Effect of Hartmann Number on Free Convective Flow in a Square Cavity with Different Positions of Heated Square Block

Abdul Halim Bhuiyan, M. A. Alim, Md. Nasir Uddin

Abstract—This paper is concerned with the effect of Hartmann number on the free convective flow in a square cavity with different positions of heated square block. The two-dimensional Physical and mathematical model have been developed, and mathematical model includes the system of governing mass, momentum and energy equations are solved by the finite element method. The calculations have been computed for Prandtl number Pr = 0.71, the Rayleigh number Ra = 1000 and the different values of Hartmann number. The results are illustrated with the streamlines, isotherms, velocity and temperature fields as well as local Nusselt number.

Keywords—Finite element method, free convection, Hartmann number, square cavity.

I. INTRODUCTION

FREE convection is one of the most important phenomena in thermal system; this is due to its wide applications in nature and engineering such as oceanic currents, sea-wind, fluid flows around shrouded heat dissipation fins, free air cooling without the aid of fans, electronics cooling, heat exchangers and so on.

MHD natural convection flow and heat transfer in a laterally heated partitioned enclosure is investigated by Kahveci and Oztuna [1]. Taghikhani and Chavoshi [2] investigated two dimensional magneto-hydrodynamics (MHD) free convection with internal heating in a square cavity and observed the effect of the magnetic field which is to reduce the convective heat transfer inside the cavity. Parvin and Nasrin [3] analyzed the flow and heat transfer characteristics for MHD free convection in an enclosure with a heated obstacle and found that, buoyancy-induced vortex in the streamlines increase as well as the thermal layer near the heated surface becomes thick with increasing Rayleigh number. Ganzarolli and Milanez [4] numerically investigated natural convection in rectangular enclosure heated from below and symmetrically cooled from the sides. Sathiyamoorty et al. [5] studied the steady natural convection flow in a square cavity with linearly heated side walls. Nawaf [6] investigated the natural convection in a square porous cavity with an oscillating wall temperature. The results are presented to demonstrate the

temporal variation of the streamlines, the isotherms and the Nusselt number. The peak value of the average Nusselt number is observed to occur at the resonance nondimensional frequency of 450 in the range considered (1–2000) for Rayleigh number 10^3. Hakan et al. [7] analyzed the laminar MHD mixed convection flow in a top sided lid-driven cavity heated by corner heater and considered the temperature of the lid is lower than that of heater. Also Hakan et al. [8] studied the effects of volumetric heat sources on Natural convection in wavy-walled enclosures are studied numerically. Bakhshan and Ashoori [9] investigated the analysis of a fluid behavior in a rectangular enclosure under the effect of magnetic field. They observed that Nusselt number rises with increasing Grashof and Prandtl numbers and decreasing Hartmann and orientation of magnetic field.

Therefore, in the light of above literatures, the aim of the present work is to investigate the effects of Hartmann number on free convective flow in a square cavity with different positions of heated square block.

II. MODEL AND MATHEMATICAL FORMULATION

Fig. 1 shows a schematic diagram and the coordinates of a two-dimensional square cavity, where the bottom wall is maintained at a uniform temperature T_h and the top wall may be maintained cooled whereas side walls are kept insulated.
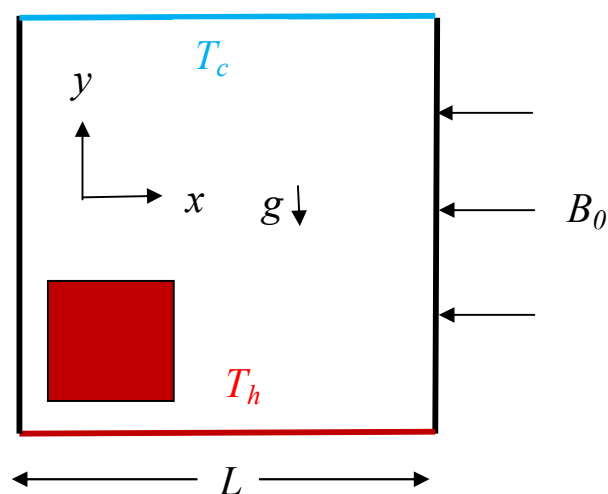

Fig. 1 The flow configuration and coordinate system

A heated square block is considered in a square cavity with different positions and Fig. 1 treats as left bottom

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configuration (LBC). The fluid is permeated by a uniform magnetic field B_0 which is applied normal to the direction of the flow and the gravitational force (g) acts in the vertically downward direction.

The fluid properties, including the electrical conductivity, are considered to be constant, except for the density, so that the Boussinesq approximation is used. Neglecting the radiation mode of the heat transfer and Joule heating, the governing equations for mass, momentum and energy of a steady tow-dimensional natural convection flow in a square cavity are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho g\beta(T - T_c) - \sigma B_0^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The governing equations are nondimensionalized using the following dimensionless variables:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad P = \frac{pL^2}{\rho\alpha^2},$$

$$\theta = \frac{T - T_c}{T_h - T_c}, \quad \sigma = \frac{\rho^2\alpha}{L^2}, \quad \alpha = \frac{k}{\rho C_p}, \quad v = \frac{\mu}{\rho}$$

Introducing the above dimensionless variables, the following dimensionless forms of the governing equations are obtained as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{5}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Ra}{\Pr}\theta - Ha^2\Pr V \tag{7}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \tag{8}$$

Here Pr is the Prandtl number, Ra is the Reyleigh number and Ha is the Hartmann number, which are defined as:

$$\Pr = \frac{v}{\alpha}, \quad Ha^2 = \frac{\sigma B_0^2 L^2}{\mu}, \quad Ra = \frac{g\beta L^3(T_h - T_c)\Pr}{v^2}$$

The corresponding boundary conditions then take the following form:

$$U = V = 0, \theta = 1 \begin{pmatrix} \text{at bottom wall of the cavity and} \\ \text{heated square block} \end{pmatrix}$$

$$U = V = 0, \theta = 0 \,(\text{at top wall})$$

$$U = V = 0, \frac{\partial \theta}{\partial N} = 0 \,(\text{at side walls})$$

$$P = 0 \begin{pmatrix} \text{fluid pressure, at the inside and} \\ \text{on the wall of the enclosure} \end{pmatrix}$$

III. NUMERICAL PROCEDURE

The Galerkin weighted residual method of finite element formulation is used to solve the dimensionless governing equations with the boundary conditions. This technique is well described by Taylor and Hood [10], and Dechaumphai [11]. In this method, the solution domain is discretized into finite element meshes and then the nonlinear governing equations are transferred into a system of integral equations by applying the Galerkin weighted residual method. Gauss quadrature method is used to perform the integration involved in each term of these equations. The nonlinear algebraic equations which are obtained are modified by imposition of boundary conditions and Newton's method is used to transform these modified equations into linear algebraic equations, and then these linear equations are solved by applying the triangular factorization method.

IV. CODE VALIDATION

In order to verify the accuracy of the numerical results which are obtained throughout the present study are compared with the previously published results. The present results of streamlines and isotherms are compared with that of Santosh et al. [12] while Pr = 0.71 and $Ra = 10^5$ and obtained good agreement which is shown in Fig. 2.
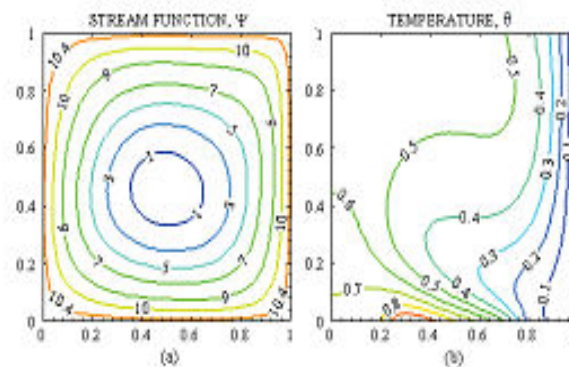

Fig. 2 (a) Obtained results by Santosh et al. [12]

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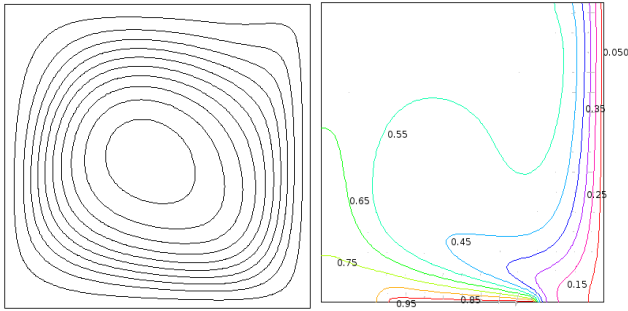Vol:8, No:2, 2014
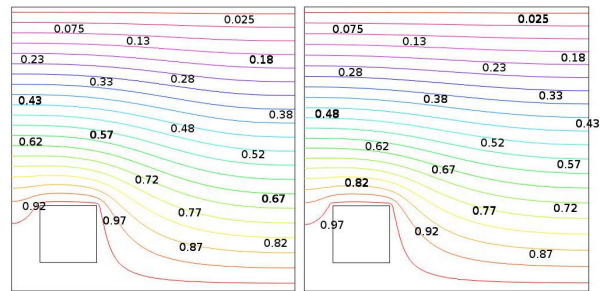
Fig. 2 (b) Obtained results by present work

V. Results and Discussions

A numerical analysis has been performed in this work to investigate the effects of various cavity configurations on the flow field while the values of Ra and Pr are fixed at 1000 and 0.71. The influence of the Hartmann number Ha (Ha = 0, 50, 100) on streamlines and isotherms are presented in Figs. 3 and 4 respectively, for left bottom configuration (LBC). From Fig. 3 (a), it is seen that, in absence of Hartmann number, one cell is formed inside the cavity. With increasing of Hartmann number, the flow strength decreases and the streamlines close to the heated square block, which are observed in Figs. 3 (b) and 3 (c). As seen the isotherms in Fig. 4, the isotherms are like as linear as well as nonlinear near the top wall and bottom wall respectively with the increase of Hartmann number. Moreover, the temperature of the flow field increases due to increase of Hartmann number Ha. Figs. 5 and 6 present the effect of Hartmann number Ha (Ha = 0, 50, 100) for right bottom configuration (RBC) on the velocity and temperature distributions while Ra = 1000 and Pr = 0.71. A small recirculation cell appears center of the square of the cavity and recirculation cell becomes smaller with the increase of Hartmann number Ha which are observed in Fig. 5. As observed in Fig. 6, the isotherms are parallel to the top wall of the square cavity but nonlinearity effect is found in the near bottom wall of the square cavity. Figs. 7 and 8 illustrate the streamlines and isotherms for the left top configuration (LTC) with variations of Hartmann number Ha (Ha = 0, 50, 100) while Ra = 1000 and Pr = 0.71. From Fig. 7, it is seen that one cell is formed center of the square cavity in absence of magnetic field. On the other hand, increasing of the Hartmann number, the cell which is formed in the square cavity, moves to the top wall and becomes bigger.
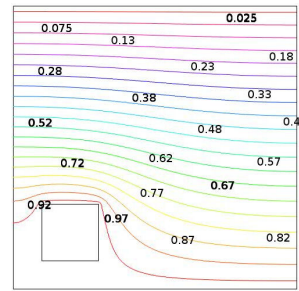

(a)   (b)
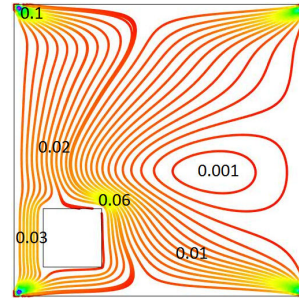
(c)

Fig. 3 Streamlines for (a) Ha = 0; (b) Ha = 50; (c) Ha = 100 while Pr = 0.71 and Ra = 1000 for LBC

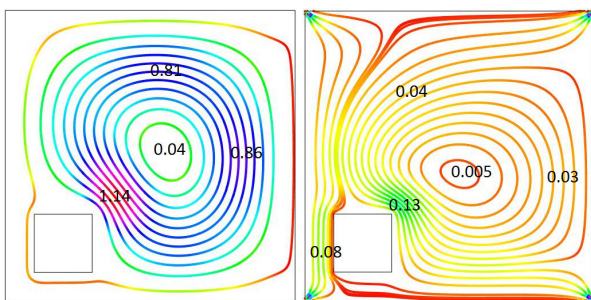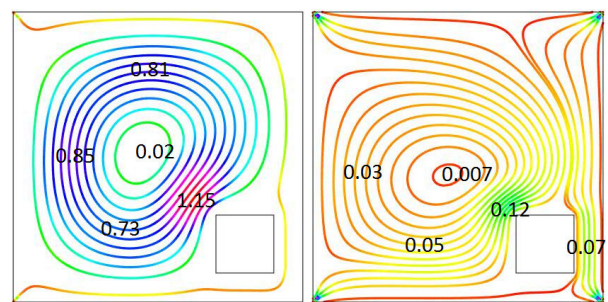

(a)   (b)

(c)

Fig. 4 Isotherms for (a) Ha = 0; (b) Ha = 50; (c) Ha = 100 while Pr = 0.71 and Ra = 1000 for LBC
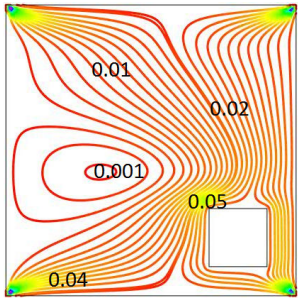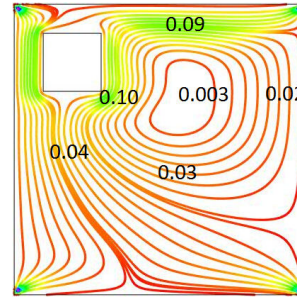

(a)   (b)

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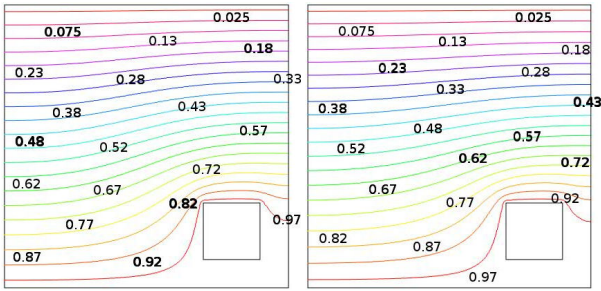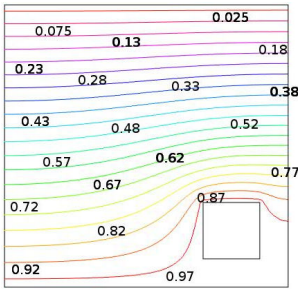Vol:8, No:2, 2014

(c)

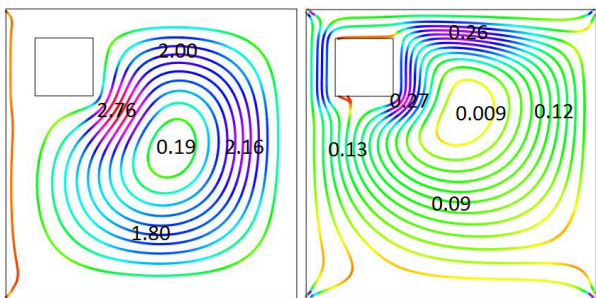Fig. 5 Streamlines for (a) *Ha* = 0; (b) *Ha* = 50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for RBC

(a)

(b)

(c)

Fig. 6 Isotherms for (a) *Ha* =0; (b) *Ha* =50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for RBC

(a)

(b)

(c)

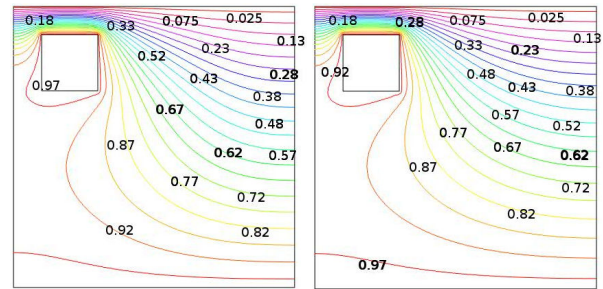Fig. 7 Streamlines for (a) *Ha* =0; (b) *Ha* =50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for LTC

The isotherms, in Fig. 8 are like as linear at the near top wall but bending at the near bottom wall with the effect of Hartmann number. Figs. 9 and 10 show the variation of Hartmann number on the square cavity for right top configuration (RTC). The cell which is formed in the square cavity is smaller comparing with the LTC in absences of *Ha* and presences of *Ha*. Increasing the Hartmann number, the Isotherms change insignificantly as observed in Fig. 10.

(a)

(b)

(c)

Fig. 8 Isotherms for (a) *Ha* =0; (b) *Ha* =50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for LTC

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(a)   (b)

(c)

Fig. 9 Streamlines for (a) *Ha* =0; (b) *Ha* =50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for RTC

(a)   (b)

(c)

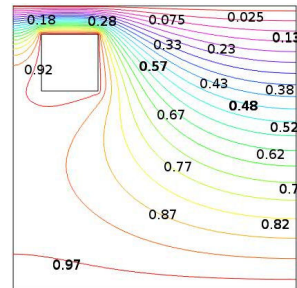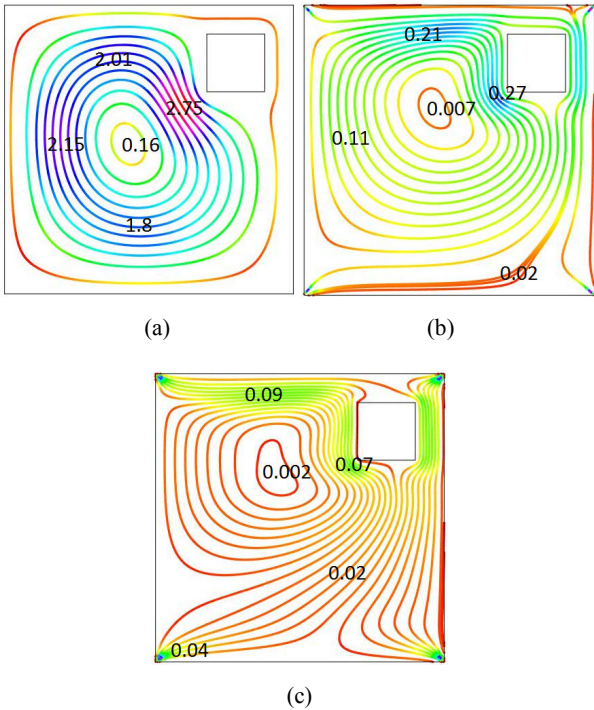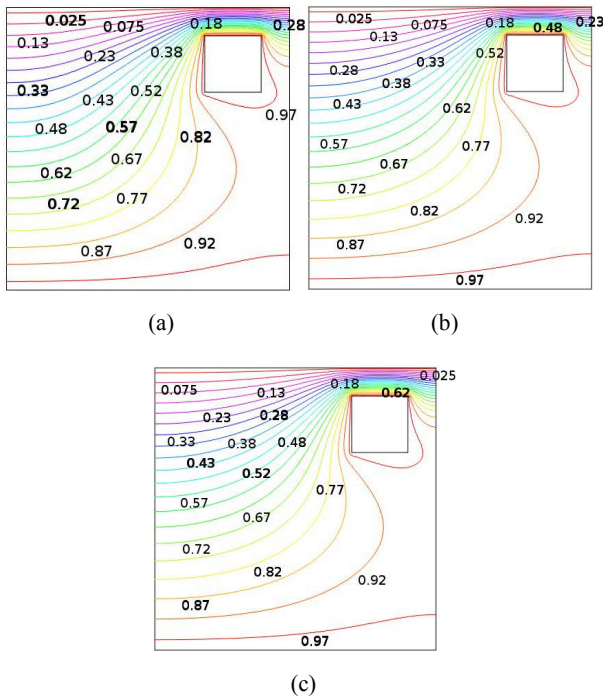Fig. 10 Isotherms for (a) *Ha* =0; (b) *Ha* =50; (c) *Ha* =100 while Pr = 0.71 and *Ra* = 1000 for RTC

Fig. 11 presents the effects of Hartmann number *Ha* on the flow field as velocity profiles for different configurations along the line *Y* = 0.5. As seen from the Fig. 11 for LBC, the velocity decreases with the increase of Hartmann number below the middle of the cavity, but the velocity increases with the increase of Hartmann number above the middle of the

cavity. This figure indicates that the velocity field becomes maximum and minimum points with *Ha* = 0, which are observed due to counterclockwise and clockwise flow directions. Similar results shows for RTC configurations and inversely for the RBC as well as LTC, respectively. The temperature fields versus the coordinate of *X* directions are plotted in Fig. 12 for different configurations. As seen from Fig. 12 for LBC, absence of Hartmann number, the maximum and minimum temperature are obtained. In presence of increasing Hartmann number, the temperature field decreases. An inverse result is observed for RBC. For LTC, the temperature field changes insignificantly with the increase of Hartmann number as *X* < 0.5 but changes significantly as *X* > 0.5 and for RTC, the temperature field changes insignificantly with the increase of Hartmann number as *X* > 0.8 but changes significantly as *X* < 0.8. The local Nusselt number at the top wall for different configurations is presented in Fig. 13 for variation of Hartmann number. From the first and fourth configurations (LBC and RTC), it is seen that the local Nusselt number decreases with the increase of Hartmann number but from the second and third configurations (RBC and LTC), it is observed that the local Nusselt number increases due to increasing Hartmann number.
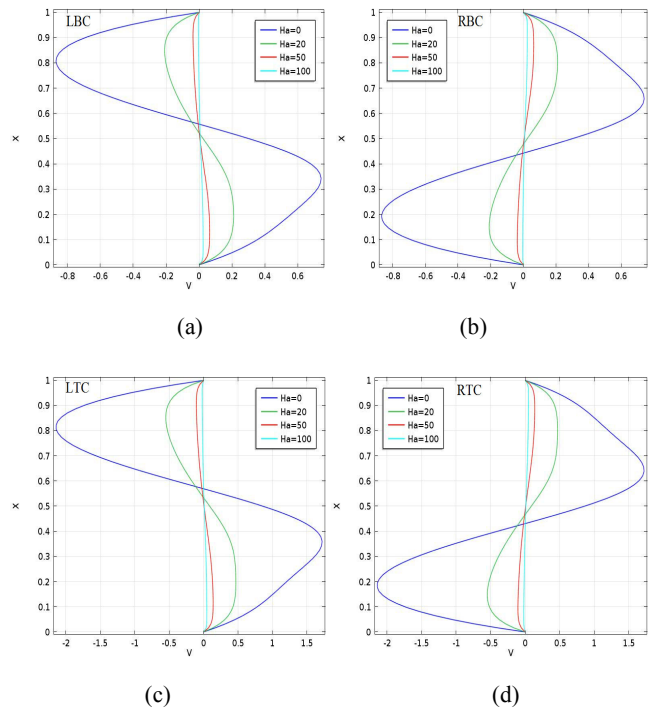

(a)   (b)

(c)   (d)

Fig. 11 Velocity profiles for different configurations along the line *Y* = 0.5 for Pr = 0.71 and *Ra* = 1000.

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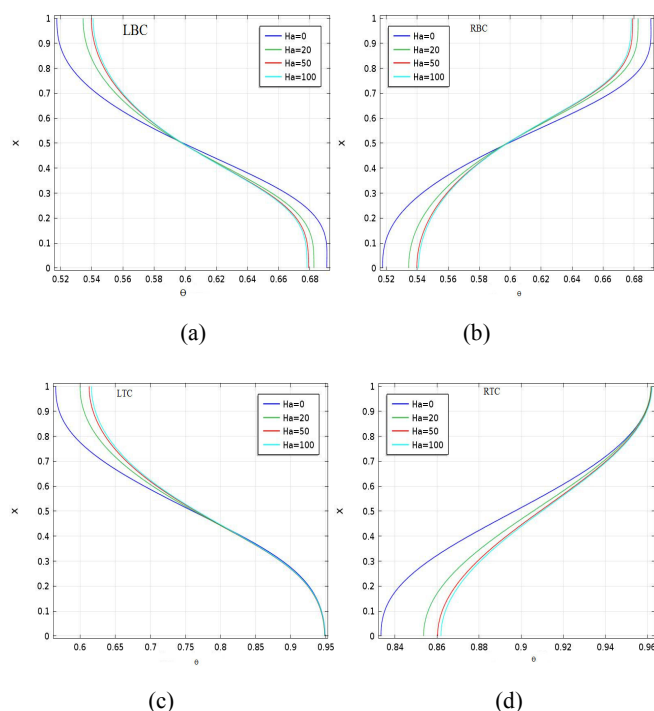Vol:8, No:2, 2014

(a)

(b)

(c)

(d)

Fig. 12 Temperature profiles for different configurations along the line $Y = 0.5$ for Pr = 0.71 and Ra = 1000
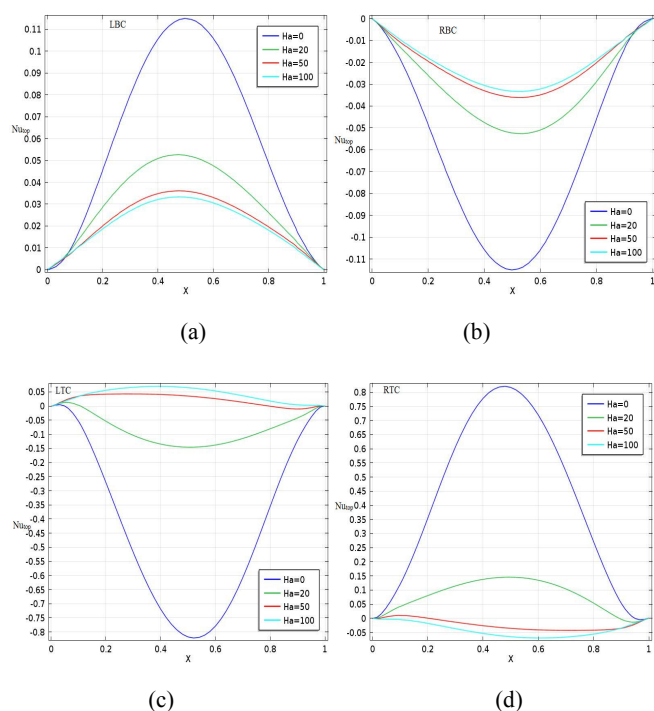

(a)

(b)

(c)

(d)

Fig. 13 Local Nusselt number for different configurations for Pr = 0.71 and Ra = 1000.

VI. CONCLUSION

Heat transfer by free convection of a heated square block in a square cavity with uniform magnetic field B_0 which is applied normal to the direction of the flow was studied numerically. The conservation of mass, momentum and energy equations were solved using the Galerkin weighted residual method of finite element formulation. As indicated above that the governing parameters were the Prandtl number Pr, the Rayleigh number Ra and the Hartmann number Ha. The effects of Hartmann number Ha due to heated square block, Prandtl number Pr and Rayleigh number Ra on the flow and temperature field have been studied in detail. From the present investigation the following conclusions may be drawn: if the Hartman number increases, the local Nusselt number at the top wall of the cavity decreases for LBC and RTC but increase for the RBC and LTC.

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