Reductions of Control Flow Graphs

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Abstract—Control flow graphs are a well-known representation of the sequential control flow structure of programs with a multitude of applications. Not only single functions but also sets of functions or complete programs can be modeled by control flow graphs. In this case the size of the graphs can grow considerably and thus makes it difficult for software engineers to analyze the control flow. Graph reductions are helpful in this situation. In this paper we define reductions to subsets of nodes. Since executions of programs are represented by paths through the control flow graphs, paths should be preserved. Furthermore, the composition of reductions makes a stepwise analysis approach possible.

Keywords—Control flow graph, graph reduction.

I. INTRODUCTION

CONTROL flow graphs are widely used since many years to represent the sequential control structure of software and for its graphical representation. Each statement is represented by a node in the graph, the edges reflect the control flow between them. Among the manifold applications of control flow graphs are white box testing, test coverage notions [11], [12], [21], [23], [28], [35], [37] and generation of test cases [4], [16], [20]. Furthermore, control flow graphs are useful in the control flow analysis in compiler construction and optimization [2], [3], [10] and in the definition and evaluation of source code metrics [13], [19], [21], [31], such as, cyclomatic complexity [24].

Control flow graphs can not only be applied to single functions in a program, but there are also approaches for interprocedural frameworks [17], [18], [22], [29] that define control flow graphs for sets of functions or for complete programs. Each node that represents a function call is split in a call node and a return node and linked to the control flow graph of the called function by newly introduced edges. The problem with interprocedural control flow graphs is their large size. Especially, when software engineers use control flow graphs in program analysis the size of the graphs makes it difficult to understand the control flow structure. Therefore, the user should be enabled to apply reductions to the graphs of sets of functions or of complete programs and shows the compositionality of reductions. Section V concludes the paper.

The remainder of this paper is organized as follows. We start with the necessary definitions about directed graphs and control flow graphs. In Section III we define the graph reduction and investigate its properties. The following chapter extends the reduction to interprocedural control flow graphs of sets of functions or of complete programs and shows the compositionality of reductions. Section V concludes the paper.

II. BASIC DEFINITIONS

In this section definitions about graphs necessary for the following are summarized. The definitions are partly taken from previous papers [14], [15], a detailed introduction to graphs can be found, e.g., in [5] or [8].

Definition 1. A directed graph (with multiple edges) is a pair \( G = (N, E) \) consisting of a finite set \( N \) of nodes and a (possibly infinite) set \( E \) of edges with \( N \cap E = \emptyset \), together with functions start : \( E \to N \) and end : \( E \to N \) that associate a start node and an end node, respectively, with each edge.

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The indegree and outdegree of a node \( n \) is the number of edges that end and start in \( n \), respectively, i.e., indegree\((n) = |\{e \in E \mid \text{end}(e) = n\}|\), outdegree\((n) = |\{e \in E \mid \text{start}(e) = n\}|\), or \( \omega \) if there are infinitely many such edges.

A node \( n \) with \( \text{indegree}(n) = 0 \) is called an entry node and a node with \( \text{outdegree}(n) = 0 \) exit node of the graph.

A path \( d \) in a graph is a non-empty, finite sequence of edges \( e_1, e_2, \ldots, e_k \) such that end\((e_i)\) = start\((e_{i+1})\) for \( i = 1, \ldots, k-1 \). The nodes start\((e_1)\), start\((e_2)\), ..., start\((e_k)\) and end\((e_k)\) in the path \( d \) form a sequence denoted by nodes\((d)\). The start node of the first edge \( e_1 \) is called the start node of \( d \), the end node of the last edge \( e_k \)—the end node of \( d \), denoted by end\((d)\). The nodes start\((e_2)\), ..., start\((e_k)\) are called inner nodes of \( d \). The concatenation of two paths \( d = e_1 e_2 \ldots e_k \) and \( d' = e'_1 e'_2 \ldots e'_m \) is defined by \( dd' = e_1 e_2 \ldots e_k e'_1 e'_2 \ldots e'_m \) if \( \text{end}(e_k) = \text{start}(e'_1) \).

We represent multiple edges between two nodes graphically as one arc with the number of edges between these two nodes as a label at the arc. Infinitely many edges are denoted by \( \omega \). If there is only one edge between two nodes the number 1 is omitted.

As mentioned in the introduction, each statement in a function is represented by a node in the control flow graph. The control flow between statements is modeled by the edges. An additional entry node \( n_{in} \) and an additional exit node \( n_{out} \) is added that identify the entry when called and the exit when the function returns. In this chapter and Chapter III we will examine only control flow graphs of single functions. Therefore the exit from and the entry to the graph when a function is called within the function are not represented as exit and entry nodes. The examples in this paper are mostly written in the programming language C, but of course control flow graphs are not restricted to this language. Which constructs are represented by nodes is not always consistent in the literature. We consider such constructs which are syntactically statements in the programming language C [14].

**Definition 2.** The control flow graph \( G_f = (N, E) \) of a function \( f \) is a directed graph that consists of

- a node \( n_a \) for each statement \( a \) in \( f \),
- one additional entry node \( n_{in} \) and one additional exit node \( n_{out} \),
- an edge \( (n_a, n_{a'}) \) if the statement \( a' \) is executed immediately after the statement \( a \),
- an edge \( (n_{in}, n_{a1}) \) for the first statement \( a_1 \) in \( f \) or an edge \( (n_{in}, n_{out}) \) if \( f \) contains no statements at all,
- edges \( (n_{a1}, n_{a2}) \) for each statement \( a' \) after which the control flow leaves the function.

The definition ensures that in the control flow graph \( G_f \) of a function \( f \) each node—with the exception of \( n_{in} \) and \( n_{out} \)—corresponds to a unique statement in the function. Between two nodes there is at most one edge. Therefore control flow graphs don’t have multiple edges and their sets of edges are finite.

As an example, the following function \texttt{find} which looks in a linked list for the last name and the first name of a person is examined:

```c
list find(list namelist, 
    string last, string first)
{
    printf("start ");
    while (namelist != NULL)
    {
        if (namelist->lastname == last) 
        {
            printf("last name found ");
            if (namelist->firstname == first) 
            {
                printf("name found ");
                return namelist;
            }
            printf("next name ");
            namelist = namelist->next;
        }
        printf("name not found ");
        return NULL;
    }
}
```

Fig. 1 shows the control flow graph of this function. For better readability the nodes in the control flow graph can be labeled. The labels “in” and “out” are assigned to the entry and exit nodes. The other nodes are labeled by “=”, “while” etc. to show which kind of statements is represented.

### III. Graph Reductions

In a graph reduction we select a subset of nodes and thus decrease the number of nodes. But paths should be preserved.
Therefore, for each path between two nodes in the subset an edge is introduced in the reduced graph. Before we define the graph reduction we need the following notation about sets of paths in directed graphs.

**Definition 3.** Let \( G = (N, E) \) be a directed graph and let \( N' \subseteq N \) be a subset of its nodes. We will examine the following sets of paths in \( G \):

\[
\begin{align*}
D & : \text{set of all paths in } G, \\
D_{N' \subseteq N'} & : \text{set of all paths in } G, \text{ whose start and end nodes are in } N', \\
D_{N \setminus N'} & : \text{set of all paths in } G, \text{ whose start and end nodes are in } N' \text{ with inner nodes in } N \setminus N'. \text{ Such paths are called } N' \setminus N' \text{-paths.}
\end{align*}
\]

The difference between the sets \( D_{N' \subseteq N'} \) and \( D_{N \setminus N'} \) is that the inner nodes in the paths in the first set are not restricted to \( N \setminus N' \) and can be any nodes from \( N \).

When we reduce a graph to a set \( N' \subseteq N \) of nodes, each \( N' \setminus N' \)-path between two nodes in \( N' \) will be replaced by an edge. This defines a bijection between \( N' \setminus N' \)-paths in \( G \) and edges in the reduced graph which keeps the start and end nodes.

**Definition 4.** Let \( G = (N, E) \) be a directed graph and let \( N' \subseteq N \) be a subset of its nodes. A directed graph \( G' = (N', E') \) is a reduction of \( G \) to \( N' \) if there is a bijective function

\[
\delta : D_{N \setminus N'} \rightarrow E'
\]

from the set of the \( N' \setminus N' \)-paths in \( G \) to the set of edges in \( G' \) such that

\[
\begin{align*}
\text{start}(d) = \text{start}(\delta(d)) \\
\text{end}(d) = \text{end}(\delta(d))
\end{align*}
\]

for all \( N' \setminus N' \)-paths \( d \in D_{N \setminus N'} \).

All reductions of \( G \) to the same subset of nodes \( N' \) are equal up to isomorphism. Therefore, we speak of the reduction of \( G \) to \( N' \).

Since there may be more than one or infinitely many \( N' \setminus N' \)-paths in \( G \) between two nodes \( n, m \in N' \)—even if \( G \) has no multiple edges—, multiple edges are necessary to represent them in the reduced graph. The number of edges in the reduced graph \( G' \) between two nodes \( n \) and \( m \) is equal to the number of different \( N' \setminus N' \)-paths in \( G \) that start in \( n \) and end in \( m \).

Clearly, the graph \( G = (N, E) \) reduced to \( N' \) is equal to \( G \).

In the control flow graph of the function \( \text{find} \) there are infinitely many paths between the entry node and the exit node described by the regular expression

\[
n_1 n_2 (n_3 n_4 (n_5 n_6 n_9 | n_9) n_{10})^* n_1 n_1 n_2 n_3 | n_1 n_2 (n_3 n_4 (n_5 n_6 n_9 | n_9) n_{10})^* n_3 n_4 n_5 n_6 n_7 n_8 n_9 n_{13}
\]

but, of course no path between the exit node and the entry node. The graph reduced to the entry node and the exit node is shown in Fig. 2.

In [14] and [15] we defined the reduction to the D-nodes, that are entry nodes, exit nodes and such nodes that represent decisions, that are, nodes with outdegree \( \geq 2 \). This approach was introduced by Paige [27] in order to support the analysis of programs by partitioning the control flow graph.

**Definition 5.** Let \( G = (N, E) \) be a directed graph. A node \( n \in N \) is called D-node if it is an entry node or an exit node or if outdegree\((n) \geq 2 \). The reduction of \( G \) to its D-nodes is called decision graph of \( G \).

A node that is not a D-node has indegree \( \geq 1 \) and outdegree exactly 1.

In [14] the \( N' \setminus N' \)-paths where \( N' \) is the set of D-nodes are called DD-paths. Since there is no branching possible after leaving a D-node \( n \) until reaching another D-node, all inner nodes in a DD-path are different [12] and there are at most outdegree\((n) \) different DD-paths that start in \( n \). The decision graph of a control flow graph has therefore only finitely many edges.

The D-nodes in the control flow graph of the function \( \text{find} \) (Fig. 1) are the entry and exit nodes \( n_1, n_13 \) and the while-and if-nodes \( n_3, n_4, n_6 \). The paths

\[
(n_1, n_2) (n_2, n_3) (n_3, n_11) (n_11, n_12) (n_12, n_13) (n_3, n_4) (n_4, n_5) (n_5, n_6) (n_6, n_9) (n_9, n_{10}) (n_{10}, n_3) (n_6, n_7) (n_7, n_8) (n_8, n_{13})
\]

between D-nodes are replaced by edges in the decision graph (when we write an edge as pair of nodes which is possible in graphs without multiple edges). In Fig. 3 the decision graph of the control flow graph of the function \( \text{find} \) is depicted.

To analyze the structure of the loops functions can be reduced to entry and exit nodes and nodes that represent loops. Fig. 4 shows the reduction of the control flow graph of the example \( \text{find} \). This graph gives a simplified picture of the
control flow, which is in many cases sufficient for the analysis:

The function starts with a while-loop. For each execution of
the loop two possibilities arise (last name found or neither last
nor first name found). After the while-loop we get two possibili-
ties. For each execution of the loop two possibilities arise (last
name found or neither last nor first name found). After the while-
loop there are two different reasons for return (last and first
name found or not found).

Each path \( d \in D_{N,N'} \) that starts and ends with a node in
\( N' \) can be uniquely written as concatenation of \( N' \) paths by
splitting it at nodes in \( N' \): \( d = d_1 \ldots d_m \) with \( m \geq 1 \). The
function \( \delta \) can be straightforwardly extended to such paths by
\( \delta(d) = \delta(d_1) \delta(d_2) \ldots \delta(d_m) \).

For example, the path

\[
d = (n_1,n_2)(n_2,n_3)(n_3,n_4)(n_4,n_5)(n_5,n_6)(n_6,n_9)
\quad(n_9,n_{10})(n_{10},n_3)
\]

in the control flow graph of the function \( \text{find} \) can be split at
the D-nodes \( n_3, n_4, n_6 \) into the DD-paths

\[
d_1 = (n_1,n_2)(n_2,n_3)
\quad d_2 = (n_3,n_4)
\quad d_3 = (n_4,n_5)(n_5,n_6)
\quad d_4 = (n_6,n_9)(n_9,n_{10})(n_{10},n_3).
\]

Furthermore, we have

\[
\delta(d_1) = (n_1,n_3)
\quad \delta(d_2) = (n_3,n_4)
\quad \delta(d_3) = (n_4,n_6)
\quad \delta(d_4) = (n_6,n_3)
\]

and

\[
\delta(d) = (n_1,n_3)(n_3,n_4)(n_4,n_6)(n_6,n_3).
\]

From

\[
\text{end}(\delta(d_j)) = \text{end}(d_j) = \text{start}(d_{j+1}) = \text{start}(\delta(d_{j+1}))
\]

for \( j = 1, \ldots, m - 1 \) it follows that \( \delta(d) \) is a path in the
control flow graph. Additionally, when we consider the sequence
of nodes in \( d \) and project it to \( N' \) we get

\[
\text{start}(d_1)\text{start}(d_2) \ldots \text{start}(d_m)\text{end}(d_m)
\]

which is equal to the sequence of nodes in \( \delta(d) \).

In the example, this sequence is

\[
n_1 n_3 n_4 n_6 n_3.
\]

The extended function \( \delta : D_{N',N'} \to D' \) from the set of
paths in \( G \) that start and end with a node in \( N' \) to the set of
paths in \( G' \) is a bijection function. This leads to the following
theorem.

**Theorem 1.** Let \( G = (N,E) \) be a directed graph, let \( N' \subseteq
N \) be a subset of its nodes and let \( G' = (N',E') \) be the
reduction of \( G \) to \( N' \) with the bijection \( \delta : D_{N,N'} \to E' \).

The extension

\[
\delta : D_{N',N'} \to D'
\]

of \( \delta \) is also bijective. For all paths \( d \in D_{N',N'} \) it holds that
\( \text{nodes}(d) \mid N' = \text{nodes}(\delta(d)) \)

where \( |N'| \) is the projection to \( N' \) and in particular

\[
\text{start}(d) = \text{start}(\delta(d))
\quad \text{end}(d) = \text{end}(\delta(d))
\]

Proof. It remains to show that \( \delta \) is bijective. Let \( d \) and \( \delta(d) \)
be two paths in \( G \) that start and end with a node in \( N' \) where
\( \delta(d) = \delta(d) = e_1 e_2 \ldots e_m \). This means that both \( d \)
and \( \delta(d) \) are the concatenation of \( m \) \( N' \) paths \( d_1 d_2 \ldots d_m \).
\( d \) and \( \delta(d) \) respectively. From \( \delta(d_j) = e_j = \delta(d_j) \) it
follows that \( d_j = d_j \) (\( d \) is bijective according to Definition
4) and \( d = \delta(d) \). Let \( e_1 e_2 \ldots e_m \) be a path in \( G' \). Then
\( \delta(d_j) = \delta^{-1}(e_1) \delta^{-1}(e_2) \ldots \delta^{-1}(e_m) \) is a path in
\( G \) that starts and ends with a node in \( N' \) with \( \delta(d) = e_1 e_2 \ldots e_m \). \( \blacksquare \)

This theorem shows that all paths in \( G \) that start and end
in a node in \( N' \) are represented in the reduced graph. In a
previous paper [14] we showed that there might be paths that
start in a node in \( N' \) but do not end in \( N' \) and furthermore
can not be prolonged to a node in \( N' \). For the reduction
to D-nodes such paths have been called unconditional loops.

One example for this situation is

\[
\text{if} \ (x) \ 	ext{label: goto label;}
\]

In the control flow graph there is a path starting in the D-node
representing the if-statement leading to the goto-node which
cannot be prolonged to a DD-path.

In a control flow graph an execution of the function that
terminates with a return-statement or at the finishing brace
induces a path that starts in the entry node \( n_{in} \) and ends in
the exit node \( n_{out} \). When the control flow graph is reduced to a
subset of its nodes that includes \( n_{in} \) and \( n_{out} \), the path and thus
the execution is—according to Theorem 1—also represented
in the reduced graph. For this reason, a reduction of a control
flow graph should always include the entry node \( n_{in} \) and the
exit node \( n_{out} \).

So far we defined the

- the reduction to the entry and exit nodes,
- the reduction to the entry, exit and loop nodes and
- the reduction to D-nodes.

Another interesting reduction is

- the reduction to the interface nodes, that are entry nodes,
  exit nodes and nodes that represent function calls.

Fig. 5 shows the control flow graph of another example
\texttt{calcSolution} and its reduction to the interface nodes.
This function calculates the solution of a system of \( N \) linear
equations with \( N \) variables using Cramer’s rule. It gets as
parameters the coefficient matrix \( A \), its determinant and the
vector of constants. The first if-statement checks the determi-
nant of the coefficient matrix. If it is zero the function returns.
The following for-statement loops through all columns of the matrix, copies the matrix to $B$ and replaces the column by the vector of constants with three more for-loops. Then a function $\text{calcDeterminant}$ is called with parameter $B$ within an if-statement. If it returns with an error, the for-loop it terminated by a break-statement and the function returns. Otherwise, the determinant of $B$ is divided by the determinant of $A$.

IV. INTERPROCEDURAL CONTROL FLOW GRAPHS

So far, we examined only single functions and their control flow graphs. If we want to represent groups of functions or whole programs graphically, we need interprocedural control flow graphs. A well-known approach, e.g., proposed by Reps, Horwitz and Sagiv [29] or by Harrold and Rothermel [17], [18], is to split each node that represents a function call into two nodes—a call node and a return node. We add an edge from the call node to the entry node of the control flow graph of the called function and from the corresponding exit node to the return node. All edges that start in the original node start then in the return node (Fig. 6).

**Definition 6.** Let $F$ be a set of functions with control flow graphs $(N_f, E_f)$ for $f \in F$ where we assume that the sets $N_f$ and also the sets $E_f$ are pairwise disjoint. The **interprocedural control flow graph** of $F$ is defined as follows. In the graph $(\bigcup_{f \in F} N_f, \bigcup_{f \in F} E_f)$, we add for each node $n$ that represents a function call (call node)

- a return node $m$,
- an edge $(m, n')$ for each edge $(n, n')$ that starts in $n$, the edges $(n, n')$ are then deleted,
- an edge that starts in $n$ and ends in the entry node of the control flow graph of the called function,
- an edge that starts in the exit node of the control flow graph of the called function and ends in $m$.

In interprocedural control flow graphs we label the arcs between a call node and the entry node of the control flow graph of the called function with the function name for better understanding.

The complete example program that calculates the solution of a system of linear equations mentioned above consists of seven functions and eight function calls, one of them a recursive call. The interprocedural control flow graph has 79 nodes. Fourteen of them are entry or exit nodes, 16 are call or return nodes. The reduction of this graph to its interface nodes, that are, entry, exit, call and return nodes, has only 30 nodes and is shown in Fig. 7 where edges between call nodes and entry nodes and edges between exit nodes and return nodes are drawn in blue color.
be followed by eliminating the inner nodes of the function \texttt{calcDeterminant} in a second reduction step. Of course, the result should be the same if we do both reductions in one step.

**Theorem 2.** Let $G = (N, E)$ be a directed graph and let $N'' \subseteq N' \subseteq N$ be two subsets of its nodes. Then it holds that

$$\text{red}(G, N'') = \text{red}(\text{red}(G, N'), N'')$$

where red is the reduction of a graph to a subset of its nodes.

**Proof.** Let us introduce the following notation:

1. $G'' = (N'', E'')$ is the reduction of $G$ to $N''$ with the bijection $\delta'' : D_{N'' \cap N'} \to E''$.
2. $G' = (N', E')$ is the reduction of $G$ to $N'$ with the bijection $\delta' : D_{N' \cap N} \to E'$.
3. $G = (N', E)$ is the reduction of $G'$ to $N''$ with the bijection $\delta' : D_{N' \cap N} \to E$.

We have to show that $G'' = \widehat{G}$. Both graphs $G''$ and $\widehat{G}$ have the same set $N''$ of nodes. Now we construct a bijection $\delta : E'' \to \widehat{E}$ that preserves the start and the end nodes.

Let $e \in E''$ be an edge in the reduced graph $G''$ (Fig. 8). Then $d = \delta'^{-1}(e) \in D_{N'' \cap N'}$ is a path in $G$.

As shown in Theorem 1, we can extend $\delta''$ from $D_{N'' \cap N'} \to E''$ to $D_{N'' \cap N'} \to D'$. This extension can be restricted to $D_{N'' \cap N'} \to D'$. A path $d \in D_{N'' \cap N'}$ is then mapped to a path $d' = \delta(d) \in D'$. From Theorem 1 we know that $\text{nodes}(d) = \text{nodes}(d')$ and $d' \in D_{N'' \cap N'}$. This function $\delta' : D_{N'' \cap N'} \to D'_{N'' \cap N'}$ is injective since $\delta : D_{N'' \cap N'} \to D'_{N'' \cap N'}$ is injective. Let $d'$ be a path in $D'_{N'' \cap N'}$. We know that it exists $d \in D_{N'' \cap N'}$ with $d = \delta'(d)$. From $\text{nodes}(d) = \text{nodes}(d')$ follows that $d$ is in $D_{N'' \cap N'}$. Together this means that $\delta : D_{N'' \cap N'} \to D'_{N'' \cap N'}$ is also injective.

If we apply $\widehat{\delta}$, we get an edge $\widehat{e} = \delta'(d')$ in $\widehat{G}$. Thus we defined a bijective function $\delta = \delta'^{-1} \circ \delta' \circ \delta : E'' \to \widehat{E}$. ■

In Fig. 9 the reduction to the interface nodes is followed by a second reduction. This time we eliminate—with the exception of entry nodes and exit nodes—all nodes with only one incoming and one outgoing arc (regardless of the multiplicity of edges) and get a more compact graph. The entry and exit nodes are kept to identify the entry and exit points of the functions. The reduced graph consists of only 19 nodes. This graph is also sufficient to understand the control flow in the program. The function \texttt{main} calls \texttt{readSystem}. In this function infinitely many paths lead from the entry to the exit because the size of the system of linear equations is not limited. Then \texttt{solveSystem} is called. Firstly, it calls \texttt{calcDeterminant}. After the return \texttt{solveSystem} might return with one of two possible causes without calculating a solution. Otherwise, it calls \texttt{calcSolution}. For this function there are two causes of exceptions that lead to its return immediately after its call. If no exception occurs, it calls in a loop \texttt{calcDeterminant}. The reason for infinitely many paths on the loop is the same as above. At last \texttt{main} calls \texttt{writeSolution}. In the function \texttt{calcDeterminant} three different exceptions may cause it to return after being called. It calls \texttt{constructSubmatrix} and itself recursively in a loop.

**Fig. 8.** Construction of the bijective function $\delta$

**Fig. 9.** Second reduction of the example program

Often library functions should not be analyzed and therefore should not be shown in the control flow graph. To model this, Definition 6 can be changed such that not all call nodes have to be split but only those that the user chooses. The control flow graphs of the corresponding called functions are excluded from
the interprocedural control flow graph. In the example, we exclude the library function calcDeterminant and get the reduced control flow graph as shown in Fig. 10. The call nodes of calcDeterminant are considered as interface nodes in this example.

V. CONCLUSION

In this paper we studied reductions of directed graphs to subsets of nodes. Paths between these nodes are replaced by edges. The reductions can be applied to control flow graphs of single functions or to interprocedural control flow graphs of sets of functions but are not restricted to control flow graphs. Executions are preserved and reductions can be done stepwise. This can help software engineers when they analyze their programs and the control flow structure.

There are, besides the applications mentioned in the introduction, also other fields where control flow graphs and reductions can be used, for example, the detection of malware [7], the analysis of the control flow to find deviations caused by attacks on the program [1], [33] or the extraction of the control flow structure from executables [32], [34]. And it seems promising to apply graph reductions to other domains, e.g., VHDL [36], workflow modeling [9], [30] or the testing of graphical user interfaces [6], [25], [26], and exploit their advantages.

In future work a prototype tool could be implemented to find a set of reductions that are most helpful to software engineers in practical applications (Fig. 11).

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REFERENCES

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