Bifurcation Study and Parameter Analyses Boost Converter

S. Ben Said, K. Ben Saad, M. Benrejeb

Abstract—This paper deals with bifurcation analyses in current programmed DC/DC Boost converter and exhibition of chaotic behavior. This phenomenon occurs due to variation of a set of the studied circuit parameters (input voltage and a reference current). Two different types of bifurcation paths have been observed as part as part of another bifurcation arising from variation of suitable chosen parameter.

Keywords—Bifurcation, Chaos, Boost converter, Current-programmed control, Initial parameters.

I. INTRODUCTION

THE last past decades, current programmed DC/DC Boost converters have been subject to intensive studies and research in nonlinear control theory [1]. More recently, it was shown that these converters can exhibit a chaotic behavior. Anterior surveys have intended that power converters may operate in different period regime from one to another as system parameters are varying [3]. Such behavior is called bifurcation. It was shown in [4] that, for Buck and Boost converters, controlled in voltage mode, it exists doubling period and hopf bifurcation. Deane and Hamill studied the theory of chaos phenomenon that happened in current-mode controlled Boost converters in Continual Conduction Mode (CCM) [5]; Banerjee discussed the chaotic phenomenon of the PWM Buck converter in CCMs [2]. Then, power converters are known by another type of bifurcation that border collision which interrupted all standard bifurcations illustrated with defeat of stability [10], no structural change and standard appearance bifurcation diagrams. In literature, authors considerate chaos a non student behavior. In [6], it has been shown that a route to chaos is possible via quasi-periodic orbits and period-doubling are possible. Essentially, by varying I_{ref} we would observe the circuit changes its qualitative behavior from a fundamental stable system to a chaotic system. We submit to as the primary bifurcation parameter which is I_{ref}.

In this paper, our objective is to study bifurcation and possible path ways from which a DC/DC convert may route to chaos via quasi-periodic orbits and period doubling. This period-Doubling phenomenon was observed long ago.

Salwa Ben Said, Kamel Ben Saad, and Mohamed Benrejeb are with LA.R.A Automatique, National School of Engineering of Tunis, Tunis, Tunisia, (e-mail: bensaid.salwa@yahoo.fr, bensaad.kamel@enit.rnu.tn, benrejebmohamed@enit.rnu.tn).

However, there has been no attempt to find out the condition that determines the type of the route to chaos for a given set of initial parameters [7]. So, our main result in this paper addresses this important issue.

The reminder of this paper is organized a follows. Section II displays the mathematical model of the DC/DC Boost converter. In Section III is presented the main idea. Section IV is devoted to present bifurcation charts. Finally some conclusions are given in Section V.

II. BOOST CONVERTER DESCRIPTION

The current-mode controlled Boost converter, given in Fig. 1, is composed by an inductor L, a switch SW, a diode D, a capacitor C and a load resistor R connected in parallel with the capacitor C. Switch SW is controlled by a feedback path constituted by an RS trigger and a comparator. The inductance current compared to a reference current I_{ref} is chosen as the programming variable, generating the on-off driving signal for the switch SW.
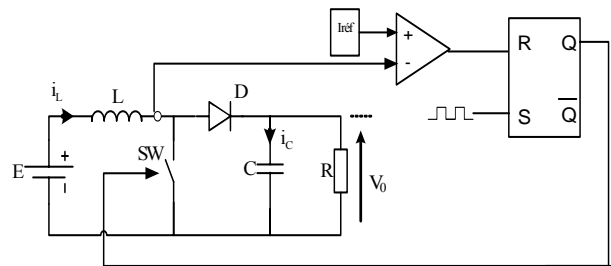

Fig. 1 Schematic diagram of the Boost converter under current-mode control

It's necessary to assume that the Boost converter operates in continuous current mode, where the inductance and switch period T are chosen such that the inductor current never falls to zero. Hence, there are two switch states toggles periodically according to whether SW is closed or open. The circuit takes the first switch states at t=nT, becomes closed at the beginning of each cycle. The inductor current rises linearly until $i_L=I_{ref}$. The second switch state arrived when $iL= I_{ref}$, open and remains open until the arrival of the next clock pulse, where it is closed again and the diode D conducts. The Boost converter is then described by a pair of coupled first-order differential equations. A clock pulse makes switch SW closed again.

When switch SW is closed, the mathematical model is given by:

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:8, No:1, 2014

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}E \\ \dfrac{dV_0}{dt} = -\dfrac{1}{RC}V_0 \end{cases} \tag{1}$$

When switch SW is opened, the model is described by:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}\left(E - V_0\right) \\ \dfrac{dV_0}{dt} = \dfrac{1}{C}\left(i_L - \dfrac{V_0}{R}\right) \end{cases} \tag{2}$$

Then, the mathematical model of the converter be rewritten in the following state space form:

$$\begin{cases} \dot{X} = AX + BE \\ V_0 = CX \end{cases} \tag{3}$$

where:

$$A = \begin{bmatrix} 0 & -\dfrac{1-d}{L} \\ \dfrac{1-d}{C} & -\dfrac{1}{RC} \end{bmatrix} \tag{4}$$

The control matrix B is defined by

$$B = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \tag{5}$$

and

$X = \begin{bmatrix} i_L & V_0 \end{bmatrix}^T$ the space vector and d is equal to 0 or 1,

III. BASIC IDEA

The bifurcation diagram is the most powerful tool to investigate the nonlinear phenomena. In a bifurcation diagram [8], a periodic steady state of the system is represented as a set of points equal to the periodicity of the system for a fixed parameter. For chaos, numerous points are plotted in the diagram because chaos means infinity periods and the points never fall at the same position. Therefore, in such a bifurcation diagram, the change of behavior of a system is clearly shown as a parameter is varied [9]. Variable, by comparing with a reference current, generates the on/off driving signal for switch. Specifically, switch is turned on at the beginning of the cycle, i.e. We assume that the circuit takes switch state one for $nT < t < (n+d)T$, and switch state two for $(n+d)T < \le t < (n+1)T$, where n is an integer, is d the duty cycle, and T is the period.

In this section, we study the bifurcation and chaos in the Boost converter with numerical method. We use Matlab to show chaos and bifurcation phenomena and to plot phase portrait, section Poincare bifurcation diagrams. Among them, bifurcation refers to sudden change of qualitative behavior of a dynamical system when parameters varied.

IV. BIFURCATION CHARTS WITH THE REFERENCE CURRENT AS THE CONTROL PARAMETER

The parameters of converter are given by Table I.

TABLE I
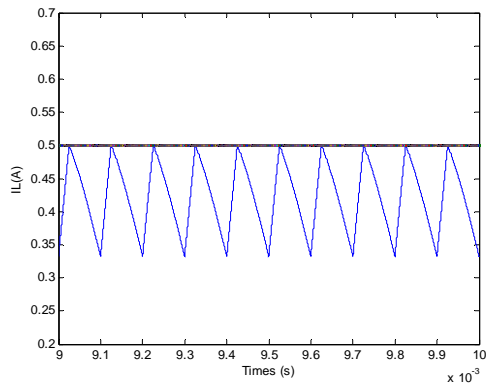PARAMETERS OF SIMULATION FOR THE BOOST CONVERTER

Parameters	Values
L	1.5 mH
R	50Ω
C	5μF
T	100μs
E	10V

As the bifurcation parameters, the reference current Iref is increasing gradually and examine the steady-state waveforms of the inductance i_L, firstly, and the output voltage V_0, secondly. Fig. 2 represents the current mode controlled Boost converter. In this figure, are given the when using a current mode regulator applied to the converter. The waveform of inductor currents are shown in Figs. 2 (a), (b), (c) for the values current I_{ref} 0.5A resp 1A, 1.5A.
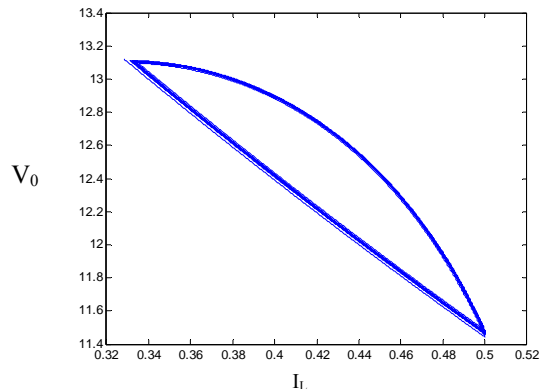
It comes that for I_{ref}=0.5A, Fig. 2 (a) shows the stable period-1 and periodic nature of the system, for I_{ref}=1, Fig. 2 (b) illustrate stable period-2 and for I_{ref}=1.5A Fig. 2 (c) the chaotic behavior under reference-current variation. Hence, the converter goes to chaos via period-doubling.

The current phase portraits shown in Fig. 3 correspond to current voltage phase, Fig. 2. It comes that for I_{ref}=0.5A stable period1, for I_{ref}=1A stable period 2 and for I_{ref}=1.5A chaotic phenomena. Then the converter presents a bifurcation periodic doubling.

In other terms, Figs. 3 and 4 show the bifurcation diagrams of the DC/DC converter. Such that, the circuit goes throuth stable 1 T orbits, stable 2 T orbits, quasi-4T orbits and finally exhibits chaos.
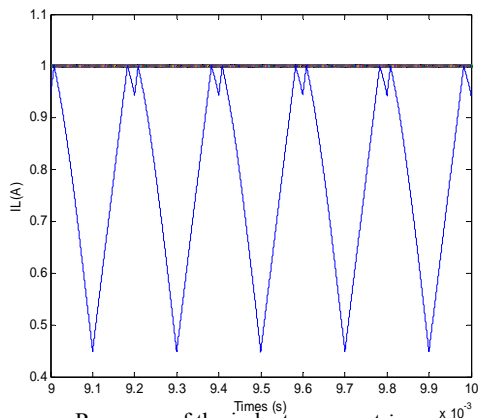
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International Journal of Electrical and Computer Engineering
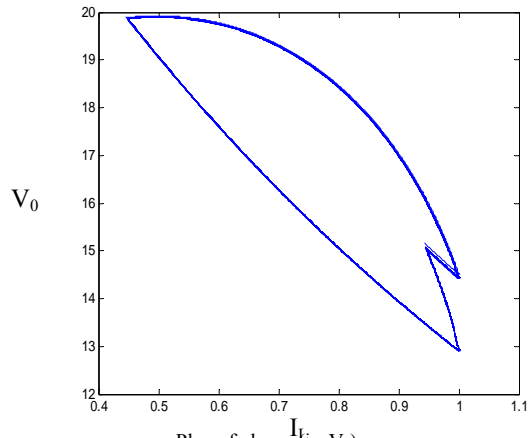Vol:8, No:1, 2014

Reponses of the inductor current i_L

Plan of phase (i_L, V_0)

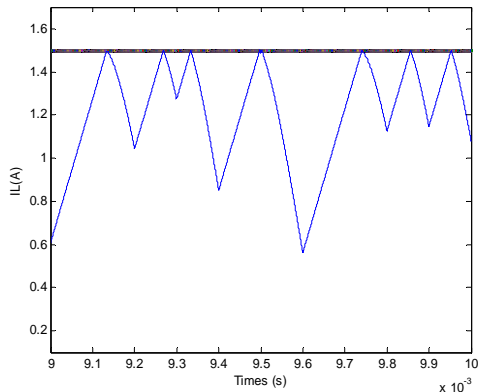(a) Stable period-1 operation for I_{ref}= 0.5A
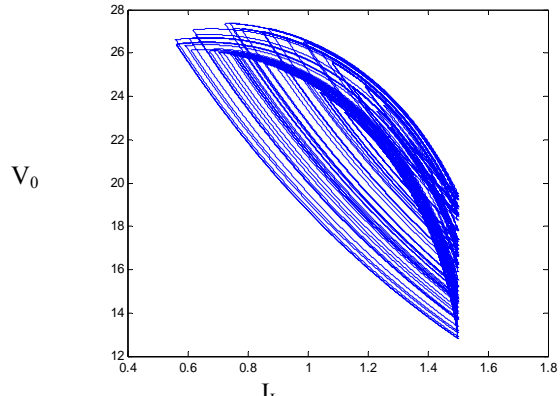
Reponses of the inductor current i_L

Plan of phase (i_L, V_0)

(b) Stable period-2 operation for I_{ref}= 1A

Reponses of the inductor current i_L

Plan of phase (i_L, V_0)

(c) Chaotic operation for I_{ref}= 1.5A

Fig. 2 Current-mode controlled Boost converter. i_L responses inductor current

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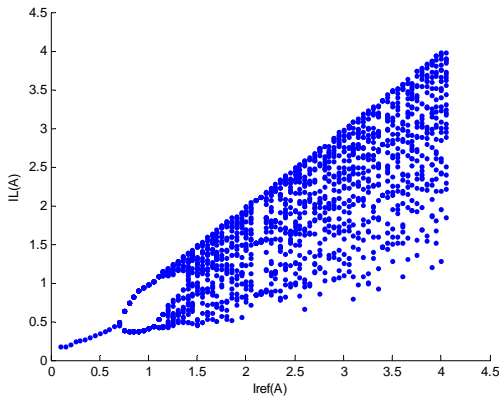International Journal of Electrical and Computer Engineering
Vol:8, No:1, 2014

Fig. 3 Bifurcation diagrams from a current-mode controlled Boost converter

I_{ref} denotes the reference peak inductor current and the inductor current value at the turn-off instant of the switch.

## V. INFLUENCE OF R, C AND E FOR THE BIFURCATION TYPE

We consider a secondary bifurcation parameter that initial parameter which is defined as the condition initial of the circuit Boost. In the first a resistor parameter in the second is a capacitor and finally the input voltage.

The standard bifurcation involves no structural change, where as border collision must involve a structural change.

In this section, is considered study of these bifurcation and, in particular, we will investigate how the choice of parameters of converter and their values can determine the type of behavior of a current-programmed boost converter, the bifurcation influence of reference current I_{ref}, resistor R, capacitor C and input voltage E.

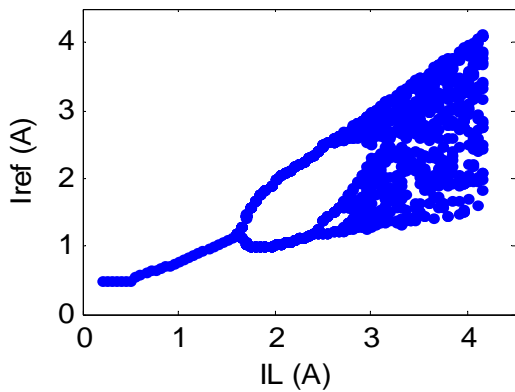The corresponding bifurcation diagrams are given in Figs. 4-6 for:

- $R \in [40\Omega, 100\Omega]$ for fixed E and C, E=10V and C=5µF.
- $C \in [5\mu F, 50\mu F]$ for fixed E and R, E=10V and R=50Ω.
- $E \in [5V, 10V]$ for fixed C and R, C=5µF and R=50Ω.
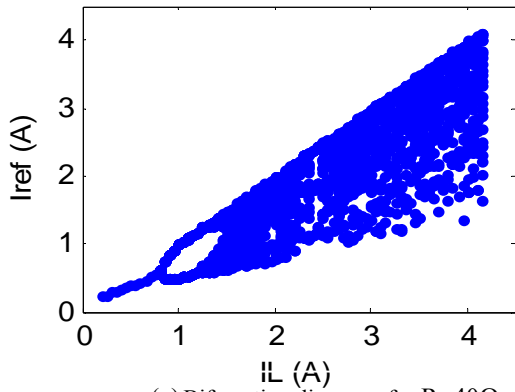
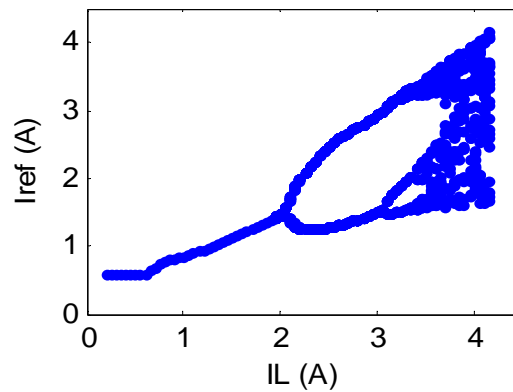
(a) Bifurcation diagrams for R=40Ω
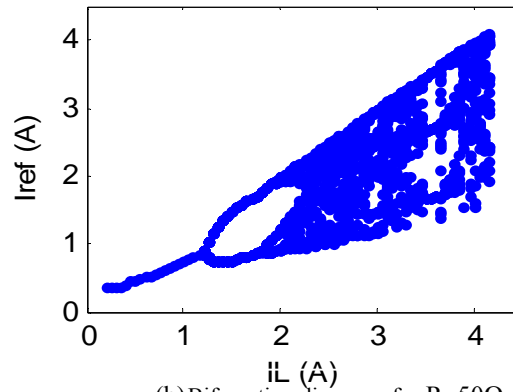
(b) Bifurcation diagrams for R=50Ω

(c) Bifurcation diagrams for R=70Ω

(d) Bifurcation diagrams for R=100Ω

Fig. 4 Period-doubling cascade interrupted by a border collision Dimensionless parameter that is relevant to this bifurcation is R. From (a) to (d), the resistor gradually changes from a small value to a large value

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:8, No:1, 2014

(a) Diagram of bifurcation for C=5μF

(b) Diagram of bifurcation for C=10μF

(c) Diagram of bifurcation for C=20μF

(d) Diagram of bifurcation for C=50μF

Fig. 5 Period-doubling cascade interrupted by a border collision Dimensionless parameter that is relevant to this bifurcation is C. From (a) to (d), the capacitor gradually changes from a small value to a large value

(d) Bifurcation diagrams for E=5V

(c) Bifurcation diagrams for E=7V

(b) Bifurcation diagrams for E=10V

(a) Bifurcation diagrams for E=15V

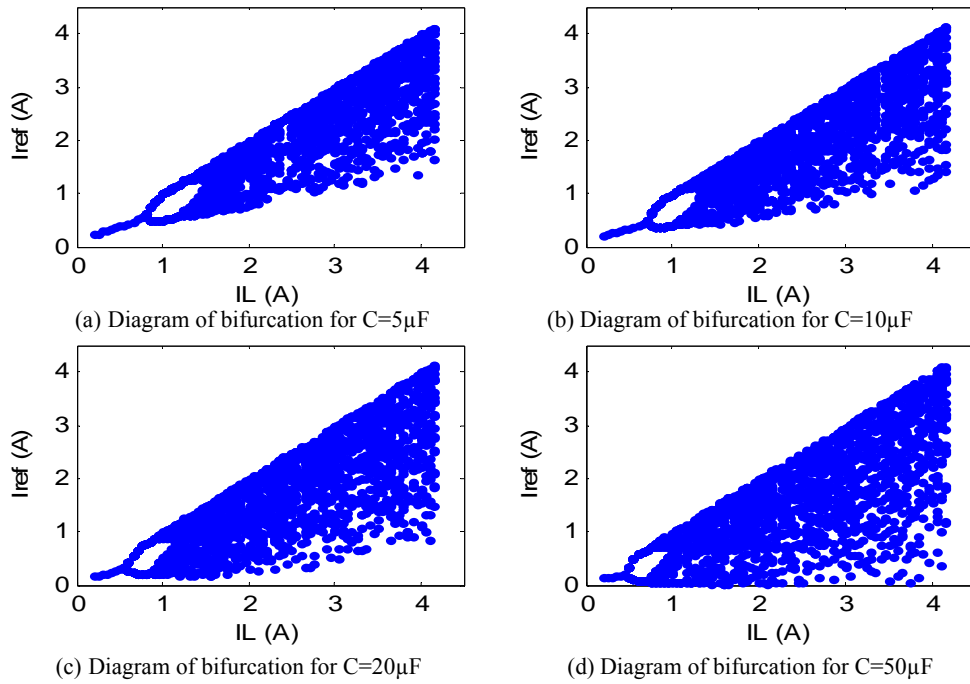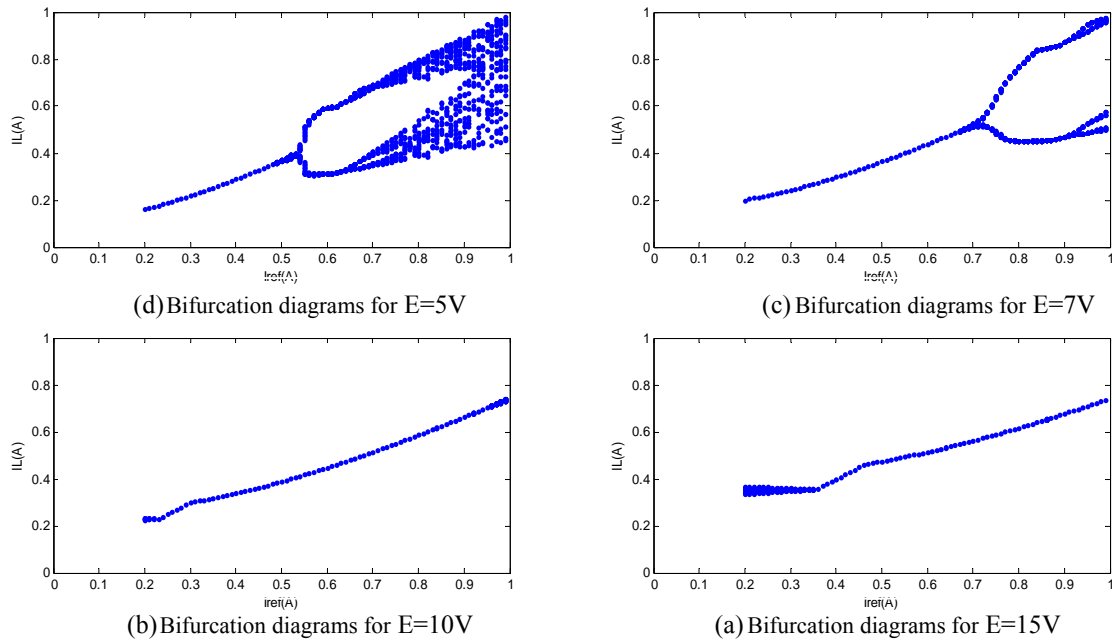Fig. 6 Period-doubling cascade interrupted by a border collision Dimensionless parameter that is relevant to this bifurcation is E. From (a) to (d), the input voltage gradually changes from a small value to a large value

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:8, No:1, 2014

The analysis of the overall diagrams shows that all considered parameters influence bifurcation and chaos characteristics. The bifurcation phenomena then the chaotic one starts as soon as the resistance value are small. As shown in Figs. 4 and 5. For the highest value of input voltage, E=10V, only the stable periodic orbit is observed. When input voltage decreases, the bifurcation and chaotic phenomena become more and more dominant.

Our methodology consists on the modification of circuit parameters and observing the phase portrait of the system (converter). Three cases are given by Figs. 4-6 that correspond to the bifurcation diagram of the converter.

From Fig. 4, it's clearly seen that the stability response of system depends on the resistor value. However, the behavior of the converter remains always chaotic for such parameters.

Fig. 5 corresponds to the capacity system changes. In this figure, are shown bifurcation diagram, as in previous case, when valey resistor is varying, we can see in Fig. 5 that capacity variations do not influence on the stability system. The converter exhibits the some behavior (chaotic) for sum valey of these parameters (capacity).

Fig. 6 shows that a variation on the input voltage allows the system to track a stable periodic behavior and then avoid chaos. For different values of the input voltage, we show that the system can exhibit a behavior from chaos to stable periodic orbits as shown in Fig. 6.

VI. CONCLUSION

Stable periodic orbit, bifurcation and chaotic phenomena identification for the studied Boost converters DC/DC in current mode controlled, is the main contribution of the present paper. Input voltage and resistor R, Capacity C and inductor L influence behavior of these converters. We have demonstrated that there exist two types of path way to chaos via regions of period doubling or quasi- periodicity. Those bifurcation paths are viewed as part of another regardless to a suitable parameter variation. The proposed contribution constitutes the first step of the order controller Boost converter design.

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Salwa Ben Said was born in Tunisia in 1984. She graduated in 2007 from « Ecole Supérieure de technologie et d'informatique. She received the Master Degree in Automatic and signal processing in 2009 from "National engineering school of Tunis (ENIT)". She is currently a PhD student in ENIT. Her research interests are in the area of advanced powers converters control design.

Kamel Ben Saad was born in 1976 in Tunis, Tunisia. He graduated in 2000 from National engineering school of Tunis (ENIT). He received the master degree also from ENIT in 2001. He received the PhD degree in Electrical Engineering in 2005 from Ecole Centrale de Lille and Ecole Nationale d'Ingnieurs de Tunis. He is currently senior lecturer at the ENIT. His research interests are in the area of machines, classical and intelligent, control.

Mohamed Benrejeb was born in Tunisia in 1950. He obtained the Diploma of Ingenieur IDN (French Grande Ecole) in 1973, The Master degree of Automatic Control in 1974, the PhD in Automatic Control of the University of Lille in 1976 and the DSc of the same University in 1980. Full Professor at Ecole Nationale d'Ingenieurs de Tunis since 1985 and at Ecole Centrale de Lille since 2003, his research interests are in the area of analysis and synthesis of complex systems based on classical and non conventional approaches.