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Stability Criteria for Uncertainty Markovian Jumping Parameters of BAM Neural Networks with Leakage and Discrete Delays

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Abstract—In this paper, the problem of stability criteria for Markovian jumping BAM neural networks with leakage and discrete delays has been investigated. Some new sufficient condition are derived based on a novel Lyapunov-Krasovskii functional approach. These new criteria based on delay partitioning idea are proved to be less conservative because free-weighting matrices method and a convex optimization approach are considered. Finally, one numerical example is given to illustrate the the usefulness and feasibility of the proposed main results.

Keywords—Stability, Markovian jumping neural networks, Time-varying delays, Linear matrix inequality.

I. Introduction

BIDIRECTIONAL associative memory (BAM) neural networks have been extensively studied in recent years due to its wide application in various areas such as image processing,pattern recognition, automatic control, associative memory,optimization problems, and so on.BAM neural network is composed of neurons arranged in two layers: the x-layer and y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer. Now, many sufficient conditions ensuring stability BAM neural networks have been derived, see, for example, [1-19] and references cited therein.

On the other hand, systems with Marvokian jumps have been attracting increasing research attention. The Marvokian jump systems have the advantage of modeling the dynamic systems subject to abrupt variation in their structures, such as operating in different points of a nonlinear plant [16]. Recently, there has been a growing interest in the study of neural networks with Marvokian jumping parameters [20-28]. In [20], the problem of stochastic stability criteria for BAM neural networks with Marvokian jumping parameters are investigated based on partitioning idea. In addition, the authors in [25] discussed the problem of BAM neural networks with constant delays in the leakage term. Moreover, Peng [26], investigated global attractive periodic solution of BAM neural networks with continuously distributed delays in the leakage terms. To the best of our knowledge, the stability analysis for Markovian jumping BAM neural networks with leakage and discrete delays has never been tackled, and such a situation motivates our present study.

In this paper, the stability analysis for Markovian jumping BAM neural networks with leakage and discrete delays is considered. Some new delay-dependent stability criteria for Markovian jumping BAM neural networks with leakage and discrete delays will be proposed by dividing the delay interval into multiple segments, and constructing new Lyapunov-Krasovskii functional. The obtained criterion are less conservative because free-weighting matrices method and a convex optimization approach are considered. Finally, one numerical example is given to illustrate the the usefulness and feasibility of the proposed main results.

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II. Problem statement

Consider the following BAM neural networks with leakage and discrete delays:

$$\begin{cases} \dot{x}_p(t) = -Ax_p(t-\sigma) + C\tilde{f}(y_q(t)) + E\tilde{f}(y_q(t-h(t))) + I_p \\ \dot{y}_q(t) = -By_q(t-\delta) + D\tilde{g}(x_p(t)) + F\tilde{g}(x_p(t-\varsigma(t))) + J_q \end{cases} \tag{1}$$

where $x_p(t) = [x_{p1}(t), x_{p2}(t), \ldots, x_{pn}(t)]^T \in R^n$ and $y_q(t) = [y_{q1}(t), y_{q2}(t), \ldots, y_{qn}(t)]^T \in R^n$ denote the state vectors; $\tilde{g}(x_p(\cdot)) = [\tilde{g}_1(x_{p1}(\cdot)), \tilde{g}_2(x_{p2}(\cdot)), \ldots, \tilde{g}_n(x_{pn}(\cdot))]^T \in R^n$ and $\tilde{f}(y_q(\cdot)) = [\tilde{f}_1(y_{q1}(\cdot)), \tilde{f}_2(y_{q2}(\cdot)), \ldots, \tilde{f}_n(y_{qn}(\cdot))]^T \in R^n$ are the neuron activation function;$A = diag\{a_i\} \in R^n$ and $B = diag\{b_i\} \in R^n$ are positive diagonal matrices;C and D are the connection weight matrices,E and F are the delayed connection weight matrices; $I_p = [I_{p1}, I_{p2}, \ldots, I_{pn}]^T \in R^n$ and $J_q = [J_{q1}, J_{q2}, \ldots, J_{qn}]^T \in R^n$ are the constant input vector; σ and δ are the leakage delays satisfying $\sigma \geq 0$ and $\delta \geq 0$, respectively.

The following assumptions are adopted throughout the paper.
Assumption 1: The delay $h(t)$ and $\varsigma(t)$ are time-varying continuous functions and satisfies:

$$0 \leq \varsigma(t) \leq \varsigma, \dot{\varsigma}(t) \leq \varsigma_D < 1, 0 \leq h(t) \leq h, \dot{h}(t) \leq h_D < 1 \tag{2}$$

where $\varsigma, h, \varsigma_D$ and h_D are constants.
Assumption 2: Neuron activation function $g_i(\cdot), f_i(\cdot)$ in (1)

satisfies the following condition:

$$l_{1i}^{-} \le \frac{\tilde{f}_i(\alpha)-\tilde{f}_i(\beta)}{\alpha-\beta} \le l_{1i}^{+}, \tilde{f}_i(0)=0$$
$$l_{2i}^{-} \le \frac{\tilde{g}_i(\alpha)-\tilde{g}_i(\beta)}{\alpha-\beta} \le l_{2i}^{+}, \tilde{g}_i(0)=0 \qquad (3)$$

for all $\alpha,\beta \in R$,$\alpha \ne \beta$,$i=1,2,\dots,n$.
Based on this assumption, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Let $x_p^*=[x_{p1}^*,x_{p2}^*,\dots,x_{pn}^*]^T$, $y_q^*=[y_{q1}^*,y_{q2}^*,\dots,y_{qn}^*]^T$ is the equilibrium point of (1) and using the transformation $x(\cdot)=x_p(\cdot)-x_p^*$,$y(\cdot)=y_q(\cdot)-y_q^*$, system (1) can be converted to the following system :

$$\begin{cases} \dot{x}(t)=-Ax(t-\sigma)+Cf(y(t))+Ef(y(t-h(t))) \\ \dot{y}(t)=-By(t-\delta)+Dg(x(t))+Fg(x(t-\varsigma(t))) \end{cases} \qquad (4)$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T$, $y(t)=[y_1(t),y_2(t),\dots,y_n(t)]^T$, $g(x(\cdot))=[g_1(x_1(\cdot)),g_2(x_2(\cdot)),\dots,g_n(x_n(\cdot))]^T$, $f(y(\cdot))=[f_1(y_1(\cdot)),f_2(y_2(\cdot)),\dots,f_n(y_n(\cdot))]^T$, $f_i(y_i(\cdot))=\tilde{f}_i(y_i(\cdot)+y_{qi}^*)-\tilde{f}_i(y_{qi}^*)$,and $g_i(x_i(\cdot))=\tilde{g}_i(x_i(\cdot)+x_{pi}^*)-\tilde{g}_i(x_{pi}^*)$,$i=1,2,\dots,n$.
From inequalities (3) and (4),one can obtain that:

$$l_{1i}^{-} \le \frac{f_i(\alpha)}{\alpha} \le l_{1i}^{+}, f_i(0)=0,$$
$$l_{2i}^{-} \le \frac{g_i(\alpha)}{\alpha} \le l_{2i}^{+}, g_i(0)=0, i=1,2,\dots,n. \qquad (5)$$

Given probability space (Ω,Υ,P), where Ω is sample space,Υ is $\sigma-$algebra of subset of the sample space, and P is the probability measure defined on Υ. Let $\{r(t),t\in[0,+\infty)\}$ be a right-continuous Markovian process on the probability space which takes values in the finite space $S=\{1,2,\dots,N\}$ with generator $\Pi=(\pi_{i\times j})_{N\times N}$ given by:

$$P\{r(t+\Delta t)=j|r(t)=i\}=\begin{cases} \pi_{ij}\Delta t+o(\Delta t) & j\ne i \\ 1+\pi_{ii}\Delta t+o(\Delta t) & j=i \end{cases} \qquad (6)$$

with transition rates $\pi_{ij} \ge 0$ for $i,j \in S, j \ne i$ and $\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}$,where $\Delta t>0$ and $\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t}=0$. Due to the disturbance frequent occurs in many applications,and combining with the discussion above, in this paper, we consider delayed BAM neural networks with uncertainty Markovian jumping parameters described by the following nonlinear differential equations:

$$\begin{cases} \dot{x}(t)=-A(r(t),t)x(t-\sigma)+C(r(t),t)f(y(t)) \\ \qquad +E(r(t),t)f(y(t-h(r(t),t))) \\ \dot{y}(t)=-B(r(t),t)y(t-\delta)+D(r(t),t)g(x(t)) \\ \qquad +F(r(t),t)g(x(t-\varsigma(r(t),t))) \end{cases} \qquad (7)$$

when $r(t)=i\in S$,and the matrix functions $A(r(t),t), B(r(t),t), C(r(t),t), D(r(t),t), E(r(t),t), F(r(t),t), h(r(t),t), \varsigma(r(t),t)$ are denoted as $A_i(t),B_i(t),C_i(t),D_i(t), E_i(t),F_i(t),h_i(t),\varsigma_i(t)$,respectively, and $h_i(t),\varsigma_i(t)$ denote the time-varying delays which satisfy $\dot{h}_i(t)\le h_{Di}<1, 0\le h_i(t)\le h_i, 0\le\varsigma_i(t)\le\varsigma_i$,$\dot{\varsigma}_i(t)\le\varsigma_{Di}<1$,$\tilde{h}=\max_{j\in S}\{h_j\}$,$\tilde{\varsigma}=\max_{j\in S}\{\varsigma_j\}$.

Assumption 3: $A_i(t)=A_i+\Delta A_i(t), B_i(t)=B_i+\Delta B_i(t), C_i(t)=C_i+\Delta C_i(t), D_i(t)=D_i+\Delta D_i(t), E_i(t)=E_i+\Delta E_i(t), F_i(t)=F_i+\Delta F_i(t)$,where the matrices $\Delta A_i(t),\Delta B_i(t),\Delta C_i(t),\Delta D_i(t),\Delta E_i(t),\Delta F_i(t)$ are the uncertainties of the system and have the form

$$[\Delta A_i(t),\Delta B_i(t),\Delta C_i(t),\Delta D_i(t),\Delta E_i(t),\Delta F_i(t)]$$
$$=G_iF_i(t)[E_{ai},E_{bi},E_{ci},E_{di},E_{ei},E_{fi}] \qquad (8)$$

where $G_i,E_{ai},E_{bi},E_{ci},E_{di},E_{ei},E_{fi}$ are known constant real matrices with appropriate dimensions and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements bounded by

$$F_i^T(t)F_i(t)\le I, \ \forall i\in S. \qquad (9)$$

Let $(x(t,\phi),y(t,\varphi))$ be the state trajectory the system (9) from the initial data $\phi\in C_{F_0}^b([-\tilde{\varsigma},0];R^n), \varphi\in C_{F_0}^b([-\tilde{h},0];R^n)$.It can be seen that system (9) admits a trivial solution $(x(t,0),y(t,0))\equiv 0$ corresponding to the initial data $\phi=0,\varphi=0$.

Definition 1 For the BAM neural network (9) and every initial condition $\phi\in C_{F_0}^b([-\tilde{\varsigma},0];R^n),\varphi\in C_{F_0}^b([-\tilde{h},0];R^n), r(0)=i_0$,the trivial solution is said to be stochastically stable if the following condition is satisfied:

$$\lim_{t\to\infty} E\left\{\int_0^t(|x(s,\phi,i_0)|^2+|y(s,\varphi,i_0)|^2)ds\right\}<\infty \qquad (10)$$

Lemma 1 [7]. For any positive semi-definite matrices $X=\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33}\end{bmatrix}>0$,the following integral inequality holds:

$$-\int_{t-\varsigma(t)}^{t}\dot{x}^T(s)X_{33}\dot{x}(s)ds$$
$$\le\int_{t-\varsigma(t)}^{t}\begin{bmatrix} x(t) \\ x(t-\varsigma(t)) \\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & 0\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\varsigma(t)) \\ \dot{x}(s)\end{bmatrix}ds \qquad (11)$$

Lemma 2 [3]. Let Z,H and S be real matrices of appropriate dimensions with H satisfying $H^TH\le I$,then for any scalar $\varepsilon>0$,the following inequality holds:

$$ZHS+(ZHS)^T\le\varepsilon^{-1}ZZ^T+\varepsilon S^TS \qquad (12)$$

III. Main results

In this section,we consider the case of $\Delta A_i(t)=\Delta B_i(t)=\Delta C_i(t)=\Delta D_i(t)=\Delta E_i(t)=\Delta F_i(t)=0$ in system (9),a new Lyapunov functional is constructed to derive the condition under which the system (9) are stochastically stable in the mean square.For representation convenience,the following notations are introduced:

$$L_1=diag\{\frac{l_{11}^{+}+l_{11}^{-}}{2},\frac{l_{12}^{+}+l_{12}^{-}}{2},\dots,\frac{l_{1n}^{+}+l_{1n}^{-}}{2}\},$$
$$\bar{L}_1=diag\{l_{11}^{+}l_{11}^{-},l_{12}^{+}l_{12}^{-},\dots,l_{1n}^{+}l_{1n}^{-}\},$$
$$L_2=diag\{\frac{l_{21}^{+}+l_{21}^{-}}{2},\frac{l_{22}^{+}+l_{22}^{-}}{2},\dots,\frac{l_{2n}^{+}+l_{2n}^{-}}{2}\},$$
$$\bar{L}_2=diag\{l_{21}^{+}l_{21}^{-},l_{22}^{+}l_{22}^{-},\dots,l_{2n}^{+}l_{2n}^{-}\}$$

Theorem 1 For any given scalars $h_i \geq 0, \varsigma_i \geq 0, h_{Di}, \varsigma_{Di}$ and integers $l \geq 1, k \geq 1$,the system (9)with leakage and discrete delays is globally asymptotically stable if there exist symmetric positive definite matrices $P_{1i}, P_{2i}, Q_{ji}, M_j, (j = 1,2,3,4), S_{ji}, (j = 1,2,\ldots,6), R_j, (j = 1,2,\ldots,8)$,positive diagonal matrices $W_{ij}, (j = 1,2,\ldots,6)$,and any matrices $X_i = \begin{bmatrix} X_{1i} & X_{2i} & X_{3i} \\ * & X_{4i} & X_{5i} \\ * & * & X_{6i} \end{bmatrix}, Y_i = \begin{bmatrix} Y_{1i} & Y_{2i} & Y_{3i} \\ * & Y_{4i} & Y_{5i} \\ * & * & Y_{6i} \end{bmatrix}, U_i = \begin{bmatrix} U_{1i} & U_{2i} & U_{3i} \\ * & U_{4i} & U_{5i} \\ * & * & U_{6i} \end{bmatrix}, V_i = \begin{bmatrix} V_{1i} & V_{2i} & V_{3i} \\ * & V_{4i} & V_{5i} \\ * & * & V_{6i} \end{bmatrix}$ with appropriate dimensions,for any $i = 1,2,\ldots,N$,such that the following LMIs holds:

$$\sum_{j=1}^{N} \pi_{ij} Q_{kj} < M_k, \ \ k = 1,2,3,4 \tag{13}$$

$$\sum_{j=1}^{N} \pi_{ij} S_{kj} < R_k, \ \ k = 1,2,\ldots,6 \tag{14}$$

$$\begin{bmatrix} X_{1i} & X_{2i} & X_{3i} \\ * & X_{4i} & X_{5i} \\ * & * & R_7 \end{bmatrix} \geq 0 \tag{15}$$

$$\begin{bmatrix} Y_{1i} & Y_{2i} & Y_{3i} \\ * & Y_{4i} & Y_{5i} \\ * & * & R_7 \end{bmatrix} \geq 0 \tag{16}$$

$$\begin{bmatrix} U_{1i} & U_{2i} & U_{3i} \\ * & U_{4i} & U_{5i} \\ * & * & R_8 \end{bmatrix} \geq 0 \tag{17}$$

$$\begin{bmatrix} V_{1i} & V_{2i} & V_{3i} \\ * & V_{4i} & V_{5i} \\ * & * & R_8 \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} \Xi + \bar{\Xi} & \tilde{\varsigma}\aleph^T R_7 & \tilde{h}\Im^T R_8 \\ * & -\tilde{\varsigma} R_7 & 0 \\ * & * & -\tilde{h} R_8 \end{bmatrix} < 0 \tag{19}$$

where

$$\aleph = \begin{bmatrix} 0 & -A_i & 0_{n\times 9n} & C_i & 0 & E_i \end{bmatrix}$$

$$\Im = \begin{bmatrix} 0_{n\times 4n} & D_i & 0 & F_i & 0 & -B_i & 0_{n\times 5n} \end{bmatrix}$$

$$\Xi = [\Xi_{mn}], \ \bar{\Xi} = [\bar{\Xi}_{mn}], \ (m,n = 1,2,\ldots,14)$$

$$\Xi_{11} = Q_{1i} - (1 - \frac{\pi_{ii}\varsigma_i}{l})\tilde{E}_1 Q_{1i}\tilde{E}_1^T + \tilde{I}_1^T(\pi_{ii}P_{1i} + Q_{3i} + S_{1i} + S_{2i} + \sigma M_3 + \tilde{\varsigma}(R_1 + R_2) - \bar{L}_2 W_{i4} + \varsigma_i X_{1i} + 2X_{3i})\tilde{I}_1 + \frac{\tilde{\varsigma}}{l} M_1$$

$$\Xi_{12} = -\tilde{I}_1^T P_{1i} A_i, \ \Xi_{13} = -(1 - \frac{\pi_{ii}\varsigma_i}{l})\tilde{E}_1 Q_{1i}\tilde{I}_3^T$$

$$\Xi_{14} = \varsigma_i \tilde{I}_1 X_{2i} - \tilde{I}_1^T X_{3i} + \tilde{I}_1^T X_{5i}^T$$

$$\Xi_{15} = \tilde{I}_1^T L_2 W_{i4}, \ \Xi_{1,12} = \tilde{I}_1^T P_{1i} C_i$$

$$\Xi_{1,14} = \tilde{I}_1^T P_{1i} E_i, \ \Xi_{22} = -Q_{3i}$$

$$\Xi_{33} = -(1 - \frac{\pi_{ii}\varsigma_i}{l})\tilde{I}_3 Q_{1i}\tilde{I}_3^T - (1 - \pi_{ii}\varsigma_i)S_{1i} - \bar{L}_2 W_{i6} + \varsigma_i Y_{4i} - 2Y_{5i}$$

$$\Xi_{34} = \varsigma_i Y_{2i}^T - Y_{3i}^T + Y_{5i}, \ \Xi_{36} = \bar{L}_2 W_{i6}$$

$$\Xi_{44} = -(1 - \varsigma_{Di})S_{2i} + \pi_{ii}\varsigma_i S_{2i} - \bar{L}_2 W_{i5} + \varsigma_i(X_{4i} + Y_{1i}) + 2Y_{3i}^T - 2X_{5i}$$

$$\Xi_{47} = L_2 W_{i5}, \ \Xi_{55} = S_{5i} + \tilde{\varsigma} R_5 - W_{i4}$$

$$\Xi_{58} = D_i^T P_{2i}\tilde{I}_2, \ \Xi_{66} = -W_{i6}$$

$$\Xi_{77} = -(1 - \varsigma_{Di})S_{5i} + \pi_{ii}\varsigma_i S_{5i} - W_{i5}, \Xi_{78} = F_i^T P_{2i}\tilde{I}_2$$

$$\Xi_{88} = Q_{2i} - (1 - \frac{\pi_{ii}h_i}{k})\tilde{E}_2 Q_{2i}\tilde{E}_2^T + \tilde{I}_2^T(\pi_{ii}P_{2i} + Q_{4i} + S_{3i} + S_{4i} + \delta M_4 + \tilde{h}(R_3 + R_4) - \bar{L}_1 W_{i1} + h_i U_{1i} + 2U_{3i})\tilde{I}_2 + \frac{\tilde{h}}{k} M_2$$

$$\Xi_{89} = -\tilde{I}_2^T P_{2i} B_i, \Xi_{8,10} = -(1 - \frac{\pi_{ii}h_i}{k})\tilde{E}_2 Q_{2i}\tilde{I}_4^T$$

$$\Xi_{8,11} = \tilde{I}_2^T(U_{2i} - U_{3i} + U_{5i}^T), \ \Xi_{8,12} = \tilde{I}_2^T L_1 W_{i1}, \Xi_{99} = -Q_{4i}$$

$$\Xi_{10,10} = -(1 - \frac{\pi_{ii}h_i}{k})\tilde{I}_4 Q_{2i}\tilde{I}_4^T - S_{3i} + \pi_{ii}h_i S_{3i} - \bar{L}_1 W_{i3} + hY_{4i} - 2V_{5i}$$

$$\Xi_{10,11} = h_i V_{2i}^T - V_{3i}^T + V_{5i}, \ \Xi_{10,13} = L_1 W_{i3}$$

$$\Xi_{11,11} = -(1 - h_{Di})S_{4i} + \pi_{ii}h_i S_{4i} - \bar{L}_1 W_{i2} + h_i U_{4i} - 2U_{5i} + 2V_{3i}^T + h_i V_{1i}$$

$$\Xi_{11,14} = L_1 W_{i2}, \ \Xi_{12,12} = S_{6i} + \tilde{h} R_6 - W_{i1}$$

$$\Xi_{13,13} = -W_{i3}, \ \Xi_{14,14} = -(1 - h_{Di})S_{6i} + \pi_{ii}h_i S_{6i} - W_{i2}$$

$$\bar{\Xi}_{11} = \tilde{I}_1^T \sum_{j\neq i} \pi_{ij} P_{1j}\tilde{I}_1 + \sum_{j\neq i} \frac{\pi_{ij}\varsigma_j}{l}\tilde{E}_1 Q_{1i}\tilde{E}_1^T$$

$$\bar{\Xi}_{13} = \sum_{j\neq i} \frac{\pi_{ij}\varsigma_j}{l}\tilde{E}_1 Q_{1i}\tilde{I}_3^T$$

$$\bar{\Xi}_{33} = \sum_{j\neq i} \frac{\pi_{ij}\varsigma_j}{l}\tilde{I}_3 Q_{1i}\tilde{I}_3^T + \sum_{j\neq i} \pi_{ij}\varsigma_j S_{1i}$$

$$\bar{\Xi}_{44} = \sum_{j\neq i} \pi_{ij}\varsigma_j S_{2i},\ \bar{\Xi}_{77} = \sum_{j\neq i} \pi_{ij}\varsigma_j S_{5i}$$

$$\bar{\Xi}_{88} = \tilde{I}_2^T \sum_{j\neq i} \pi_{ij} P_{2j} \tilde{I}_2 + \sum_{j\neq i} \frac{\pi_{ij} h_j}{k} \tilde{E}_2 Q_{2i} \tilde{E}_2^T$$

$$\bar{\Xi}_{8,10} = \sum_{j\neq i} \frac{\pi_{ij} h_j}{k} \tilde{E}_2 Q_{2i} \tilde{I}_4^T$$

$$\bar{\Xi}_{10,10} = \sum_{j\neq i} \pi_{ij} h_j S_{3i} + \sum_{j\neq i} \frac{\pi_{ij} h_j}{k} \tilde{I}_4 Q_{2i} \tilde{I}_4^T$$

$$\bar{\Xi}_{11,11} = \sum_{j\neq i} \pi_{ij} h_j S_{4i},\ \bar{\Xi}_{14,14} = \sum_{j\neq i} \pi_{ij} h_j S_{6i}$$

All the other items in matrix Ξ and $\bar{\Xi}$ are 0.

$$\tilde{I}_1 = \begin{bmatrix} I_n & 0_{n\times(l-1)n} \end{bmatrix},\ \tilde{I}_2 = \begin{bmatrix} I_n & 0_{n\times(k-1)n} \end{bmatrix}$$

$$\tilde{I}_3 = \begin{bmatrix} 0_{n\times(l-1)n} & I_n \end{bmatrix},\ \tilde{I}_4 = \begin{bmatrix} 0_{n\times(k-1)n} & I_n \end{bmatrix}$$

$$\tilde{I}_5 = \begin{bmatrix} I_{n\times(l+6)n} & 0_{n\times(k+6)n} \end{bmatrix}$$

$$\tilde{I}_6 = \begin{bmatrix} 0_{n\times(l+6)n} & I_{n\times(k+6)n} \end{bmatrix}$$

$$\tilde{E}_1 = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix}_{ln\times ln}$$

$$\tilde{E}_2 = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix}_{kn\times kn}$$

Proof: Construct the following Lyapunov-Krasovskii functional:

$$V(x_t, y_t, r(t)) = \sum_{m=1}^{8} V_i(x_t, y_t, r(t))$$

with

$$V_1(x_t, y_t, r(t)) = x^T(t) P_1(r(t)) x(t) + y^T(t) P_2(r(t)) y(t)$$

$$V_2(x_t, y_t, r(t)) = \int_{t-\frac{\varsigma_i}{l}}^{t} \gamma_1^T(s) Q_1(r(t)) r_1(s) ds + \int_{t-\frac{h_i}{k}}^{t} \gamma_2^T(s) Q_2(r(t)) r_2(s) ds$$

where

$$\gamma_1^T(s) = \begin{bmatrix} x^T(s) & x^T(s-\frac{\varsigma_i}{l}) & \dots & x^T(s-\frac{(l-1)\varsigma_i}{l}) \end{bmatrix},$$
$$\gamma_2^T(s) = \begin{bmatrix} y^T(s) & y^T(s-\frac{h_i}{k}) & \dots & y^T(s-\frac{(k-1)h_i}{k}) \end{bmatrix}$$

$$V_3(x_t, y_t, r(t)) = \int_{t-\sigma}^{t} x^T(s) Q_3(r(t)) x(s) ds + \int_{t-\delta}^{t} y^T(s) Q_4(r(t)) y(s) ds$$

$$\begin{aligned} V_4(x_t, y_t, r(t)) &= \int_{t-\varsigma(r(t))}^{t} x^T(s) S_1(r(t)) x(s) ds \\ &+ \int_{t-\varsigma(r(t),t)}^{t} x^T(s) S_2(r(t)) x(s) ds \\ &+ \int_{t-h(r(t))}^{t} y^T(s) S_3(r(t)) y(s) ds \\ &+ \int_{t-h(r(t),t)}^{t} y^T(s) S_4(r(t)) y(s) ds \end{aligned}$$

$$V_5(x_t, y_t, r(t)) = \int_{t-\varsigma(r(t),t)}^{t} g^T(x(s)) S_5(r(t)) g(x(s)) ds + \int_{t-h(r(t),t)}^{t} f^T(y(s)) S_6(r(t)) f(y(s)) ds$$

$$\begin{aligned} V_6(x_t, y_t, r(t)) &= \int_{-\frac{\tilde{\varsigma}}{l}}^{0} \int_{t+\theta}^{t} \gamma_1^T(s) M_1 \gamma_1(s) ds d\theta \\ &+ \int_{-\frac{\tilde{h}}{k}}^{0} \int_{t+\theta}^{t} \gamma_2^T(s) M_2 \gamma_2(s) ds d\theta \\ &+ \int_{-\sigma}^{0} \int_{t+\theta}^{t} x^T(s) M_3 x(s) ds d\theta \\ &+ \int_{-\delta}^{0} \int_{t+\theta}^{t} y^T(s) M_4 y(s) ds d\theta \end{aligned}$$

$$\begin{aligned} V_7(x_t, y_t, r(t)) &= \int_{-\tilde{\varsigma}}^{0} \int_{t+\theta}^{t} x^T(s)(R_1+R_2) x(s) ds d\theta \\ &+ \int_{-\tilde{h}}^{0} \int_{t+\theta}^{t} y^T(s)(R_3+R_4) y(s) ds d\theta \\ &+ \int_{-\tilde{\varsigma}}^{0} \int_{t+\theta}^{t} g^T(x(s)) R_5 g(x(s)) ds d\theta \\ &+ \int_{-\tilde{h}}^{0} \int_{t+\theta}^{t} f^T(y(s)) R_6 f(y(s)) ds d\theta \end{aligned}$$

$$V_8(x_t, y_t, r(t)) = \int_{-\tilde{\varsigma}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_7 \dot{x}(s) ds d\theta + \int_{-\tilde{h}}^{0} \int_{t+\theta}^{t} \dot{y}^T(s) R_8 \dot{y}(s) ds d\theta$$

Then, taking the derivative of $V(x_t, y_t, r(t))$ with respect to t along the system (7) yields

$$\begin{aligned} LV_1(x_t, y_t, i) &= 2x^T(t) P_{1i} \dot{x}(t) + x^T(t) \left(\sum_{j=1}^{N} \pi_{ij} P_{1j} \right) x(t) \\ &+ 2y^T(t) P_{2i} \dot{y}(t) + y^T(t) \left(\sum_{j=1}^{N} \pi_{ij} P_{2j} \right) y(t) \end{aligned} \quad (20)$$

$$\begin{aligned}LV_2(x_t,y_t,i) &= r_1^T(t)Q_{1i}r_1(t) - r_1^T(t-\frac{\varsigma_i}{l})Q_{1i}r_1(t-\frac{\varsigma_i}{l})\\ &+\sum_{j=1}^{N}\frac{\pi_{ij}\varsigma_j}{l}r_1^T(t-\frac{\varsigma_i}{l})Q_{1i}r_1(t-\frac{\varsigma_i}{l})\\ &+r_2^T(t)Q_{2i}r_2(t)-r_2^T(t-\frac{h_i}{k})Q_{2i}r_2(t-\frac{h_i}{k})\\ &+\sum_{j=1}^{N}\frac{\pi_{ij}h_j}{k}r_2^T(t-\frac{h_i}{k})Q_{2i}r_2(t-\frac{h_i}{k})\\ &+\int_{t-\frac{\varsigma_i}{l}}^{t}\gamma_1^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{1j}\right)\gamma_1(s)ds\\ &+\int_{t-\frac{h_i}{k}}^{t}\gamma_2^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)\gamma_2(s)ds\end{aligned} \quad (21)$$

$$\begin{aligned}LV_3(x_t,y_t,i) &= x^T(t)Q_{3i}x(t)-x^T(t-\sigma)Q_{3i}x(t-\sigma)\\ &+\int_{t-\sigma}^{t}x^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{3j}\right)x(s)ds\\ &+y^T(t)Q_{4i}y(t)-y^T(t-\delta)Q_{4i}y(t-\delta)\\ &+\int_{t-\delta}^{t}y^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)y(s)ds\end{aligned} \quad (22)$$

$$\begin{aligned}LV_4(x_t,y_t,i) &\le x^T(t)S_{1i}x(t)-x^T(t-\varsigma_i)S_{1i}x(t-\varsigma_i)\\ &+\left(\sum_{j=1}^{N}\pi_{ij}\varsigma_j\right)x^T(t-\varsigma_i)S_{1i}x(t-\varsigma_i)\\ &+\int_{t-\varsigma_i}^{t}x^T(s)\left(\sum_{j=1}^{N}\pi_{ij}S_{1j}\right)x(s)ds\\ &+x^T(t)S_{2i}x(t)-(1-\varsigma_{Di})x^T(t-\varsigma_i(t))S_{2i}x(t-\varsigma_i(t))\\ &+\left(\sum_{j=1}^{N}\pi_{ij}\varsigma_j(t)\right)x^T(t-\varsigma_i(t))S_{2i}x(t-\varsigma_i(t))\\ &+y^T(t)S_{3i}y(t)-y^T(t-h_i)S_{3i}y(t-h_i)\\ &+\left(\sum_{j=1}^{N}\pi_{ij}h_j\right)y^T(t-h_i)S_{3i}y(t-h_i)\\ &+\int_{t-h_i}^{t}y^T(s)\left(\sum_{j=1}^{N}\pi_{ij}S_{3j}\right)y(s)ds\\ &+y^T(t)S_{4i}y(t)-(1-h_{Di})y^T(t-h_i(t))S_{4i}y(t-h_i(t))\\ &+\left(\sum_{j=1}^{N}\pi_{ij}h_j(t)\right)y^T(t-h_i(t))S_{4i}y(t-h_i(t))\\ &+\int_{t-\varsigma_i(t)}^{t}x^T(s)\left(\sum_{j=1}^{N}\pi_{ij}S_{2j}\right)x(s)ds\\ &+\int_{t-h_i(t)}^{t}y^T(s)\left(\sum_{j=1}^{N}\pi_{ij}S_{4j}\right)y(s)ds\end{aligned} \quad (23)$$

$$\begin{aligned}LV_5(x_t,y_t,i) &\le g^T(x(t))S_{5i}g(x(t))+f^T(y(t))S_{6i}f(y(t))\\ &-(1-\varsigma_{Di})g^T(x(t-\varsigma_i(t)))S_{5i}g(x(t-\varsigma_i(t)))\\ &-(1-h_{Di})f^T(y(t-h_i(t)))S_{6i}f(y(t-h_i(t)))\\ &+\left(\sum_{j=1}^{N}\pi_{ij}\varsigma_j(t)\right)g^T(x(t-\varsigma_i(t)))S_{5i}g(x(t-\varsigma_i(t)))\\ &+\left(\sum_{j=1}^{N}\pi_{ij}h_j(t)\right)f^T(y(t-h_i(t)))S_{6i}f(y(t-h_i(t)))\\ &+\int_{t-\varsigma_i(t)}^{t}g^T(x(s))\left(\sum_{j=1}^{N}\pi_{ij}S_{5j}\right)g(x(s))ds\\ &+\int_{t-h_i(t)}^{t}f^T(y(s))\left(\sum_{j=1}^{N}\pi_{ij}S_{6j}\right)f(y(s))ds\end{aligned} \quad (24)$$

$$\begin{aligned}LV_6(x_t,y_t,i) &= \frac{\tilde{\varsigma}}{l}\gamma_1^T(t)M_1\gamma_1(t)+\frac{\tilde{h}}{k}\gamma_2^T(t)M_2\gamma_2(t)\\ &+\sigma x^T(t)M_3x(t)+\delta y^T(t)M_4y(t)\\ &-\int_{t-\frac{\tilde{\varsigma}}{l}}^{t}\gamma_1^T(s)M_1\gamma_1(s)ds-\int_{t-\frac{\tilde{h}}{k}}^{t}\gamma_2^T(s)M_2\gamma_2(s)ds\\ &-\int_{t-\sigma}^{t}x^T(s)M_3x(s)ds-\int_{t-\delta}^{t}y^T(s)M_4y(s)ds\end{aligned} \quad (25)$$

$$\begin{aligned}LV_7(x_t,y_t,i) &= \tilde{\varsigma}x^T(t)(R_1+R_2)x(t)+\tilde{h}y^T(t)(R_3+R_4)y(t)\\ &+\tilde{\varsigma}g^T(x(t))R_5g(x(t))+\tilde{h}f^T(y(t))R_6f(y(t))\\ &-\int_{t-\tilde{\varsigma}}^{t}x^T(s)(R_1+R_2)x(s)ds\\ &-\int_{t-\tilde{h}}^{t}y^T(s)(R_3+R_4)y(s)ds\\ &-\int_{t-\tilde{\varsigma}}^{t}g^T(x(s))R_5g(x(s))ds\\ &-\int_{t-\tilde{h}}^{t}f^T(y(s))R_6f(y(s))ds\end{aligned} \quad (26)$$

$$\begin{aligned}LV_8(x_t,y_t,i) &\le \tilde{\varsigma}\dot{x}^T(t)R_7\dot{x}(t)+\tilde{h}\dot{y}^T(t)R_8\dot{y}(t)\\ &-\int_{t-\varsigma_i}^{t}\dot{x}^T(s)R_7\dot{x}(s)ds\\ &-\int_{t-h_i}^{t}\dot{y}^T(s)R_8\dot{y}(s)ds\end{aligned} \quad (27)$$

Using Lemma 1 and (15)-(18),one can obtain the following inequalities

$$-\int_{t-\varsigma_i}^{t}\dot{x}^T(s)R_7\dot{x}(s)ds \le \int_{t-\varsigma_i(t)}^{t}\begin{bmatrix}x(t)\\x(t-\varsigma_i(t))\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}X_{1i}&X_{2i}&X_{3i}\\ *&X_{4i}&X_{5i}\\ *&*&0\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\varsigma_i(t))\\\dot{x}(s)\end{bmatrix}ds + \int_{t-\varsigma_i}^{t-\varsigma_i(t)}\begin{bmatrix}x(t-\varsigma_i(t))\\x(t-\varsigma_i)\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Y_{1i}&Y_{2i}&Y_{3i}\\ *&Y_{4i}&Y_{5i}\\ *&*&0\end{bmatrix}\begin{bmatrix}x(t-\varsigma_i(t))\\x(t-\varsigma_i)\\\dot{x}(s)\end{bmatrix}ds \quad (28)$$

$$-\int_{t-h_i}^{t}\dot{y}^T(s)R_8\dot{y}(s)ds \le \int_{t-h_i(t)}^{t}\begin{bmatrix}y(t)\\y(t-h_i(t))\\\dot{y}(s)\end{bmatrix}^T\begin{bmatrix}U_{1i}&U_{2i}&U_{3i}\\ *&U_{4i}&U_{5i}\\ *&*&0\end{bmatrix}\begin{bmatrix}y(t)\\y(t-h_i(t))\\\dot{y}(s)\end{bmatrix}ds + \int_{t-h_i}^{t-h_i(t)}\begin{bmatrix}y(t-h_i(t))\\y(t-h_i)\\\dot{y}(s)\end{bmatrix}^T\begin{bmatrix}V_{1i}&V_{2i}&V_{3i}\\ *&V_{4i}&V_{5i}\\ *&*&0\end{bmatrix}\begin{bmatrix}y(t-h_i(t))\\y(t-h_i)\\\dot{y}(s)\end{bmatrix}ds \quad (29)$$

For positive diagonal matrices $W_{ij}, j = 1, 2, \ldots, 6$,we can get from (5) that

$$\begin{bmatrix}y(t)\\f(y(t))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_1W_{i1}&L_1W_{i1}\\ *&-W_{i1}\end{bmatrix}\begin{bmatrix}y(t)\\f(y(t))\end{bmatrix}\ge 0 \quad (30)$$

$$\begin{bmatrix}y(t-h_i(t))\\f(y(t-h_i(t)))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_1W_{i2}&L_1W_{i2}\\ *&-W_{i2}\end{bmatrix}\begin{bmatrix}y(t-h_i(t))\\f(y(t-h_i(t)))\end{bmatrix}\ge 0 \quad (31)$$

$$\begin{bmatrix}y(t-h_i)\\f(y(t-h_i))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_1W_{i3}&L_1W_{i3}\\ *&-W_{i3}\end{bmatrix}\begin{bmatrix}y(t-h_i)\\f(y(t-h_i))\end{bmatrix}\ge 0 \quad (32)$$

$$\begin{bmatrix}x(t)\\g(x(t))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_2W_{i4}&L_2W_{i4}\\ *&-W_{i4}\end{bmatrix}\begin{bmatrix}x(t)\\g(x(t))\end{bmatrix}\ge 0 \quad (33)$$

$$\begin{bmatrix}x(t-\varsigma_i(t))\\g(x(t-\varsigma_i(t)))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_2W_{i5}&L_2W_{i5}\\ *&-W_{i5}\end{bmatrix}\begin{bmatrix}x(t-\varsigma_i(t))\\g(x(t-\varsigma_i(t)))\end{bmatrix}\ge 0 \quad (34)$$

$$\begin{bmatrix}x(t-\varsigma_i)\\g(x(t-\varsigma_i))\end{bmatrix}^T\begin{bmatrix}-\bar{L}_2W_{i6}&L_2W_{i6}\\ *&-W_{i6}\end{bmatrix}\begin{bmatrix}x(t-\varsigma_i)\\g(x(t-\varsigma_i))\end{bmatrix}\ge 0 \quad (35)$$

From (13)-(14) and (20)-(35),one can obtain $LV(x_t, y_t, i) \le \xi^T(t)\Sigma_i\xi(t)$.
where

$\Sigma_i = \Xi + \bar{\Xi} + \tilde{\varsigma}\aleph^T R_7\aleph + \tilde{h}\Im^T R_8\Im$

$\xi^T(t) = \begin{bmatrix}\xi_1^T(t) & \xi_2^T(t)\end{bmatrix}$

$\xi_1^T(t) = [\gamma_1^T(t), x^T(t-\sigma), x^T(t-\varsigma_i), x^T(t-\varsigma_i(t)), g^T(x(t)), g^T(x(t-\varsigma_i)), g^T(x(t-\varsigma_i(t)))]$

$\xi_2^T(t) = [\gamma_2^T(t), y^T(t-\delta), y^T(t-h_i), x^T(t-h_i(t)), f^T(y(t)), f^T(y(t-h_i)), f^T(y(t-h_i(t)))]$

According to (19) and Schur complement,we can get $\Sigma_i < 0$,let $\lambda_1 = \min \lambda_{min}\{-\Sigma_i\}, i \in S$,so $\lambda_1 > 0$.Then, by Dynkin's formula, we have

$$E\{V(x_t, y_t, i)\} - E\{V(\phi, \varphi, i_0)\} \le -\lambda_1 E\left\{\int_0^t(|x(s)|^2 + |y(s)|^2)ds\right\}$$

and,hence

$$E\left\{\int_0^t(|x(s)|^2 + |y(s)|^2)ds\right\} \le \frac{1}{\lambda_1}E\{V(\phi, \varphi, i_0)\}$$

Based on Definition 1, the system (7) are stochastically stable and the proof is completed. ■

Remark 1 Theorem 1 proposes an improved stochastically stability criterion for Markovian jumping BAM neural networks with leakage and discrete delays. The main idea is to divide the delay interval into multiple segments ,and the thinner the delay is partitioned, the more obviously the conservatism can be reduced.

Based on Theorem 1,we have the following result for uncertainty Markovian jumping parameters of BAM neural networks with leakage and discrete delays.

Theorem 2 For any given scalars $h_i \ge 0, \varsigma_i \ge 0, h_{Di}, \varsigma_{Di}$ and integers $l \ge 1, k \ge 1$,the system (7)with leakage and discrete delays is globally asymptotically stable if there exist two scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$,symmetric positive definite matrices $P_{1i}, P_{2i}, Q_{ji}, M_j, (j = 1, 2, 3, 4), S_{ji}, (j = 1, 2, \ldots, 6), R_j, (j = 1, 2, \ldots, 8)$,positive diagonal matrices $W_{ij}, (j = 1, 2, \ldots, 6)$,and any matrices $X_i = \begin{bmatrix}X_{1i}&X_{2i}&X_{3i}\\ *&X_{4i}&X_{5i}\\ *&*&X_{6i}\end{bmatrix}, Y_i = \begin{bmatrix}Y_{1i}&Y_{2i}&Y_{3i}\\ *&Y_{4i}&Y_{5i}\\ *&*&Y_{6i}\end{bmatrix}, U_i = \begin{bmatrix}U_{1i}&U_{2i}&U_{3i}\\ *&U_{4i}&U_{5i}\\ *&*&U_{6i}\end{bmatrix}, V_i = \begin{bmatrix}V_{1i}&V_{2i}&V_{3i}\\ *&V_{4i}&V_{5i}\\ *&*&V_{6i}\end{bmatrix}$ with appropriate dimensions,for any $i = 1, 2, \ldots, N$,such that the following LMIs holds:

$$\sum_{j=1}^{N}\pi_{ij}Q_{kj} < M_k, \quad k = 1, 2, 3, 4 \quad (36)$$

$$\sum_{j=1}^{N}\pi_{ij}S_{kj} < R_k, \quad k = 1, 2, \ldots, 6 \quad (37)$$

$$\begin{bmatrix}X_{1i}&X_{2i}&X_{3i}\\ *&X_{4i}&X_{5i}\\ *&*&R_7\end{bmatrix}\ge 0 \quad (38)$$

$$\begin{bmatrix}Y_{1i}&Y_{2i}&Y_{3i}\\ *&Y_{4i}&Y_{5i}\\ *&*&R_7\end{bmatrix}\ge 0 \quad (39)$$

$$\begin{bmatrix}U_{1i}&U_{2i}&U_{3i}\\ *&U_{4i}&U_{5i}\\ *&*&R_8\end{bmatrix}\ge 0 \quad (40)$$

$$\begin{bmatrix} V_{1i} & V_{2i} & V_{3i} \\ * & V_{4i} & V_{5i} \\ * & * & R_8 \end{bmatrix} \geq 0 \quad (41)$$

$$\begin{bmatrix} \Xi+\bar{\Xi} & \tilde{\varsigma}\aleph^T R_7 & \tilde{h}\Im^T R_8 & \aleph_{22} & \sqrt{\varepsilon_1}\aleph_{11}^T & \Im_{22} & \sqrt{\varepsilon_2}\Im_{11} \\ * & -\tilde{\varsigma}R_7 & 0 & \frac{1}{2}R_7G_i & 0 & 0 & 0 \\ * & * & -\tilde{h}R_8 & 0 & 0 & \frac{1}{2}R_8G_i & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (42)$$

where

$$\aleph_{11} = \begin{bmatrix} 0 & -E_{ai} & 0_{n\times 9n} & E_{ci} & 0 & E_{ei} \end{bmatrix}$$

$$\aleph_1 = \begin{bmatrix} \aleph_{11} & 0 & 0 \end{bmatrix}$$

$$\aleph_{22} = \begin{bmatrix} G_i^T P_{1i}\tilde{I}_1 & 0_{n\times 13n} \end{bmatrix}^T$$

$$\aleph_2 = \begin{bmatrix} \aleph_{22}^T & \frac{1}{2}G_i^T R_7 & 0 \end{bmatrix}^T$$

$$\Im_{11} = \begin{bmatrix} 0_{n\times 4n} & E_{di} & 0 & E_{fi} & 0 & -E_{bi} & 0_{n\times 5n} \end{bmatrix}$$

$$\Im_1 = \begin{bmatrix} \Im_{11} & 0 & 0 \end{bmatrix}$$

$$\Im_{22} = \begin{bmatrix} 0_{n\times 7n} & G_i^T P_{2i}\tilde{I}_2 & 0_{n\times 6n} \end{bmatrix}^T$$

$$\Im_2 = \begin{bmatrix} \Im_{22}^T & 0 & \frac{1}{2}G_i^T R_8 \end{bmatrix}^T$$

Proof: Replacing $A_i, B_i, C_i, D_i, E_i, F_i$ in (19) with $A_i + G_iF_i(t)E_{ai}, B_i + G_iF_i(t)E_{bi}, C_i + G_iF_i(t)E_{ci}, D_i + G_iF_i(t)E_{di}, F_i + G_iF_i(t)E_{fi}$,respectively,(19) is equivalent to the following condition:

$$\begin{bmatrix} \Xi+\bar{\Xi} & \tilde{\varsigma}\aleph^T R_7 & \tilde{h}\Im^T R_8 \\ * & -\tilde{\varsigma}R_7 & 0 \\ * & * & -\tilde{h}R_8 \end{bmatrix} + \aleph_1^T F_i^T(t)\aleph_2^T + \aleph_2 F_i(t)\aleph_1 + \Im_1^T F_i^T(t)\Im_2^T + \Im_2 F_i(t)\Im_1 < 0 \quad (43)$$

According to Lemma 2,(43) is true if there exist two scalars $\varepsilon_1, \varepsilon_2 > 0$ such that the following inequality holds:

$$\begin{bmatrix} \Xi+\bar{\Xi} & \tilde{\varsigma}\aleph^T R_7 & \tilde{h}\Im^T R_8 \\ * & -\tilde{\varsigma}R_7 & 0 \\ * & * & -\tilde{h}R_8 \end{bmatrix} + \varepsilon_1^{-1}\aleph_2\aleph_2^T + \varepsilon_1\aleph_1^T\aleph_1 + \varepsilon_2^{-1}\Im_2\Im_2^T + \varepsilon_2\Im_1^T\Im_1 < 0 \quad (44)$$

Using the Schur complement shows that (44) is equivalent to (42).This completes the proof. ■

Remark 2 In this paper,Theorem 1 and Theorem 2 require the upper bound of the derivative of time-varying h_{Di}, ς_{Di} known.However,in practice,h_{Di}, ς_{Di} are unknown.Considering this situation,we can set $S_{ji} = 0, j = 1, 2, \ldots, 6$ in Theorem 1 and Theorem 2.

TABLE I
MAXIMUM VALUE OF $\tilde{\varsigma}$ WITH DIFFERENT l, k,UNKNOWN ς_D, BY THEOREM 1 IN EXAMPLE 1

Method	$h_D = 0.1$	$h_D = 0.3$	$h_D = 0.5$
$l = 1, k = 1$	0.692	0.541	0.437
$l = 1, k = 2$	1.573	1.268	1.025
$l = 2, k = 3$	1.917	1.901	1.873
$l = 3, k = 4$	2.589	2.448	2.098

TABLE II
MAXIMUM VALUE OF $\tilde{h}$ WITH DIFFERENT l, k,UNKNOWN h_D, BY THEOREM 1 IN EXAMPLE 1

Method	$\varsigma_D = 0.1$	$\varsigma_D = 0.3$	$\varsigma_D = 0.5$
$l = 1, k = 1$	0.753	0.675	0.542
$l = 1, k = 2$	1.178	1.025	0.978
$l = 2, k = 3$	1.769	1.561	1.252
$l = 3, k = 4$	2.364	2.237	2.034

IV. EXAMPLE

In this section,we provide one numerical example to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

Example 1 Consider delayed BAM neural networks with uncertainty Markovian jumping parameters as follows:

$$\begin{cases} \dot{x}(t) = -A_i x(t-\sigma) + C_i f(y(t)) + E_i f(y(t-h_i(t))) \\ \dot{y}(t) = -B_i y(t-\delta) + D_i g(x(t)) + F_i g(x(t-\varsigma_i(t))) \end{cases}$$

where

$$A_1 = \begin{bmatrix} 1.8 & 0 \\ 0 & 2.2 \end{bmatrix}, A_2 = \begin{bmatrix} 2.3 & 0 \\ 0 & 1.6 \end{bmatrix}, B_1 = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.9 & 0 \\ 0 & 3.1 \end{bmatrix}, C_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.4 & -0.3 \\ -0.8 & 0.1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, D_2 = \begin{bmatrix} -0.6 & -0.8 \\ 0 & 0.1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.5 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.3 & 0.6 \\ -0.5 & -0.9 \end{bmatrix}, F_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}, F_2 = \begin{bmatrix} -0.4 & 0.1 \\ 0.1 & -0.7 \end{bmatrix},$$

$$\pi = \begin{bmatrix} -7 & 7 \\ 6 & -6 \end{bmatrix}$$

In this example,we assume condition $\sigma = \delta = 0.1$.In Table I,we consider the case of $h_1 = h_2 = 0.1$,the upper bound of $\tilde{\varsigma}$ with different l, k,unknown ς_D.In Table II,we consider the other case of $\varsigma_1 = \varsigma_2 = 0.3$,the upper bound of $\tilde{h}$ with different l, k,unknown h_D.According to this two Tables,we can see this example shows that the stability condition gives much less conservative results in this paper.

V. CONCLUSION

In this present paper,we have investigated the problem of stability for uncertainty Markovian jumping parameters of BAM neural networks with leakage and discrete delays.Two sufficient conditions have been presented.The obtained criteria are less conservative because free-weighting matrices method and a convex optimization approach are considered.Finally,one example has been given to illustrate the effectiveness of the proposed method.

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REFERENCES

[1] Y.H.Du, S.M.Zhong, N.Zhou, L.Nie, W.Q.Wang, Exponential passivity of BAM neural networks with time-varying delays. Applied Mathematics and Computation 221 (2013) 727-740.

[2] H.W.Wang,Q.K.Song,C.G.Duan,LMI criteria on exponential stability of BAM neural networks with both time-varying delays and general activation functions, Math.Comput.Simul. 81 (2010) 837-850.

[3] Yang,R,Gao,H,Shi,P.Novel robust stability criteria for stochastic Hopfield neural networks with time delays.IEEE Transactions on Systems,Man and Cybernetics,Part B,(39)2,(2009)467-474.

[4] H. D. Qi, L. Qi, Deriving sufficient conditions for global asymptotic stability of delayed neural networks via nonsmooth analysis, IEEE Trans. Neural Netw., 2004, 15(1), pp. 99-109.

[5] H. Ye, N. Micheal, K. Wang, Robust stability of nonlinear time-delay systems with applications to neural networks, IEEE Trans. Circuits Syst. I, Fundam. Theory Appl., 1996, 43(7), pp. 532-543.

[6] Y.Ou, H.Liu, Y.Si, Z.Feng, Stability analysis of discrete-time stochastic neural networks with time-varying delay, Neurocomputing, 72 (2010) 740-748.

[7] Liu PL. Robust exponential stability for uncertain time-varying delay systems with delay dependence.Journal of The Franklin Institute 2009;346(10):958-968.

[8] Y. Liu, Z. Wang, and X. Liu, On global exponential stability of generalized stochastic neural networks with mixed time-delays, Neurocomputing, 2006, 70(1-3), pp. 314-326, Dec. 2006.

[9] S. Arik,Global asymptotic stability of hybrid bidirectional associative memory neural networks with time delays. Phys. Lett.A 351(2006) 85-91.

[10] Q.K. Song, Z.J. Zhao, Y.M. Li, Global exponential stability of BAM neural networks with distributed delays and reaction diffusion terms, Phys. Lett.A 335(2005) 213-225.

[11] J.H.Park,O.M.Kwon,Further results on state estimation for neural networks of neutral-type with time-varying delay,App. Math.. Comput. 208(2009) 69-57.

[12] Z.G.Wu, P.Shi, H.Su, J.Chu, Delay-dependent stability analysis for switched neural networks with time-varying delay,IEEE Trans. Syst.Man Cybern. Part B: Cybern, 41 (6) (2011) 1522-1530.

[13] J.K.Tian, S.M.Zhong, Improved delay-dependent stability for neural networks with time-varying delay, Appl.Math.Comput. 217 (2011) 10278-10288.

[14] Q. Zhang, X. Wei, and J. Xu, Delay-dependent exponential stability of cellular neural networks with time-varying delays, Chaos, Solitons Fractals, 2005, 23(4), pp. 1363-1369.

[15] K. Gu, V. K. Kharitonov, and J. Chen, Stability of Time-Delay Systems. Boston, MA: Birkhauser, 2003.

[16] M.S.Mahmoud, P.Shi, Robust stability,stabilization and H_∞ control of time-delay systems with Markovian jump parameters,Int.J. Robust Nonlinear Control 13(2003)755-784.

[17] O.M.Kwon, J.H.Park, Improved delay-dependent stability criterion for networks with time-varying, Phys Lett. A 373(2009) 529-535.

[18] X.F. Liao, G. Chen, E.N.Sanchez, Delay-dependent exponential stability analysis of delayed neural networks: an LMI approach, Neural Netw. 15(2002) 855-866.

[19] P. Park, J. W. Ko, and C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, Automatica, 2011, 47(1), pp. 235-238.

[20] L.Xie,Stochastic robust stability analysis for Markovian jumping neural networks with delays,in:Proceedings IEEE International Conferences on Networking, Sensing and Control,vol. 22,2005,pp. 923-928.

[21] Z.Wang, Y.Liu,X.Liu,State estimation for jumping recurrent neural networks with discrete and distributed delays, Neural Netw. 22(2009)41-48.

[22] X.Lou, B.Cui,Delay-dependent stochastic stability of delayed Hopfield neural networks with Markovian jump paramters, J.Math. Anal. Appl. 328(2007)316-326.

[23] K.Gu,An integral inequality in the stability problem of time delay systems, in: Proceedings of the 39th IEEE Conference on Decision Control,2000,pp.2805-2810.

[24] H.Liu, Y.Ou, J.Hu, T.Liu, Delay-dependent stability analysis for continuous-time BAM neural networks with Markovian jump paramters, Neural Netw.23(2010)315-321.

[25] K.Gopalsamy, Leakage delay in BAM,J.Math. Anal. 325(2007)1117-1132.

[26] S.Peng, Global attractive periodic solutions of BAM neural networks with continously distributed delays in the leakage terms, Nonlinear Anal. Real World Appl.11(2010)2141-2151.

[27] H.Bao, J.Cao, Stochastic global exponential for neutral-type impulsive impilsive neural networks with mixed time-delays and Markovian jumping paramters. Commun. Nonlinear Sci.Numer.Simul. 16(2011)3786-3791.

[28] W.Han, Y.Liu, L.Wang, Robust exponential stability of Markovian jumping neural networks with mode-dependent delay,Commun.Nonlinear Sci Numer.Simul 15(2010)2529-2535.

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