Unsteady Transient Free Convective Flow of an Incompressible Viscous Fluid under Influence of Uniform Transverse Magnetic Field

Praveen Saraswat, Vipin Kumar Verma, Rudraman Singh

Abstract**—**The unsteady transient free convection flow of an incompressible dissipative viscous fluid between parallel plates at different distances have been investigated under porous medium. Due to presence of heat flux under the influence of uniform transverse magnetic field the velocity distribution and the temperature distribution, is shown graphically. Since exact solution is not possible so we find parametrical solution by perturbation technique. The result is shown in graph for different parameters. We notice that heat generation effects fluid velocity keeping in which of free convection which cools.

Keywords**—**Transient, Convection, MHD, Viscous, Porous.

I. Introduction

ENGINEERING problems in which fluid supports an exothermic chemical or nuclear reactions are very common today. It is important in technologies and physical problems.

The unsteady MHD free convective flows of dissipative fluids were first showed by Siegel [5] and later on, Eckert find it experimentally. Gebhart and Mollendrof [3] have also dealt with this case of small temperatures or in high Prandtl number P_r or even high gravitation field. Raptis [4] considered free convection flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Soundalgekar [6] have studied free convective flow of an incompressible viscous dissipative fluid.

Varshney and Kumar [7] discussed the problem by dealing with incompressible viscous dissipative fluid for MHD free convection flow through porous medium by finite difference method. Amakiri and Ogulu [2] are of the view that heat generators increase with fluid velocity when the free convective current cools the plate. Agrawal and Kishore [1] used thermal and mass diffusion on MHD natural convection flow between two infinite vertical moving with oscillatory porous parallel plates.

Here, we have considered parallel infinite plates between distance (d) and then studied free convection flow of viscous fluid between plates through porous medium.

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II. Mathematical Analysis

The governing equations of MHD flow between parallel plates are given as follows:

$$\frac{\partial u'}{\partial t'} = v' \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_D'\right) - \sigma \frac{\mu_e^2}{e} H_0^2 u' - \frac{vu'}{k'} \quad (1)$$

$$e.\,C_p \frac{\partial T'}{\partial t'} = \frac{K' \partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \quad (2)$$

Here, u' is the velocity of fluid, g is the acceleration due to gravity, K' is thermal conductivity of fluid, p is the coefficient of volume expansion, H_0 is constant magnetic field, μ is the viscosity of fluid, σ is electrical conductivity, C_p specific heat, k' is the permeability of porous medium.

The boundary conditions are as follows:

$$u' = u_0,\ T' = T_\omega' \xrightarrow{at} y' = 0$$
$$u' \to 0,\ T' \to T_D' \xrightarrow{at} y' = d$$

Let us take non-dimensional variables as follows:

$$t = \frac{t'}{T_r},\ y = \frac{y'}{D},\ u = \frac{u'}{u_0},\ \theta = \frac{T' - T_D'}{T_w' - T_D'} \quad (3)$$

$\Delta T = T_w' - T_D',\ u_0 = \left(\upsilon g\beta\Delta T\right)^{1/3}$:Reference Velocity

L= $\left(\frac{g\beta\Delta T}{\upsilon^2}\right)^{-1/3}$: Reference Temperature,

$T_r = \left(\frac{g\beta\Delta T}{\upsilon^{-1/3}}\right)^{-2/3}$: Thermal temperature,

$E = \frac{u_0^2}{C_p\Delta t}$: Eckert Number,

$P_r = \frac{hC_p}{k'}$: Prandtl Number

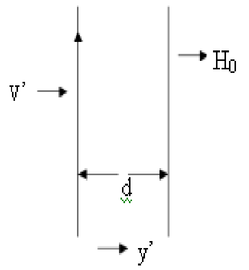

Fig. 1 Viscous Fluid between Parallel Infinite Plates through Porous Medium

BC's are reduced as: t > 0

$$u = 1,\ \theta = 1 \quad at \quad y = 0$$
$$u \to 0 \ \ \theta \to 0 \quad at \quad y = d$$

The boundary conditions are reduced as follows:

$$u_0 = 1,\ \ u_1 = 0,\ \ \theta_0 = 1,\ \theta_1 = 0 \text{ (at y =0)}$$
$$u_0 = 0,\ \ u_1 = 0,\ \ \theta_0 = 0,\ \theta_1 = 0 \text{ (at y= d).}$$

In multi-parameter perturbation technique and $\in << 1$ We take:

$$u_0 = u_{00} + E\ u_{01} \tag{4}$$

$$\theta_0 = \theta_{00} + E\ \theta_{01} \tag{5}$$

$$u_1 = u_{10} + E\ u_{11} \tag{6}$$

$$\theta_1 = \theta_{10} + E\ \theta_{11} \tag{7}$$

Finally we have:

$$u''_{00} - u_{00}\left(M + \frac{1}{k}\right) = -\ \theta_{00} \tag{8}$$

$$u''_{01} - u_{01}\left(M + \frac{1}{k}\right) = -\ \theta_{01} \tag{9}$$

$$u''_{10} - u_{10}\left(M + \frac{1}{k}\right) - i\omega u_{10} = -\ \theta_{10} \tag{10}$$

$$u''_{11} - u_{11}\left(M + \frac{1}{k}\right) - i\omega u_{11} = -\ \theta_{11} \tag{11}$$

$$\theta''_{00} = 0 \tag{12}$$

$$\theta''_{01} + P_r\ u'^2_{00} = 0 \tag{13}$$

$$\theta''_{10} - P_r\, i\omega\, \theta_{10} = 0 \tag{14}$$

$$\theta''_{11} - P_r\, i\omega\, \theta_{11} = -2\, P_r\, u'_{00}\ u'_{10} \tag{15}$$

Boundary conditions are reduced is:

$$u_{00} = 1,\ u_{01} = 0,\ u_{10} = 0,\ u_{11} = 0,\ \ \theta_{00} = 1,\ \theta_{01} = 0 \text{ (at y = 0),}$$

$$u_{00} = 0, u_{01} = 0,\ \theta_{00} = 0, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, u_{10} = 0, u_{11} = 0 \text{ (at y = d),}$$

Keeping in view of boundary condition's, we get the solutions as:

$$\theta_{00} = 1 - \frac{y}{d} \tag{16}$$

$$u_{00} = -\ A_1\, e^{\alpha y} + A_2\, e^{-\alpha y} + A_3 y - A_4 \tag{17}$$

$$\theta_{01} = -\ B_1\, e^{2\alpha y} - B_2\, e^{-2\alpha y} + B_3 y^2 - B_4\, e^{\alpha y} + B_5\, e^{-\alpha y} \tag{18}$$

$$u_{01} = C_1\ e^{2\alpha y} + C_2\ e^{-2\alpha y} + y^2\ C_3 - C_4\ y e^{\alpha y} - C_5\, y\, e^{-\alpha y} + C_6\, e^{\alpha y} + C_7\ e^{-\alpha y} \tag{19}$$

$$u_{10} = 0,\ \theta_{10} = 0,\ u_{11} = 0,\ \ \theta_{11} = 0 \tag{20}$$

Solving the above equations using BC's, we obtain:

$$\theta = 1 - \frac{y}{d} + E\left(-B_1\, e^{2\alpha y} - B_2\, e^{-2\alpha y} + B_2 y^2 + B_4\, e^{\alpha y} + B_5\, e^{-\alpha y}\right) \tag{21}$$

$$u = -A_1 e^{\alpha y} + A_2 e^{-\alpha y} + A_3 y - A_4 + E\left(C_1 e^{2\alpha y} + C_2 e^{-2\alpha y} + C_3 y^2 + C_4 y e^{\alpha y} - C_5 y e^{-\alpha y}\right) \tag{22}$$

TABLE I
THE PARTICULAR VALUES OF PARAMETERS

	Different Values of Parameters			
P_r	0.025	0.05	0.075	0.1
α^2	10	21	7	5
A_1	0.2586	1.994	0.6074	0.8299
A_2	1.3932	1.1901	1.5315	1.6916
A_3	0.5	0.233	0.71	1
A_4	0.1	0.473	0.142	0.2
B_1	0.00042	0.0497	0.00692	0.01722
B_2	0.01213	0.0177	0.0439	0.072
B_3	0.0316	0.545	0.2034	0.3028
B_4	0.0204	0.0102	0.0244	0.0742
B_5	0.011	0.00605	0.06164	1.513
C_1	0.00014	0.00079	0.00033	0.00114
C_2	0.0004	0.00028	0.0021	0.0048
C_3	0.0032	0.0259	0.0291	0.0765
C_4	0.0102	0.0051	0.0127	0.0371
C_5	0.0055	0.0031	0.031	0.756
C_6	–28.75	–1.7E–13	1.31E–08	3.53E–07
C_7	28.75	–.0031	–.031	–.756
λ_1	0.0293	0.00332	0.0153	–.3992
λ_2	0.0055	0.0031	0.031	.756

III. SOLUTION OF THE PROBLEM

$$u_{at\,m=0} = -.2586\,e^{\sqrt{10}.y} + 1.3932\,e^{-\sqrt{10}\,y} + .5y - .1 + E\Big[.000014e^{2\sqrt{10}\,y} + .0004\,e^{-2\sqrt{10}y} + .0032y^2 - .0102x\,e^{\sqrt{10}\,y}.y - .0055\,x\,e^{2\sqrt{21}\,y}.y\Big] \quad (23)$$

$$u_{at\,m=1} = -1.994\,e^{\sqrt{21}\,y} + 1.1101\,e^{-\sqrt{21}y} + .233y - .0473 + E\Big[.00014e^{2\sqrt{21}y} + .00028\,e^{-2\sqrt{21}y} + .0259y^2 - .0051y\,e^{-\sqrt{21}y} - .0031\,y\,e^{\sqrt{21}\,y}\Big] \quad (24)$$

$$u_{at\,m=2} = -.6074\,e^{\sqrt{7}\,y} + 1.5315\,e^{-\sqrt{7}y} + .71y - .142 + E\Big[.00033e^{2\sqrt{7}\,y} - .0021e^{-2\sqrt{7}y} + .0291y^2 - .0127y\,e^{\sqrt{7}y} - .031\,y\,e^{-\sqrt{7}\,y}\Big] \quad (25)$$

$$u_{at\,m=3} = -.8299\,e^{\sqrt{5}.y} + 1.6916\,e^{-\sqrt{5}y} + 1y - .2 + E\Big[.00114e^{2\sqrt{5}.y} + .0048\,e^{-2\sqrt{5}y} + .0765y^2 - .0371y\,e^{\sqrt{5}.y} - .756\,y\,e^{-\sqrt{5}.y}\Big] \quad (26)$$

$$\theta_{at\,m=0} = 1 - 5y + E\Big[.0316y^2 - .00042\,e^{2\sqrt{10}.y} - .01213\,e^{2\sqrt{10}.y} + .0204\,e^{\sqrt{10}\,y} + .011e^{-\sqrt{10}\,y}\Big] \quad (27)$$

$$\theta_{at\,m=1} = 1 - 5y + E\Big[.545y^2 - .0497\,e^{2\sqrt{21}\,y} - .0177\,e^{-2\sqrt{21}\,y} + .0102\,e^{\sqrt{21}\,y} + .0061\,e^{-\sqrt{21}\,y}\Big] \quad (28)$$

$$\theta_{at\,m=2} = 1 - 5y + E\Big[.2034y^2 - .0069\,e^{2\sqrt{7}.y} - .0439\,e^{\sqrt{7}.y} + .0616e^{-\sqrt{7}.y}\Big] \quad (29)$$

$$\theta_{at\,m=3} = 1 - 5y + E\Big[.0328y^2 - .0172\,e^{2\sqrt{5}\,y} - .072\,e^{\sqrt{5}.y} + .0616\,e^{-\sqrt{5}.y}\Big] \quad (30)$$

IV. RESULTS AND DISCUSSION

From Fig. 2, we observe that velocity increases with the increase of Eckert Number and also there is an increase with the distance from first plate. Effect of magnetic field shows that the velocity first decreases and then began to increase.

From Fig. 3, we can say that increase of Eckert Number causes an increase in temperature very nearer to plate and then it begins to decrease later on.

With the increase in magnetic field, temperature decreased slowly and then it had a very slight increase.

In Fig. 4, temperature is affected due to distance from first plate as is clear from graphs.

In Fig. 5, it shows that velocity first decrease with increase in magnetic field and began to increase slightly.

TABLE II
VELOCITY (U) PROFILES AT DIFFERENT VALUES OF ECKERT NUMBER WITH CHANGING DISTANCE BETWEEN PLATES

	VELOCITY (U) at M=0			
y	u at E=.1	u at E=.2	u at E=.3	u at E=.4
0	1.034641	1.034683	1.034724	1.034641
1	27.17373	27.15071	27.1277	27.17373
2	633.0785	632.3775	631.6765	633.0785
3	15151.28	15353.95	15556.61	15151.28
4	487284.3	621546.7	755809.1	487284.3
5	83899687	1.59E+08	2.35E+08	83899687

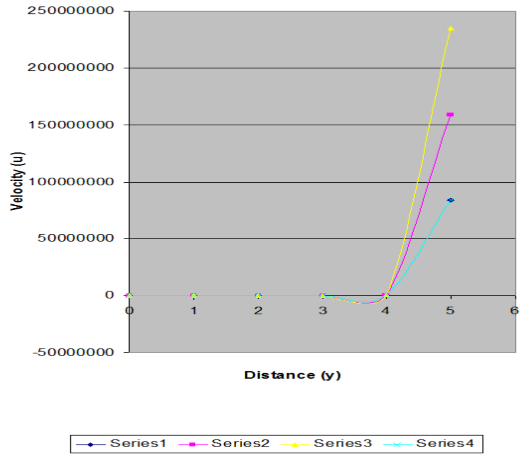

Fig. 2 Velocity (U) Profiles at Different Values of Eckert Number E = .1, .2, .3, .4 and at M=0

TABLE III
TEMPERATURE (θ) PROFILES AT DIFFERENT VALUES OF ECKERT NUMBER WITH RESPECT TO DISTANCE BETWEEN PLATES

	Temperature (θ) AT M = 0			
y	θ at E=.1	θ at E=.2	θ at E=.3	θ at E=.4
0	0.99184	0.98368	0.97552	0.96736
1	4.645953	5.291907	5.937860265	6.583814
2	395.2239	781.4479	1167.671816	1553.896
3	215199.6	430385.1	645570.6926	860756.3
4	1.20E+08	2.39E+08	358696565.8	4.78E+08
5	6.64E+10	1.33E+11	-1.99E+11	2.66E+11

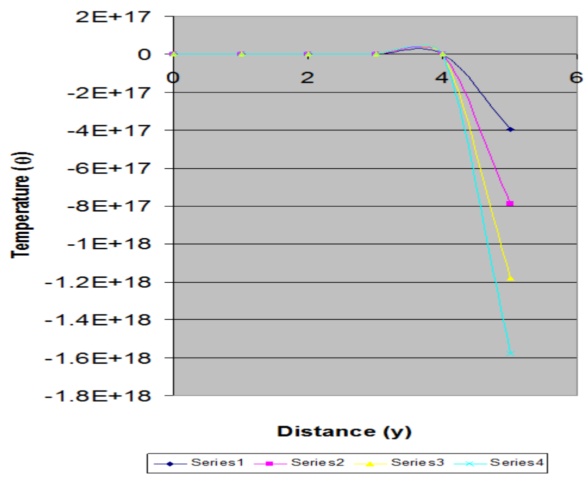

Fig. 3 Temperature (θ) Profiles at Different Values of Eckert Number E = .1,.2,.3,.4 and at M=0

TABLE IV
TEMPERATURE (Θ) PROFILES WITH RESPECT TO MAGNETIC FIELD (M)

	TEMPERATURE (θ)			
M	θ at E=.1	θ at E=.2	θ at E=.3	θ at E=.4
0	0.99184	0.98368	0.97552	0.96736
1	0.99489	0.98978	0.98467	0.97956
2	1.001089	1.002178	1.003267	1.004356
3	0.99724	0.99448	0.99172	0.98896

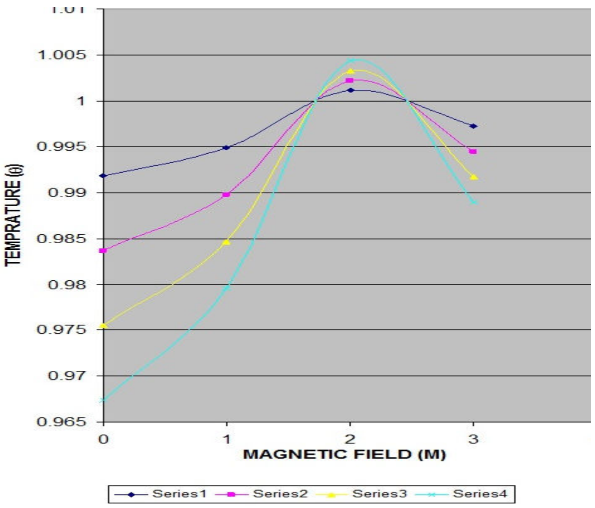

Fig. 4 Temperature (θ) Profiles with respect to Magnetic Field (M) at Different Values of Eckert Number E=.1,.2,.3,.4.

TABLE V
VELOCITY (U) PROFILES WITH RESPECT TO MAGNETIC FIELD (M)

	VELOCITY (U)			
M	u at E=.1	u at E=.2	u at E=.3	u at E=.4
0	1.034641	1.034683	1.034724	1.034641
1	-0.931171	-0.931141	-0.931112	-0.931082
2	0.782154	0.782208	0.782262	0.782316
3	0.662294	0.662888	0.663482	0.664076

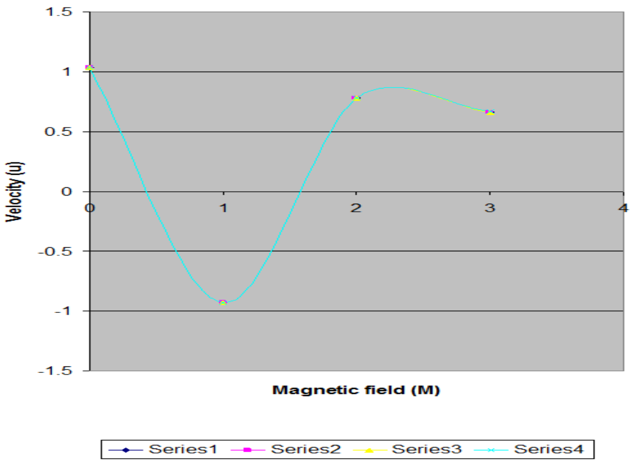

Fig. 5 Velocity (u) Profiles with respect to Magnetic Field (M) at Different Values of Distance Parameter Y =0,1,2,3

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