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Precision Assessment of the Orthometric Heights Determination in the Northern Part of Libya

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***Abstract*—**The Global Positioning System (GPS), satellite-based technology, has been utilized extensively in the last few years in a wide range of Geometrics and Geographic Information Systems' (GIS) applications. One of the main challenges dealing with GPS-based heights consists of converting them into Mean Sea Level (MSL) heights, which is used in surveys and mapping.

In this research's work, differences in heights of 50 points, in northern part of Libya has been carried out by using both ordinary leveling (in which Geoid is the reference datum) and GPS techniques (in which Ellipsoid is the reference datum). In addition, this study utilized the EGM2008 model to obtain the undulation values between the ellipsoidal and orthometric heights. From these values of ellipsoidal heights can be obtained from GPS observations to compute the orthomteric heights. This research presents a suitable alternative, from an economical point of view, to substitute the expensive traditional leveling technique, particularly, for topographic mapping.

***Keywords*—**Geoid undulation, GPS, ordinary and geodetic leveling, orthometric height.

I. INTRODUCTION

THE Number of quantities has to be measured in usual survey's works. These quantities may be bearings, angles, or distances (horizontal, vertical or inclined). Reduction of these values leads to points' determination, relative position.

Leveling is the name had been given to the process of measuring the difference in elevation between two or more points. The heights of points relative to a chosen surface are known as the reduced levels of these points. Reference surface is usually known as a datum. Leveling has many applications in engineering surveying that used in all stages of construction projects from the initial site survey to the final setting out [5].

Leveling process can be carried out by using different equipments and techniques of survey. It may be precise; when precise optical or digital levels had been used, or it may be ordinary; upon using ordinary optical, automatic or digital levels. Moreover it could be carried out trigonometrically or barometrically.

II. ORDINARY AND GEODETIC LEVELING

For short distances, the difference in level between two points can easily be determined by setting a staff vertically in each point and then constructing a horizontal line by using optical or digital level. The difference in the intersection points, of the horizontal line with both staves, produces the difference in level between these ground points Fig. 1. This technique is known as ordinary leveling [9].

For long distances, considering geoid undulation of the earth, direct leveling doesn't solve the problem and it will be more complicated. This is so because horizontal line doesn't solve the problem directly. Leveling technique, taking the geoid undulation into account, is usually known as geodetic leveling.

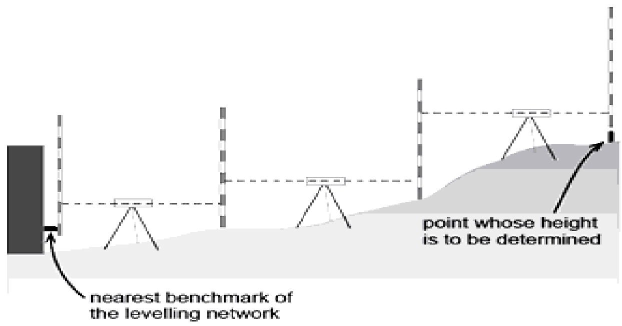

Fig. 1 Ordinary leveling

III. OVERVIEW OF GPS

GPS is a satellite navigation system established by the United State Department of Defense (DoD), designed to provide 3-dimensional positioning, velocity, and time information, regardless of the weather, at any time and almost anywhere on the globe. There are at least 24 GPS satellites in 6 orbital planes, each satellite transmits two signals continuously, which are created from a fundamental signal (frequency=10.15 MHz), driven by the GPS satellite's atomic clock. The initial frequencies are L1 and L2. The L1 carrier signal is formed by multiplying the fundamental signal by 154 (frequency=1575.42 MHz, wavelength 19.0cm). The L2 carrier signal is formed by multiplying the fundamental signal by 120 (frequency =1227.60 MHz, wavelength 24.0cm). The L2 signal is used for calibration of signal delay due to the Earth's ionosphere [3]. Additionally, the new civil GPS signals have been developed for different purposes. These signals are L2C, L5 and L1 [6]. Table I shows main information of these signals.

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TABLE I
NEW GPS SIGNALS

Signal	Benefits	Number of satellites broadcasting now	Availability on 24 satellites
L2C	Meets commercial needs for ionosphiric correction	10	~2018
L5	Meets requirements for safety of life transportation	3	~2021
L1C	GNSS interoperability; performance improvements in challenged environments	Begins lunching in 2015 with GPS III	~2026

IV. Ellipsoid and Geoid

The geoid is a surface of constant potential energy that coincides with the mean sea level over the oceans. This definition is not very rigorous. Firstly, mean sea level is not quite a surface of constant potential due to dynamic processes within the ocean. Secondly, the actual equipotential surface under continents is warped by the gravitational attraction of the overlying mass. But geodesists define the geoid as though that mass were always underneath the geoid instead of being above it. The main function of the geoid in geodesy is to serve as a reference surface for leveling. The elevation, measured by leveling, is relative to the geoid.

Regular shape could be approximated by the Ellipsoid. Ellipsoid is a mathematical surface obtained by revolving an ellipse at about its minor axis. Leveling, based on taking the geoid as a reference datum, is then termed geodetic leveling. The dimensions of the ellipse are selected to give the best fit of the ellipsoid to the geoid over a large area, and are based upon surveys made in the area.

The two-dimensional views, which illustrate conceptually the geoid and ellipsoid, are shown in Fig. 2. As illustrated, the geoid contains non uniform undulations, and therefore it's not readily defined mathematically. Ellipsoids, which approximate the geoid and can be defined mathematically, are therefore used to compute positions of widely spaced points that are located.

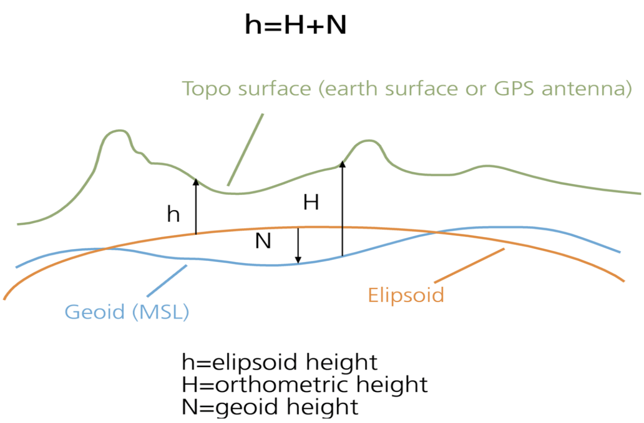

Fig. 2 Geoid and ellipsoid surfaces

V. Previous Libyan Ellipsoid and Geiod

A. Libyan Ellipsoid

The Libyan first datum, defined for geodetic network and mapping, is ELD79 based on Hayford International Ellipsoid 1924 (a = 6378388, f =1/297). In the 1980's, Libya established a Doppler network, which was initially defined in the WGS-72 datum and then in the International Terrestrial Reference Frame 2000 (ITRF00) datum. Surveying Department of Libya (SDL), later introduced LGD2006, again based on Hayford International ellipsoid 1924 (a = 6378388, f = 1/297).

For this purpose, some of 61 stations were GPS surveyed in 2006 and tied to IGS stations (Epoch: 2006.3822). Thus, precise coordinates were determined in the ITRF00 datum, based on GRS80 ellipsoid (a=6378137.0 m, f=1/298.25722101) [8].

B. Libyan Geoid

Two practical attempts had been tried to compute the Libyan geoid [1]. None of these attempts are included the usage of gravimetric data. The used data and the computation methods are concisely summarized in this section. The plotted results of these two attempts are also provided as below:

1- A geoid map, based on 19 leveled Doppler points, was computed by IGN for the benefit of the Surveying Department of Libya. In its computation, IGN had been used as the only source of data, the deflections of the vertical, which obtained from the comparison of the astronomical coordinates and their respective WGS 72 to the leveled Doppler points above, the same ellipsoid. Observation relations, linking the undulations of the geoid to the ones directly obtained from subtracting the orthometric height from the ellipsoidal height, were formed. A geoid computation program called GEOIDE that solves the system of normal equations and predicts the geoid undulations, was used. From the result of these computations, IGN drew the map of Libyan geoidal undulations [4] (see Fig. 3).

2- A second geoid map was computed by an American company called the Aero-Service Corporation (ASC). Two geoid maps, with relation to two different datums , were computed: one with relation to the local ELD79 datum (see Fig. 4), and the other with relation to WGS 72 datum (see Fig. 5) [2]. In its method, the company used a modal, generated from comparing the vertical deflections differences of each the available 155 Laplace point from all other Laplace points within 1.5 weighted least squares solution considered at IGN stations (see above), where the undulations, considered to be known, were obtained the weight of each observation was taken as inversely proportional to the linear separation between the station.

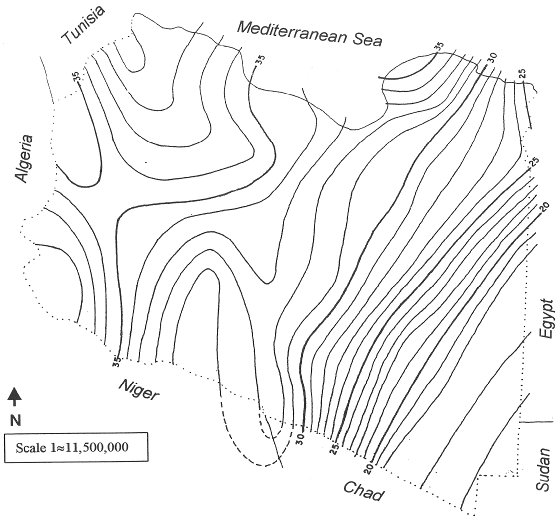

Fig. 3 IGN Libyan geoid undulation map (WGS72 datum)

Fig. 4 ASC Libyan geoid undulation map (ELD79 datum)

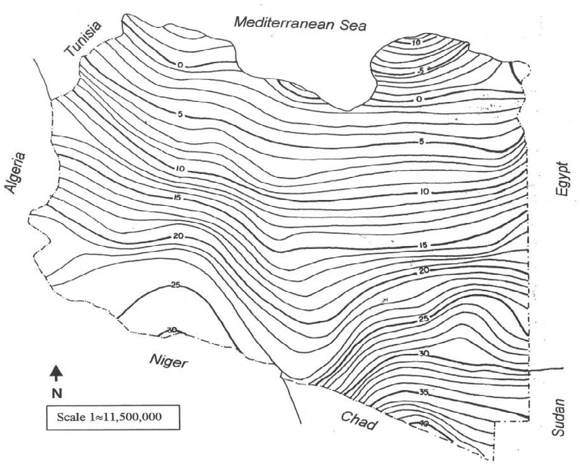

Fig. 5 ASC Libyan geoid undulation map (WGS72 datum)

VI. The EGM2008 Geopotential Model

The U.S. National Geospatial-Intelligence Agency (NGA) has released publicly in mid 2008. This gravitational model be completed to spherical harmonic degree and order 2159, contains additional coefficients extending to degree 2190 and order 2159. The WGS 84 constants are used to define the reference ellipsoid and the associated normal gravity field, to which the geoid undulations are referenced as follow:

- a = 6378137.00m (semi-major axis of WGS 84 ellipsoid),
- f = 1/298.257223563 (flattening of WGS 84 ellipsoid),
- GM = 3.986004418 x 1014 m3s-2 (Product of the Earth's mass and the Gravitational Constant),
- ω = 7292115 x 10-11 radians/sec (Earth's angular velocity).

All synthesis software, coefficients and pre-computed geoid grids assume a tide free system, as far as permanent tide is concerned [7].

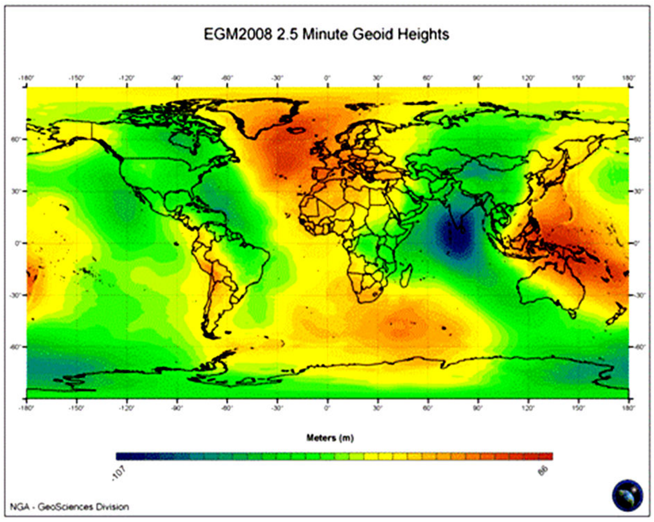

Fig. 6 Gravitational Model EGM2008

VII. Measurements and Results

In this research's work, ordinary leveling was carried out for 50 points in northern part of Libya. Automatic level had been used to do the job. Level line was started at a known benchmark and end at another one. Observations had been reduced and adjusted. Orthometric heights (H) were obtained on Libyan reference datum. GPS observations were carried out along the same line. Observations had been processed and adjusted to compute ellipsoidal height (h) of all points. Comparison between orthometric height of points and ellipsoidal heights of the same points, can be carried out by computing the undulation (N) between both results as shown in Table II. These differences represent the geoid ellipsoid separation in the area.

TABLE II
GEIOD ELLIPSIOD SEPARATION (N)

Point No.	Ellipsoid height (h) (m)	Orthometric height (H) (m)	Undulation N=h-H (m)
1	61.615	33.394	28.221
2	66.155	37.884	28.271
3	87.859	59.541	28.318
4	44.589	16.098	28.491
5	45.125	16.896	28.229
6	39.1	10.964	28.136
7	68.873	40.822	28.051
8	37.573	9.628	27.945
9	37.409	9.586	27.823
10	45.052	17.338	27.714
11	28.954	1.356	27.598
12	34.299	6.814	27.485
13	37.925	10.561	27.364
14	41.553	14.334	27.219
15	39.187	12.214	26.973
16	40.969	14.122	26.847
17	34.521	8.022	26.499
18	27.475	1.001	26.474
19	43.508	17.027	26.481
20	53.396	27.037	26.359
21	56.474	30.117	26.357
22	26.702	0.414	26.288
23	42.656	16.429	26.227
24	44.857	18.637	26.22
25	25.545	-0.705	26.25
26	47.947	21.674	26.273
27	25.407	-0.868	26.275
28	41.539	15.341	26.198
29	30.622	4.403	26.219
30	21.776	-3.33	25.106
31	24.048	-2.272	26.32
32	23.954	-2.431	26.385
33	52.214	25.651	26.563
34	45.777	19.06	26.717
35	36.301	8.44	27.861
36	33.468	6.473	26.995
37	43.846	16.787	27.059
38	53.269	26.052	27.217
39	43.963	16.527	27.436
40	50.049	22.419	27.63
41	56.033	28.224	27.809
42	57.92	29.953	27.967
43	43.573	15.318	28.255
44	46.379	17.944	28.435
45	32.694	4.09	28.604
46	64.241	35.437	28.804
47	61.329	32.367	28.962
48	43.033	14.037	28.996
49	41.806	12.581	29.225
50	51.68	22.234	29.446

Practically, ellipsoidal heights can be reduced to orthometric heights, if geoid ellipsoid separation (N) is known. In this work, EGM2008 geodetic model was used to compute geoid ellipsoid separation (undulation). Reduced geoid undulations for the observed points are listed in Table III.

TABLE III
REDUCED EGM2008 GEIOD ELLIPOSOID SEPARATION (N)

Point No.	Latitude °	′	″	Longitude °	′	″	EGM2008 (N) m
1	30	51	29.692	17	50	10.028	28.559
2	30	49	58.673	17	55	23.937	28.614
3	30	48	10.548	18	3	12.222	28.621
4	30	46	36.144	18	7	50.176	28.574
5	30	44	31.639	18	12	5.973	28.55
6	30	41	25.325	18	14	59.211	28.496
7	30	37	52.07	18	18	21.17	28.388
8	30	34	53.268	18	24	44.099	28.199
9	30	31	30.841	18	29	40.448	28.07
10	30	28	19.84	18	33	0.3996	27.968
11	30	23	51.456	18	38	0.5252	27.808
12	30	21	7.3842	18	43	34.214	27.667
13	30	19	32.803	18	48	27.402	27.554
14	30	17	41.672	18	54	19.479	27.394
15	30	14	36.803	18	59	54.007	27.184
16	30	14	10.335	19	4	24.513	27.068
17	30	14	16.709	19	8	50.72	26.953
18	30	14	58.808	19	14	34.88	26.836
19	30	15	16.165	19	20	32.67	26.73
20	30	16	3.8092	19	26	38.396	26.604
21	30	16	0.8909	19	26	44.251	26.602
22	30	19	22.739	19	30	20.854	26.558
23	30	22	14.979	19	35	5.8542	26.51
24	30	24	35.576	19	40	19.721	26.5
25	30	26	40.087	19	45	30.756	26.555
26	30	24	52.447	19	49	59.427	26.532
27	30	28	26.127	19	55	10.322	26.518
28	30	30	51.757	20	0	17.532	26.475
29	30	34	3.7792	20	4	48.96	26.453
30	30	36	0.5795	20	6	47.5	26.445
31	30	40	10.148	20	10	0.7958	26.516
32	30	42	22.292	20	11	30.135	26.575
33	30	46	59.044	20	16	11.233	26.748
34	30	51	30.405	20	14	16.114	26.891
35	30	55	50.756	20	11	13.98	27.049
36	30	58	42.276	20	11	6.4954	27.146
37	31	2	28.56	20	12	46.662	27.265
38	31	6	31.2	20	14	1.2283	27.374
39	31	12	43.531	20	14	14.97	27.575
40	31	17	58.601	20	13	5.2596	27.762
41	31	22	24.836	20	11	19.442	27.939
42	31	27	42.459	20	9	24.568	28.164
43	31	32	14.802	20	4	47.472	28.324
44	31	38	1.2252	20	1	42.487	28.544
45	31	42	12.595	19	57	36.465	28.651
46	31	46	39.198	20	1	0.0107	28.852
47	31	51	34.659	20	1	2.161	29.018
48	31	56	0.208	19	58	8.0644	29.087
49	32	0	56.727	20	3	6.2773	29.373
50	32	3	15.747	20	6	33.715	29.58

From the obtained results, the average separation can be computed to be 27.588m. Ortometric heights are then calculated by adding geodetic heights (H) to geoid separation (N) by using the following equation:

$$N = H - h \tag{1}$$

$$\text{or} \quad H=N+h \tag{2}$$

where, H represents orthometric height, h represents Ellipsoidal height, N represents geoid ellipsoid undulation.

Computed orhtometric heights, obtained by using (2) and Table III and then compared with actual ortometric heights Table IV, shows the difference between computed heights and actual ortometric heights.

TABLE IV
DIFFERENCE BETWEEN COMPUTED AND ACTUAL ORTHOMETRIC HEIGHTS

Point No.	Computed ortometric height(H) (m)	Actual ortometric height(H) (m)	Difference
1	33.056	33.394	0.338
2	37.541	37.884	0.343
3	59.238	59.541	0.303
4	16.015	16.098	0.083
5	16.575	16.896	0.321
6	10.604	10.964	0.36
7	40.485	40.822	0.337
8	9.374	9.628	0.254
9	9.339	9.586	0.247
10	17.084	17.338	0.254
11	1.146	1.356	0.21
12	6.632	6.814	0.182
13	10.371	10.561	0.19
14	14.159	14.334	0.175
15	12.003	12.214	0.211
16	13.901	14.122	0.221
17	7.568	8.022	0.454
18	0.639	1.001	0.362
19	16.778	17.027	0.249
20	26.792	27.037	0.245
21	29.872	30.117	0.245
22	0.144	0.414	0.27
23	16.146	16.429	0.283
24	18.357	18.637	0.28
25	-1.01	-0.705	0.305
26	21.415	21.674	0.259
27	-1.111	-0.868	0.243
28	15.064	15.341	0.277
29	4.169	4.403	0.234
30	-4.669	-3.33	1.339
31	-2.468	-2.272	0.196
32	-2.621	-2.431	0.19
33	25.466	25.651	0.185
34	18.886	19.06	0.174
35	9.252	8.44	-0.812
36	6.322	6.473	0.151
37	16.581	16.787	0.206
38	25.895	26.052	0.157
39	16.388	16.527	0.139
40	22.287	22.419	0.132
41	28.094	28.224	0.13
42	29.756	29.953	0.197
43	15.249	15.318	0.069
44	17.835	17.944	0.109
45	4.043	4.09	0.047
46	35.389	35.437	0.048
47	32.311	32.367	0.056
48	13.946	14.037	0.091
49	12.433	12.581	0.148
50	22.1	22.234	0.134
Average Difference=			0.216
Root Mean Square Error of Difference =			0.233

VIII. CONCLUSION

Referring to the results obtained above and analysis carried out. It could be directly concluded that:

1. Success of direct utilization of GPS observation in leveling depends on the value of geoid ellipsoid separation in the area. This separation was estimated in the study area with around 28m.
2. Orthometric heights can be reduced from GPS observation by obtaining the geoid ellipsoid separation (geoid undulation) with aid of one of available geoid models. This can easily be done by subtracting resultant undulation from GPS heights.
3. Applying EGM2008 geoid model to GPS observation in our study area, can successfully produces results estimated with 0.216m accuracy in orthometric heights. This accuracy in heights can be used to produce contour map scales of 1:5000 and smaller according to surveying department of Libya specification.

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