-2-5-14-6-9-10-12	35 min	790.00	93.12	100
6	0-1-2-5-3-11	27 min	399.16	96.01	100

Table IV presents a comparison between this study analysis results and highlighted previous work that tackled Mandl's transit network [27] in their solution as a benchmark problem. The first row indicates the source of the solutions to the benchmark problem. The second row gives the previous work year. The third row identifies the solutions search tool methods to the problem. Fourth to twelve rows denote the route network characteristics (planning parameters).

The comparison of Table IV shows with the proposed methodology provides a significantly better solution than others, particularly in terms of percentage of direct demand coverage, which makes the transit network more attractive to the users. This study proposes a less number of routes than other studies (except Mandl [27]) in the process of seeking more direct routes.

We considered the number of routes generated -in regard with other parameters of comparison- the most important criteria in judging the efficiency of any of these search methods. Less number of routes demonstrates the methodology strength in assembling candidate links in one continuous bus route and entails – accordingly – less amount computational effort ofthe bus and driver scheduling part. TableV presents a comparison of this study result and other studies which produced the same number of routes for Mandl's network [27].

TABLE IV
COMPARISON OF APPROACHES FOR MANDL'S BENCHMARK NETWORK PROBLEM [27]

Problem source	Mandl [27]		Shih and Mahmassani [11]		Bagloee & Ceder [30]	
Year	1980		1994		2011	
Search method	Mandl[1]	This study	S&M[2]	This study	GI& AS[3]	This study
Number of routes	4	6	8	6	12	6
-0- transfer trips % (D_o)	69.94	96.01	87.73	96.01	83.66	96.01
One transfer trips% (D_{o1})	29.93	3.99	12.27	3.99	15.21	3.99
Two transfer trips% (D_{o2})	0.13	0	0	0	0.95	0
Total Demand satisfied (D_{tot})	100	100	100	100	99.82	100
Network directness $d(R)$	1.05	1.15	1.03	1.15	RNR[4]	1.15
Transfer directness[5]	1.3	1.05	1.12	1.05	RNR	1.05
Total route time length (min)	82	205	151	205	261	205
Direct trips / route length (pass./min)	132.8	74.35	90	74.35	49.9	74.35

1Mandl's method [27]
2 Shih and Mahmassani's [11]
3Gravity Index & Ant [30]
4Results not Reported
5Calculated as d(R) but for transfer trips only

TABLE V
COMPARISON OF APPROACHES WITH THE SAME NUMBER OF ROUTES

Problem source	Baaj and Mahmassani [9]		Chakroborty [4]		Zhao [14]		Fan &Mumford [28]	
Year	1991		2003		2006		2008	
Search method	B&M[1]	This study	GA[2]	This study	SAFD[3]	This study	SA[4]	This study
Number of routes	6	6	6	6	6	6	6	6
-0- transfer trips %(D_o)	78.61	96.01	86.04	96.01	94.03	96.01	92.48	96.01
-1- transfer trips% (D_{o1})	21.39	3.99	13.96	3.99	5.97	3.99	7.52	3.99
Total route length (min)	122	205	187	205	RNR	205	RNR	205

1 Baaj and Mahmassani's [9] 2 Genetic Algorithm [4] 3 Simulated annealing and FD [14] 4 Simulated Annealing [28]

VI. CONCLUSION

Providing transport community with simple and effective transit route design technique is the main purpose of this paper. Solution methodology depending only on linear and integer operators was presented to solve Transit route Network design problem. The proposed solution methodology goes beyond pervious traditional heuristics and Meta – heuristics (approximate) approaches. It is highly depended on demand matrix. It is a generic method since it confirms to several network routes configurations. It doesn't bias towards any existing transit network. It is flexible; since planner can classify generated bus routes according to demand coverage, which enables the operator to execute selected routes according to available existing resources. The key indicator of good transit planning is to achieve maximum direct demand coverage with keeping the good value of other network parameters. This would encourage more people to select public transport and consequently achieving one of the most important goals of transit planning. In future work, meta-heuristic would be adopted to improve results to approach global optimal solution.

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