Effect of Carbon Nanotube Reinforcement in Polymer Composite Plates under Static Loading

S. Madhu, V. V. Subba Rao

Abstract**—**In the implementation of Carbon Nanotube Reinforced Polymer matrix Composites in structural applications, deflection and stress analysis are important considerations. In the present study, a multi scale analysis of deflection and stress analysis of carbon nanotube (CNT) reinforced polymer composite plates is presented. A micromechanics model based on the Mori-Tanaka method is developed by introducing straight CNTs aligned in one direction. The effect of volume fraction and diameter of CNTs on plate deflection and the stresses are investigated using classical laminate plate theory (CLPT). The study is primarily conducted with the intention of observing the suitability of CNT reinforced polymer composite plates under static loading for structural applications.

Keywords**—**Carbon Nanotube, Micromechanics, Composite plate, Multi-scale analysis, Classical Laminate Plate Theory.

I. Introduction

THE discovery of carbon nanotubes (CNTs) in 1991 by Iijima [1], has envisioned CNTs as important materials capable of dominating the 21st-century revolution in nanotechnology. CNTs are unique nanostructured materials with remarkable physical and mechanical properties [1]-[5]. For example, an elastic modulus higher than 1 TPa and strength near 200 GPa make CNTs attractive candidates to act as reinforcements in polymer-based composite materials. Polymer composite systems with CNTs as filler materials are ultra-light structural materials with enhanced electrical, thermal and optical characteristics [6], [7]. The prospect of obtaining advanced nanocomposites with multifunctional features, e.g., materials used for structures and electrical conductors, has attracted the efforts of researchers in both academia and industry [8]-[10]. Industry in particular recognizes many potential applications such as electro-statically dissipative materials and aerospace structural materials [8]-[13].

Based on different assumptions for displacement fields, different theories for plate analysis have been devised. These theories can be divided into three major categories, the individual layer theories, the equivalent single-layer (ESL) theories, and the three-dimensional elasticity solution procedures. These categories are further divided into sub-theories by the introduction of different assumptions. For example, the second category includes the CLPT, the first-order and higher-order shear deformation theories (FSDT and HSDT). Most studies on carbon nanotubes have focused on their material properties and modeling aspects some examples of which can be found in [14]–[20]. Even though these studies are quite useful in establishing the properties of the nanomaterials, their use in actual structural applications is the ultimate purpose for the development of this advanced class of materials. However, much of work in this direction is not accomplished. As such there is a need to observe the macro behavior of the material in actual structural elements such as a beams and plates. Deflections of beams are studied by J. Wuite and S. Adali [21] for different volume of fractions of CNTs and nanotube diameters.

The deflections and stresses of a composite plate depend on a variety of parameters like properties of reinforcement. Variation of stresses across the plate thickness is studied [25].

For the implementation of CNT reinforced polymers (CNRP) in structural applications, accurate property–microstructure relations are required in the form of micromechanics models. In the present investigation, micro-mechanical properties of CNRP are computed using Mori-Tanaka method as given in [21]-[24]. The effects of the characteristics developed in these models are investigated on the performance plates.

The present paper investigates the deflection and stresses of a nanocomposite plates with a view towards assessing the effectiveness of these materials in the design of structural materials.

II. Micromechanical Model

The micromechanics model involves the elastic behavior of CNRP reinforced with aligned nanotubes which are straight and infinitely long. In the micromechanics model, the SWNTs are considered to be solid fibers with anisotropic material properties and the values of the elastic constants are taken from Popov et al. [23]. The plate is composed of polystyrene as the matrix with the young modulus and Poisson's ratio of E_m = 1.9 GPa ν_m = 0.3 respectively. The nanotube radius is assumed to be 10 A^0 for all the cases otherwise mentioned for which the representative values of the elastic constants of SWCNTs are: n_r = 450 GPa, k_r = 30 GPa, $m_r = p_r$ = 1 GPa and l_r= 10 GPa. The bonding at the nanotube–polymer interface is taken to be perfect. The composite is considered as transversely isotropic. The bonding at the nanotube–polymer interface is taken to be perfect. The composite is considered as transversely isotropic. The elastic behavior of an elementary cell of the composite material can be expressed as

S. Madhu is an Associate Professor with the Department of Mechanical Engineering, Turbomachinery Institute of Technology & Sciences, Hyderabad, Andhra Pradesh, India (phone: +91-9959626210; e-mail: salumari@gmail.com).

Dr. V. V. Subba Rao is a Professor & Head of Mechanical Engineering, University College of Engineering Kakianada, Jawaharlal Nehru Technological University Kakinada, Andhra Pradesh, India (+91-9440555477; e-mail: rao703@yahoo.com).

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix} n & l & l & 0 & 0 & 0\\ l & k+m & k-m & 0 & 0 & 0\\ l & k-m & k+m & 0 & 0 & 0\\ 0 & 0 & 0 & 2m & 0 & 0\\ 0 & 0 & 0 & 0 & 2p & 0\\ 0 & 0 & 0 & 0 & 0 & 2p\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{23}\\ \varepsilon_{13}\\ \varepsilon_{12}\end{bmatrix} \quad (1)$$

where k, l, m, n and p are the Hill's elastic moduli; n is the uni-axial tension modulus in the fibre direction, k is the plane-strain bulk modulus normal to the fibre direction, l is the associated cross modulus, m and p are the shear moduli in planes normal and parallel to the fibre direction, respectively, as specified in J. Wuite and S. Adali [21]. A composite with a reinforcing phase volume fraction c_r, matrix Young's modulus E_m and matrix Poisson's ratio ν_m is considered. Using the Mori–Tanaka method, the Hill's elastic moduli are found to be

$$k = \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1+c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]}$$

$$l = \frac{E_m\{c_m\nu_m[E_m+2k_r(1+\nu_m)]+2c_r l_r(1-\nu_m^2)\}}{(1+\nu_m)[2c_m k_r(1-\nu_m-2\nu_m^2)+E_m(1+c_r-2\nu_m)]}$$

$$n = \frac{E_m^2 c_m(1+c_r-c_m\nu_m)+2c_m c_r(k_r n_r - l_r^2)(1+\nu_m)^2(1-2\nu_m)}{(1+\nu_m)\{2c_m k_r(1-\nu_m-2\nu_m^2)+E_m(1+c_r-2\nu_m)\}} + \frac{E_m[2c_m{}^2 k_r(1-\nu_m)+c_r n_r(1-2\nu_m+c_r)+4c_m c_r l_r \nu_m]}{2c_m k_r(1-\nu_m-2\nu_m{}^2)+E_m(1+c_r-2\nu_m)}$$

$$p = \frac{E_m[E_m c_m + 2(1+c_r)p_r(1+\nu_m)]}{2(1+\nu_m)[E_m(1+c_r)+2c_m p_r(1+\nu_m)]}$$

$$m = \frac{E_m[E_m c_m + 2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m+4c_r(1-\nu_m)]+2c_r m_r(3-\nu_m-4\nu_m{}^2)\}} \quad (2)$$

where k_r, l_r, m_r, n_r and p_r are the Hill's elastic moduli for the reinforcing phase. The expressions for the moduli of the CNRP as functions of the stiffness constants are determined for a unidirectional composite as follows:

$$E_L = n - \frac{l^2}{k}, \quad E_T = \frac{4m(kn-l^2)}{kn-l^2+mn} \quad G_{LT} = 2p \quad \text{and} \quad \nu_{LT} = \frac{l}{2k} \quad (3)$$

III. Basic Equations

The deflection and stress behavior of a laminated beam made of carbon nanotube–reinforced polymer is studied and the basic equations for a symmetric laminated beam are summarized in this section. The basic relations for laminated beam under bending loads are taken from Berthelot [24] as

$$\begin{bmatrix} M_x\\ M_y\\ M_{xy}\end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{11} & D_{16} & D_{66}\end{bmatrix}\begin{bmatrix} k_x\\ k_y\\ k_{xy}\end{bmatrix} \quad (4)$$

where M_x, M_y and M_{xy} are the bending moments, D_{ij} are the bending stiffnesses and k_x, k_y and k_{xy} are the curvatures. The differential equation governing the deflection of the beam with bending along the x-axis can be expressed as

$$\frac{d^2 w_0}{dx^2} = -D_{11}^* M_x \quad (5)$$

where w_0 is the deflection and D^*_{11} is given by

$$D_{11}^* = \frac{1}{\Delta}\left(D_{22}D_{66} - D_{26}^2\right) \quad (6)$$

with Δ being the determinant of the matrix [*Dij*] in (4), viz.

$$\Delta = D_{11}D_{22}D_{66} + 2D_{12}D_{16}d_{26} - D_{11}D_{26}^2 - D_{22}D_{16}^2 - D_{66}D_{12}^2 \quad (7)$$

The differential equation (2) can be written in terms of the effective bending modulus E_x, moment of inertia I and the bending moment M defined as

$$E_x = \frac{12}{h^3 D_{11}^*}, \; I = I_{xy} = \frac{bh^3}{12}, \; M = bM_x \quad (8)$$

From (2) and (5), it follows that

$$\frac{d^2 w_0}{dx^2} = -\frac{M}{E_x I} \quad (9)$$

A. Maximum Deflection and Stresses of Laminated Plate

The basic equations developed in the previous section are now applied to a simply supported beam subjected to a load P acting at the mid-point of the beam to validate against publish results. The same basic equations are also applied to simply supported plate subjected to uniformly distributed load. The maximum deflection of the beam is given by

$$w_{\max} = \frac{PL^3}{48E_x I} = \frac{PL^3}{48b}D_{11}^* \quad (10)$$

The maximum deflection of the plate is given by

$$w_0(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{Q_{mn}}{d_{mn}}\sin\alpha x \; \sin\beta y \quad (11)$$

The in-plane stresses can be computed from

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}^{(k)} = -z\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & 0\\ 0 & 0 & \bar{Q}_{66}\end{bmatrix}^{(k)}\begin{Bmatrix}\dfrac{\partial^2 w_0}{\partial x^2}\\ \dfrac{\partial^2 w_0}{\partial y^2}\\ 2\dfrac{\partial^2 w_0}{\partial x \partial y}\end{Bmatrix}$$

$$= z\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn}\begin{Bmatrix}\left(\bar{Q}_{11}^{(k)}\alpha^2+\bar{Q}_{12}^{(k)}\beta^2\right)\sin\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\\ \left(\bar{Q}_{12}^{(k)}\alpha^2+\bar{Q}_{22}^{(k)}\beta^2\right)\sin\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\\ -2\bar{Q}_{66}^{(k)}\alpha\beta\cos\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\end{Bmatrix} \quad (12)$$

The maximum normal stresses occur at (x, y, z) = (a/2, b/2, h/2).

For the case of mechanical loading, the stresses are normalized as follows:

$$\bar{\sigma}_{xx} = \sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right) \qquad \bar{\sigma}_{yy} = \sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right) \quad (13)$$

IV. Deflection Analysis

A. Beam

A systematic MATLAB programme is developed for the static analysis of composite beams and plates. To validate the code, a simply supported beam problem is considered whose results are available in the literature. The beam is subjected to concentrated load at the center. The deflections are computed for various stacking sequences using the developed program and presented in Fig. 1 along with the published results [21]. The present results and published results are found in good agreement. Fig. 1 indicates curves of maximum deflection plotted against CNT volume fraction for various stacking sequences for a simply supported beam subjected to concentrated load in the middle.

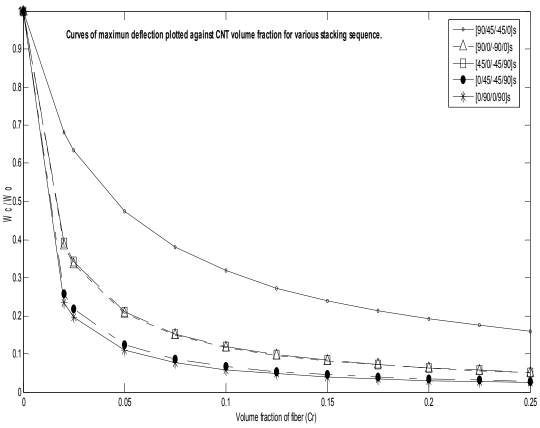

Fig. 1 Curves of maximum deflection plotted against CNT volume fraction for various stacking sequences for a simply supported beam subjected to CPL

B. Plate

An attempt is made to analyze CNT reinforced laminated composite plates as the results for the plates are not available in the literature. Having validated the beam results, the source code is extended to two dimensional problems of thin plates. A parametric study is carried out to know the response of the CNT reinforced plates for different volume fractions of CNT and the effect of stacking sequence. The CNT reinforced polymer composite plates are analyzed under different loading and boundary conditions.

For a simply supported plate, the results show that there is a decrease in the maximum deflection as the volume fraction of nanotubes increases.

The results also indicate that the small volume fractions of CNTs influence the deflections to a great extent. The effect of stacking sequence on the deflection can also be seen from Fig. 2 which shows the curves of w_c/w_o against c_r where w_o is the maximum deflection of the plate without CNT reinforcement.

The minimum deflections are obtained for the stacking sequence $[45^0/-45^0/45^0/-45^0]$s and the highest one for the stacking sequences $[0^0/90^0/0^0/90^0]$s and $[90^0/0^0/90^0/0^0]$s emphasizing the optimal design with respect to lay-up. It is also observed that as the number of layers increase, the deflection decreases indicating that the stiffness of the plate has increased (Fig. 3). The developed procedure is further extended to the static response of composite plates reinforced with carbon nanotubes. In the present study, a simply supported plate subjected to concentrated load at the centre and UDL are considered. It is observed from Fig. 4, that lower nanotube diameters lead to lower deflections indicating higher stiffness. As the CNT radius is varying, it is observed from Fig. 5, that the stacking sequence $[45^0/-45^0/45^0/-45^0]$s has the minimum deflections.

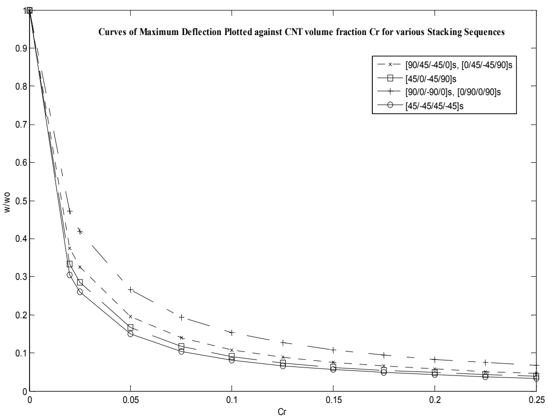

Fig. 2 Curves of maximum deflection plotted against CNT volume fraction for various stacking sequences for a Simply Supported plate subjected to CPL

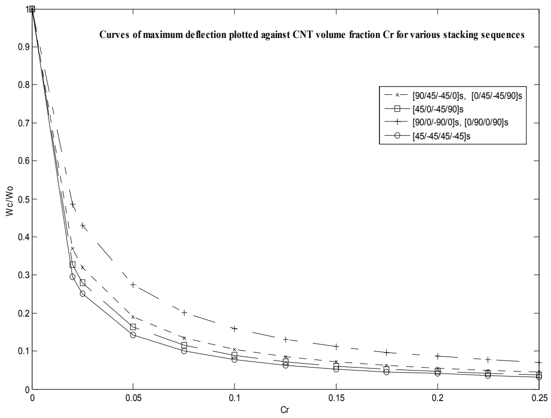

Fig. 3 Curves of maximum deflection plotted against CNT volume fraction for various stacking sequences for a Simply Supported plate subjected to UDL

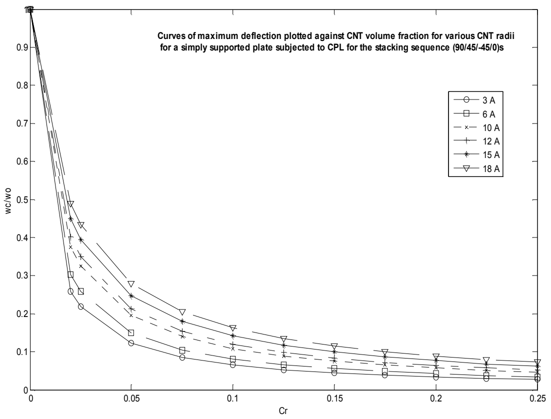

Fig. 4 Curves of maximum deflection plotted against CNT volume fraction for various CNT radii for a simply supported plate subjected to CPL for the stacking sequence [90/45/-45/0]s

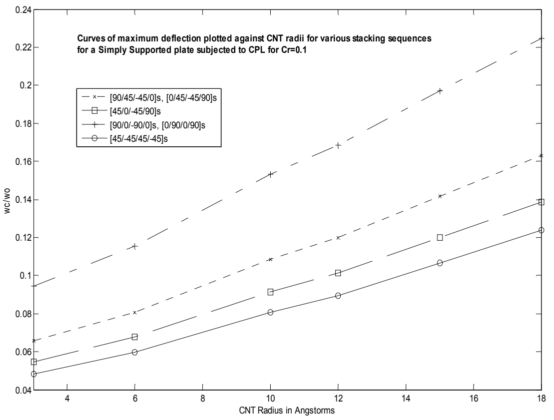

Fig. 5 Curves of maximum deflection plotted against CNT radii for various stacking sequences for a Simply Supported plate subjected to CPL for Cr=0.1

V. Stress Analysis

Stresses of a simply supported plate under UDL are investigated. It is observed that the stress distribution depends upon the stacking sequence.

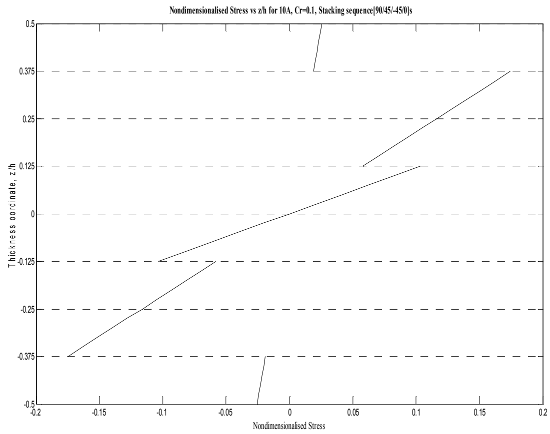

Fig. 6 Variation of Nondimensionalised maximum stress (σ_{xx}) through thickness coordinate (z/h) of square laminate under uniformly distributed load for the stacking sequence [90/45/-45/0]s

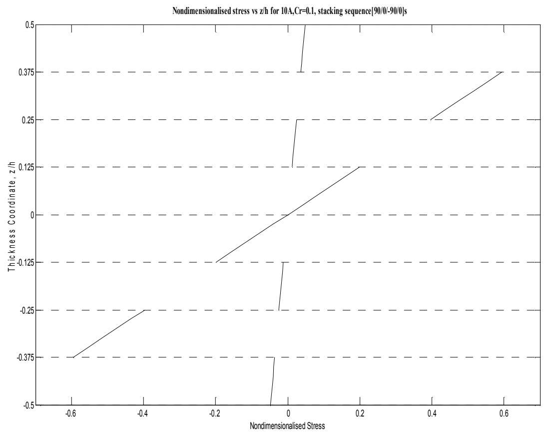

Fig. 7 Variation of Nondimensionalised maximum stress (σ_{xx}) through thickness coordinate (z/h) of square laminate under uniformly distributed load for the stacking sequence [90/0/-90/0]s

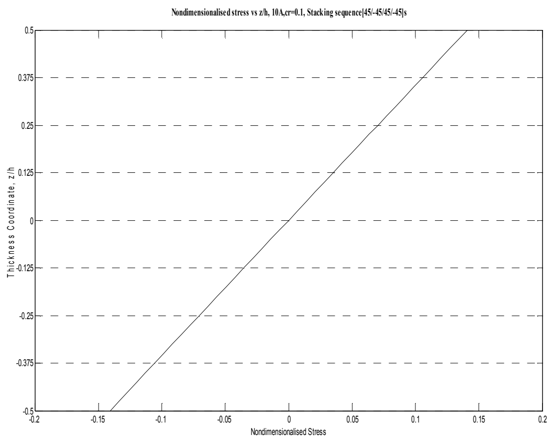

Fig. 8 Variation of Nondimensionalised maximum stress (σ_{xx}) through thickness coordinate (z/h) of square laminate under uniformly distributed load for the stacking sequence [45/-45/45/-45]s

The above graphs (Figs. 6-8) indicate stress distributions σ_{xx} normalized with respect to the corresponding stress of an isotropic plate for various stacking sequences with Cr=0.1 and nanotube diameter of 10.0 A^0. A comparison of stress distributions for various stacking sequences show the effect of the lay up on the maximum stress which can be reduced substantially by lay-up optimization with the stacking sequence [90/0/-90/0]s.

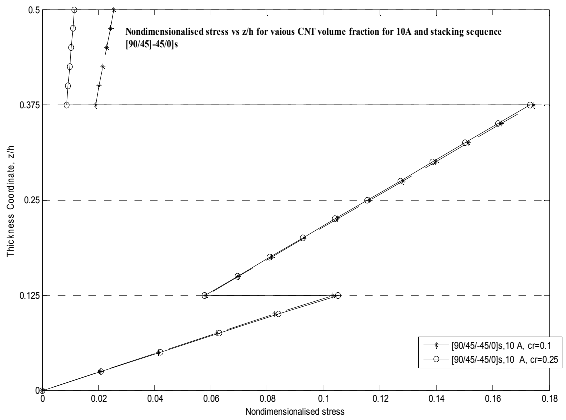

Fig. 9 Variation of Nondimensionalised maximum stress (σ_{xx}) through thickness coordinate (z/h) of square laminate under uniformly distributed load for various CNT fiber volume fraction (Cr) for the stacking sequence [90/45/-45/0]s

From the above graph it can be seen that as the volume fraction of the CNT increases, the maximum stress decreases.

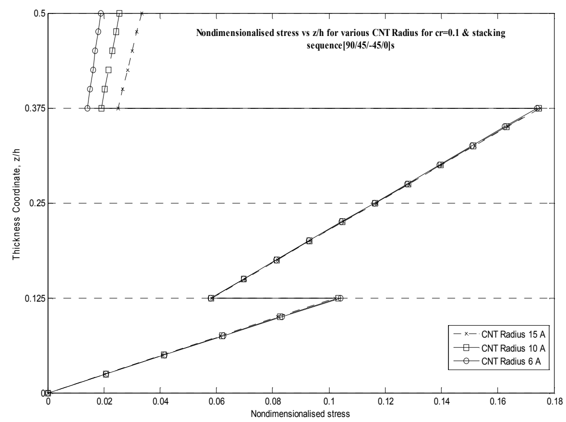

Fig. 10 Variation of Nondimensionalised maximum stress (σ_{xx}) through thickness (z/h) of square laminate under uniformly distributed load for various CNT Radii for the stacking sequence [90/45/-45/0]s

From the above graph, it is observed that, as the CNT radius increases, the maximum stress also increases.

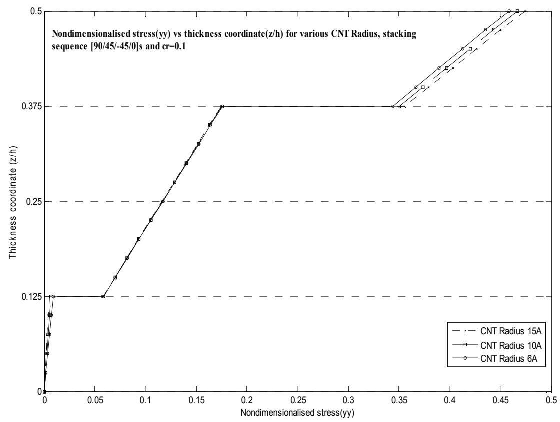

Fig. 11 Variation of Nondimensionalised maximum stress (σ_{yy}) through thickness coordinate (z/h) of square laminate under uniformly distributed load for various CNT Radii for the stacking sequence [90/45/-45/0]s

Like in the case of σ_{xx}, Fig. 11 indicates that σ_{yy} also increases with the increase in the CNT radius.

VI. Conclusion

The deflections and stresses of simply supported nanocomposite plates were examined by using micromechanics relations to determine the elastic constants in terms of nanotube volume fraction. The effect of the stacking sequence was studied on the maximum deflection and stresses.

It was observed that a small percentage of nanotube reinforcement leads to significant improvements in plate stiffness. It was shown that stacking sequence is an important parameter for reducing the maximum deflection. It was also observed that as the carbon nanotube radius increases, the stiffness reduces indicating an increase in the deflection.

With respect to stresses, it was observed that as CNT radius increases, the maximum stress also increases. Further, it is seen that the effect of the lay up on the maximum stress can be reduced substantially by lay-up optimization. Also as the volume fraction of the CNT increases, the maximum stress decreases.

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