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Design of Adaptive Controller Based On Lyapunov Stability for a CSTR

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***Abstract*—**Nonlinearity is the inherent characteristics of all the industrial processes. The Classical control approach used for a generation often fails to show better results particularly for non-linear systems and in the systems, whose parameters changes over a period of time for a variety of reasons. Alternatively, adaptive control strategies provide very good performance. The Model Reference Adaptive Control based on Lyapunov stability analysis and classical PI control strategies are designed and evaluated for Continuous Stirred Tank Reactor, which shows appreciable dynamic nonlinear characteristics.

***Keywords*—**Adaptive Control, CSTR, Lyapunov stability, MRAS, PID.

I. Introduction

ALMOST all the industrial process parameters change over time for various reasons like equipment change, change in operating conditions of the units, change in market demand. Consequently, a conventional control technique may not provide effective control of complex processes where process parameter changes can occur significantly, but cannot be measured or anticipated [5]. The classical control methods are normally a feedback method relies on monitoring the change in the process variable with respect to the set point and control designed for worst case conditions. Alternatively, adaptive control strategies are available where controller parameters and/or control structure are modified online as conditions change.

The continuous Stirred Tank Reactor (CSTR) is one of the primary unit operations in many chemical industries, exhibit reasonably high nonlinear behavior. Hence a CSTR modeling and its multiple operating conditions are studied and MRAS concepts are demonstrated through simulation.

In this paper, a Model Reference Adaptive System is designed to make use of Lyapunov stability analysis. The Lyapunov method attempts to find the Lyapunov function and an adaptation mechanism such a way the error between plant and model goes to zero.

II. Model Reference Adaptive Control

The basic philosophy of designing a linear controller (in a deterministic environment) assumes knowledge of the plant dynamic model and of the desired performances. In most cases the desired performances of the feedback control system can be specified in terms of the characteristics of a dynamic system that is a "realization" of the desired behavior of the closed-loop system [4], [6]. For example, a tracking objective can be specified in terms of the desired input-output behavior by a given transfer function. A regulation objective can be specified in terms of the evolution of the output starting from an initial disturbed value by specifying the desired pole location of the closed loop (i.e., by a given transfer function). The controller is designed such that for a given plant model the closed-loop control system has the characteristics of the desired dynamic system.

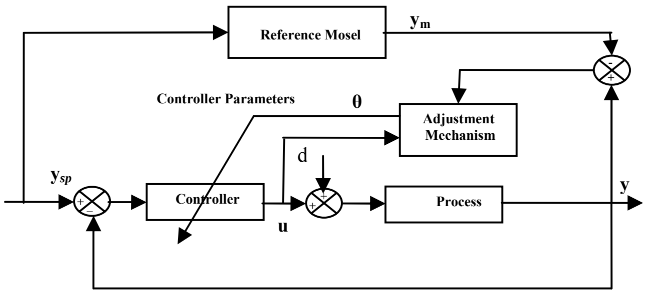

Fig. 1 Block Diagram of Model Reference Adaptive systems

The controller is now designed such that (1) the error between the output of the plant and the output of the reference model is identically zero for identical conditions, and (2) an initial error will vanish with a certain dynamics. When the plant parameters are unknown or change with time, in order to achieve the desired performances an adaptive control approach has to be considered. Fig. 1 shows the block diagram of MRAS. The reference model is the realization of system with the desired performance. This scheme is based on the observation that the difference between the output of the plant and the output of the reference model (subsequently called plant-model error) is a measure of the difference between the real and the desired performance. This information is used through the adaptation mechanism (which also receives other information) to adjust the parameters of the controller automatically in order to force asymptotically the plant-model error to zero [1].

III. MRAC with Lyapunov Stability Method

Usually the reference model is assumed as a first order system with a differential equation shown below

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \tag{1}$$

the process to be controlled is described as a first order model

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$$\frac{dy}{dt} = -ay + bu \quad (2)$$

let the controller be

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \quad (3)$$

and the error be

$$e = y - y_m \quad (4)$$

Hence

$$\frac{de}{dt} = -a_m - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \quad (5)$$

the error goes zero when the parameters

$$\theta_1 = \theta_1^0 = \frac{b_m}{b} \quad (6)$$

$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b} \quad (7)$$

To ascertain Lyapunov stability, while parameter adjustment mechanism drive the parameters $\theta_1$and θ_2 to their desired values, a quadratic function is introduced.

$$V(e, \theta_{1,}\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right) \quad (8)$$

This function is zero when e is zero and the controller parameters are equal to the correct values. If the derivative

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$
$$= -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right) + \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + \gamma u_c e\right) (9)$$

is negative, the above mentioned quadratic function is said to be a Lyapunov function. If the parameters are updated as

$$\frac{d\theta_1}{dt} = -\gamma u_c e \quad (10)$$

$$\frac{d\theta_2}{dt} = \gamma y e \quad (11)$$

the derivative

$$\frac{dV}{dt} = -a_m e^2 \quad (12)$$

is thus negative semi definite. This implies that $V(t) \leq V(0)$ and hence e,θ_1 and $\theta_2$must be bounded. As a result the output of the system $y = e + y_m$ is also bounded.

The Lyapunov stability based method avoids the stability problems present in the gradient approaches.

The adjustment law based on Lyapunov stability is given by

$$\frac{d\theta}{dt} = -\gamma e \theta \quad (13)$$

The major difference between the gradient rule and the Lyapunov method is that the sensitivity of the error to a specified parameter $\frac{\partial e}{\partial \theta}$ has been replaced by the actual value of the parameter, θ.

TABLE I
MODEL PARAMETERS AND ASSUMPTIONS [2]

Symbol	Quantity
Ea	32400 BTU/lbmol
Kc	15312 hr^{-1}
dH	-45000 BTU/lbmol
U	75 BTU/hr-ft^2-°F
R	1.987 BTU/lbmol°F
V	750 ft^3
F	3000 ft^3/hr
Caf	0.132 lbmol/ft^3
Tf	60 °F
A	1221 ft^2
dH	-45000 BTU/lbmol
U	75 BTU/hr-ft^2-°F
R	1.987 BTU/lbmol°F

The adaptation law shown in the above discussion is commonly used for first or second system but it is proved that it can be applied for a much wider range of systems. A key result of this is that a different adaptation law need not be calculated when changing to a different plant or model, unless the performance of the adaptation law is proven to be insufficient.

IV. Simulation Results

A. The Continuous Stirred Tank Reactor

The mathematical model of this reactor comes from balances inside the reactor. Notice that, a jacket surrounding the reactor also has fed and exit streams. The jacket is assumed to be perfectly mixed. Energy passes through the reactor walls into the jacket, removing the heat generated by the reaction. The control objective is to keep the temperature of the reacting mixture T, constant at the desired value. The only manipulated variable is the coolant or jacket temperature. A simplified modeling equation for a CSTR can be obtained by making following assumptions:

1. Perfect mixing inside reactor and jacket
2. Constant volume reactor and jacket and
3. Constant parameter values

In addition, to develop a simplified model, it is assumed that the jacket temperature can be directly manipulated. This assumption is very good, if a boiling heat transfer fluid is used, for example; changing the pressure on the jacket side would result in an instantaneous change in jacket temperature. Even for the re-circulating heat transfer system, the assumption of the jacket temperature being directly manipulated can be good if the jacket dynamics are rapid compared to the reactor dynamics. The following dynamic equation of CSTR is obtained [2]

$$\frac{dC_A}{dt} = f_1(C_A,T) = \frac{F}{V}(C_{AF} - C_A) - K_0 \exp\left(\frac{-E_a}{R(T+460)}\right) C_A \quad (14)$$

$$\frac{dT}{dt}= f_2(C_A,T)=\frac{F}{V}(T_f\text{-}T)+\left(\frac{-\Delta H}{\rho C_p}\right) K_0\exp\left(\frac{-E_a}{R(T+460)}\right) C_A\text{-}\frac{UA}{V\rho C_p}(T\text{-} T_j) \quad (15)$$

A simple irreversible exothermic reaction A →B is assumed in the CSTR. The concentration (C_A) of a product inside the reactor is assumed to be a function of temperature (T) in the reactor. Also, it is assumed that, the jacket temperature (T_j) is considered as a manipulated variable and the reactor temperature (T) is the controlled variable.

The steady state characteristic of a physical non-linear model of a CSTR is obtained for various value of jacket temperature (T_j). Table II shows the model parameter and operating points assumption CSTR model equation.

The steady state response of the CSTR with multiple operating points, which impose complication in the controller design shown in the Fig. 2.

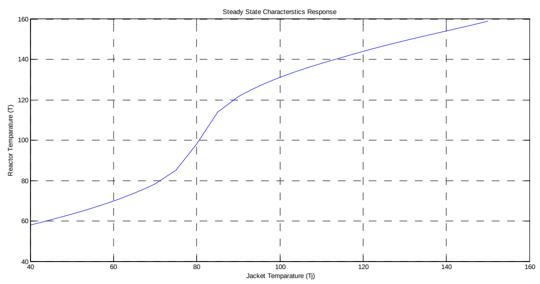

Fig. 2 The steady state characteristics of CSTR showing multiple operating regimes

B. Model Reference Adaptive Control

A Model Reference Adaptive controller is designed based on the explanation provided in the Section III and the performance is evaluated through extensive simulation for a CSTR using MATLAB/SIMULINK software. The adaptation laws are developed based on Gradient and Lyapunov methods. A first order and second order reference models are assumed for those methods.

The first order reference model is taken based on the assumption that, the closed loop system behaves like a first order system with time constant equivalent to the open loop response of the CSTR obtained by giving step change in the jacket temperature of the reactor.

The open loop response of the CSTR at various operating points is shown Fig. 2 and the corresponding transfer function models are given in Table III.

TABLE II
OPENLOOP STEP RESPONSE MODELS AND CORRESPONDING CONTROLLER SETTINGS

SI.No	Operating Regions (° F)	Transfer Function Model	Controller Settings
1	40-80	$\frac{1}{0.6s+1}$	Kc=1 Ti=0.65
2	80-120	$\frac{16.32}{s^2+5.4s+16.32}$	Kc=1.65 Ti=0.33 Td=0.68
3	120-160	$\frac{0.48}{0.12s+1}$	Kc=2.08 Ti=0.12

The second order reference model is selected based on the expected closed loop behaviour of the system by considering general second order transfer function:

$$G_m(s) = \frac{\omega_n^2}{s^2+2\omega_n\zeta s+\omega_n^2} \quad (16)$$

The damping coefficient ζ and the natural frequency ω_n are computed based on the desired overshoot and settling time of closed loop system.For temperature control of CSTR the allowable Peak overshoot M_p is chosen as 5% and settling time (T_s) as around 1min.The damping coefficient and natural frequency are given by [3]

$$\zeta = \frac{\ln\left(\frac{M_p}{100}\right)}{-\pi}\sqrt{\frac{1}{1+\left(\frac{\ln\left(\frac{M_p}{100}\right)}{-\pi}\right)^2}} \quad (17)$$

and

$$\omega_n = \frac{3}{T_s\zeta} \quad (18)$$

From the above equations the reference model is computed as

$$G_m = \frac{16.32}{s^2+5.4s+16.32} \quad (19)$$

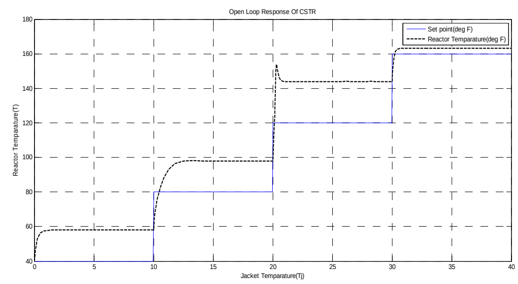

Fig. 3 The open loop response of CSTR

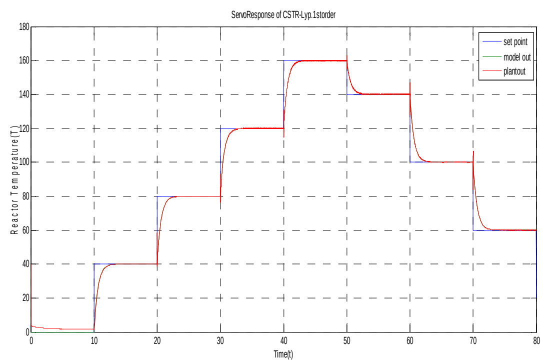

Fig. 4 Servo response of an MRAS with Lyapunov stability method with 1st order reference Model

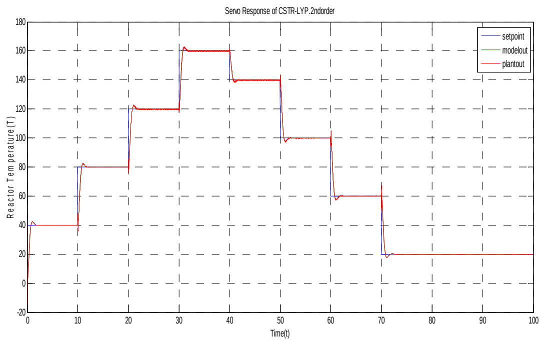

Fig. 5 Servo response of an MRAS with Lyapunov stability method with 2nd order reference Model

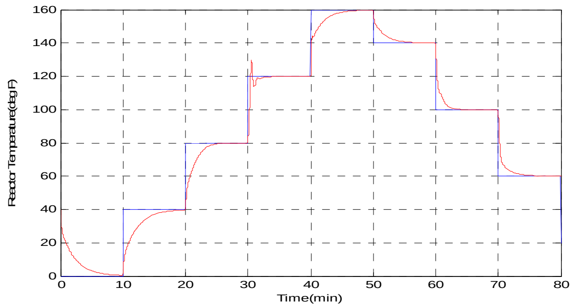

Fig. 6 Servo response of a conventional PI controller

TABLE III
COMPARISON OF PERFORMANCE OF MRAS AND CONVENTIONAL PI CONTROLLER

Operating Points	Controller	Settling Time(t_s) min	$\%M_p$	ISE
40-80	**Conventional PI**	6.3	NIL	506.4
80-120		4.2	8.3	412.3
120-140		8	NIL	862.2
40-80	**Lyapunov Stability** (with 1st order ref model)	3.97	NIL	301.3
80-120		3.98	NIL	307.8
120-140		3.49	NIL	287
40-80	**Lyapunov Stability** (with 2nd order ref model)	2.85	2.4	192.3
80-120		2.5	2.4	180.1
120-140		3.5	2.5	220

V. CONCLUSION

A model reference adaptive control strategy has been developed for CSTR, which involves mechanisms to adapt itself for nonlinearities in the system. The performance of MRAS and classical PI control are evaluated by classical performance indices like settling time, overshoot and ISE. The MRAS by Lyapunov stability perform very well compared to classical control.

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