

# Longitudinal Shear Modulus of Single Aramid, Carbon and Glass Fibres by Torsion Pendulum Tests

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**Abstract**—The longitudinal shear moduli of a single aramid, carbon and glass fibres are measured in the present study. A popularly known concept of freely oscillating torsion pendulum has been used to characterize the torsional modulus. A simple freely oscillating torsional pendulum setup is designed with two different types of plastic discs: horizontal and vertical, as the known mass of the pendulum. The time period of the torsional oscillation is measured to determine the torsional rigidity of the fibre. Then the shear modulus of the fibre is calculated from its torsional rigidity. The mean shear modulus of aramid, carbon and glass fibres measured are  $6.22 \pm 0.09$ ,  $18.5 \pm 0.91$ ,  $38.1 \pm 3.55$  GPa by horizontal disc pendulum and  $6.19 \pm 0.13$ ,  $18.1 \pm 1.34$  and  $39.5 \pm 1.83$  GPa by vertical disc pendulum, respectively. The results obtained by both pendulums differed by less than 5% and agreed well with the results reported in literature for these three types of fibres. A detailed uncertainty calculations are carried out for the measurements. It is seen that scatter as well as uncertainty (or error) in the measured shear modulus of these fibres is less than 10%. For aramid fibres the effect of gauge length on the shear modulus value is also studied. It is verified that the scatter in measured shear modulus value increases with gauge length and scatter in fibre diameter.

**Keywords**—Aramid; Carbon; Glass fibres, Longitudinal shear modulus, Torsion pendulum.

## I. INTRODUCTION

**T**HE unidirectional fibrous composites are gaining importance in critical applications in aerospace, automobile and other fields. The accurate prediction of effective thermomechanical and strength properties of these composites by theoretical considerations and experiments, is an important issue from design and analysis point of view. The prediction of these properties of composites by theoretical methods requires the thermomechanical and strength properties of both fibre and matrix material. In general, these methods are termed as micromechanics based methods. The simplest way of predicting the properties is the rule of mixtures. However, there are a class of such micromechanics based models are available in open literature (for example see [1], [2]). Another regime where the properties of fibre and matrix are important is damage mechanics of these composites (see [3]-[7] as examples for micromechanics based damage modeling). Most of the damage mechanics models are based on micromechanical considerations where thermomechanical and strength properties of individual fibre and matrix materials are used.

The fibres considered in this study are aramid, carbon and glass fibres. The former two are transversely isotropic in

nature, while the later one is isotropic in nature. The aim of the present study is to measure the longitudinal shear modulus ( $G_{LT}$ ) of these fibres by direct measurements. These fibres are used to fabricate blades of an autonomous mini helicopter at this institute. Further, a damage mechanics group at this institute is looking at the micromechanics based damage models for the composites made from these fibres (see [3]-[7]). The present work is a first step to characterize single fibres by direct measurements and a part of damage mechanics study carried out at this institute [8]-[10].

In general, two main approaches are used to assess the shear properties of single fibres. The first one, called as indirect measurements, uses various mathematical models. In this, measurements are made on a simple composites like unidirectional lamina or fibres are embedded in a matrix with known mechanical properties [11]. These methods yield a mean shear property. Hence, for accurate measurement of shear properties of individual fibres the direct measurements are imperative. It is seen that the most of the direct measurement methods for shear modulus of single fibre use torsion tests. The measurement of the longitudinal shear modulus at room temperature by torsion pendulum is very popular. The early use of this concept can be seen in the measurement of shear moduli of boron fibres in [12]. The development of torsion pendulum for torsional testing of fine filaments due to Gloor is reported in [13]. Further, the use of the torsion pendulum apparatus for measurement of dynamic longitudinal shear modulus and damping in carbon fibres is seen in [14], [15]. The use of torsion pendulum for torsional behaviour study has been reported for Kevlar 49 (aramid) fibres [16], [17], carbon fibres [14]-[20] and ceramic fibres [18], [21]. Further, this concept has been used to study the torsional behaviour of various fibres like PBO [22], PPXTA [23], optical [24] and A265 fibres [25]. Recently, researchers are attempting to characterize the natural fibres for various applications. The spider silk is such one of the natural fibres. The torsional characterization of variety of spider silks by torsion pendulum has been attempted in [26]-[28]. The use for torsion pendulum for testing the shear properties of human hair is very popular in cosmetics community (see [29]-[34]). The shear property is treated as a measure to distinguish surface and bulk effects of activities in hair formulations, damage and loss of pigmentation. Other applications can be seen in [35] and [36]. The complete characterization of advanced fibres has been attempted in [9]. A similar study can also be seen in [37].

In the present study a torsional pendulum setup has been developed to measure longitudinal shear modulus of a single

fibre. Here, three types of fibres have been studied. They are: aramid, carbon and glass fibres. The details of the experimental procedure and calculations have been presented in the following sections.

## II. EXPERIMENTAL PROCEDURE

In this section we present the theoretical background of the use of torsion pendulum for measurement of the longitudinal shear modulus. The concept given in [38] has been employed in the present development.

### A. Theory of the Experiment

The longitudinal shear modulus plays an important role in the torsional vibration. The value of the longitudinal shear modulus is measured by the measurement of the period of the free oscillations. The fibre forms an important part of the pendulum. The pendulum is set into free oscillations under the conditions that the amplitude of the oscillation is small and there are no external disturbances like air current.

Consider a fibre of length  $L$ , suspended at one end with a known mass attached at the other end. Assume the fibre material to be linear elastic, isotropic (transversely isotropic for aramid and carbon fibres) and homogeneous. This pendulum can be set manually in torsional oscillations governed by the following differential equation with constant coefficients

$$J \ddot{\theta} + c \dot{\theta} + k \theta = 0 \quad (1)$$

where  $J$  is the mass moment of inertia of the mass attached to the fibre end;  $\ddot{\theta}$ ,  $\dot{\theta}$  and  $\theta$  are the angular acceleration, angular velocity and angular amplitude, respectively of the mass attached at the end of the fibre;  $c$  is the damping coefficient of the system and  $k$  is the torsional rigidity of the fibre. Since, the effect of damping is negligible in this case (see [38]), it can be neglected in order to simplify the equations. Thus, (1) becomes

$$J \ddot{\theta} + k \theta = 0 \quad (2)$$

The torsion pendulum can be treated as a single degree of freedom undamped system under free vibration. The angular frequency  $\omega$  and time period of oscillation  $T$  are given as

$$\omega = \sqrt{\frac{k}{J}} \quad (3)$$

$$T = 2\pi \sqrt{\frac{J}{k}} \quad (4)$$

Now the stiffness of the system in terms of the time period is

$$k = \frac{4\pi^2 J}{T^2} \quad (5)$$

Considering the fibre as a cylinder under torsion, the torsional rigidity is given by

$$k = \frac{G_{LT} I_p}{L} \quad (6)$$

where,  $G_{LT}$  is the shear modulus of the fibre and  $I_p$  is its polar moment of inertia given as

$$I_p = \frac{\pi d^4}{32} \quad (7)$$

Here,  $d$  is the average fibre diameter. It is assumed that the fibre is circular in cross section. Equating (5) and (6) and substituting (7), we get,

$$G_{LT} = \frac{128 \pi J L}{d^4 T^2} \quad (8)$$

Thus, from (8) one can determine  $G_{LT}$  of the fibre.

### B. Measurement of Fibre Diameter

From (7) and (8) it is clear that the measurement of the fibre diameter plays a crucial role in the final value of the shear modulus. The error in the measurement of  $d$  affects significantly the error in the measurement of  $G_{LT}$ . Hence, an accurate measurement of the fibre diameter is an important factor in this study.

An advanced optical microscope is used to measure diameter of glass and carbon fibres. A magnification of 200X is used to capture the fibre specimen pictures. The diameter is directly measured from the image obtained. A total of five fibres of each type are measured. About 3 to 5 measuring points are taken along the length of each fibre specimen. The mean diameter is determined and used for all the calculations.

### C. Testing of Specimen

The specimen for this test is prepared by using a very small polythene sheet as end tabs, so that the tab would be stiff enough to be inserted into the slit made in the discs used as the mass of the pendulum. The gauge lengths chosen ranges from 10 to 50 mm for aramid fibres, 15 to 25 mm for carbon fibres and 10 to 30 mm for glass fibres.

The fibres are tested with two different discs. In case of horizontal disc, the axis of oscillation is set to be normal to the plane of the disc as shown in Fig. 1(a) and for the vertical disc, the axis is parallel to its plane which is shown in Fig. 1(b). The mass of horizontal and vertical disc are measured to be 180 mg and 220 mg and their mass moment of inertia about the axis of oscillation is calculated to be 2.409 g mm<sup>2</sup> and 3.094 g mm<sup>2</sup>, respectively. One end of the specimen tab is held by a clip and other end is inserted into the slit of the horizontal disc.

Immense care is taken while using horizontal disc pendulum to ensure that the disc is completely horizontal and the axis of fibre coincides with the axis of rotation of the disc. In case of the vertical disc, the lower end of the fibre is directly bonded to the surface of the disc with suitable light weight adhesive, ensuring the coincidence of the axis of fibre with axis of rotation of the disc. Further, the entire setup is enclosed in an airtight transparent box to avoid external disturbance due to air currents, etc.

A small twist is given to the fibre by manually setting the pendulum in free torsional oscillation. The period of oscillation of the resulting motion is then measured with a stop watch

of 0.01 s resolution, by following the motion of a reference mark on the disc relative to a similar mark on the platform mounted below the disc. The measured time period is used in (8) to calculate the shear modulus of the fibre. 15 fibres each of aramid, carbon and glass are tested using the horizontal disc pendulum and 10 fibres each are tested with the vertical disc pendulum.

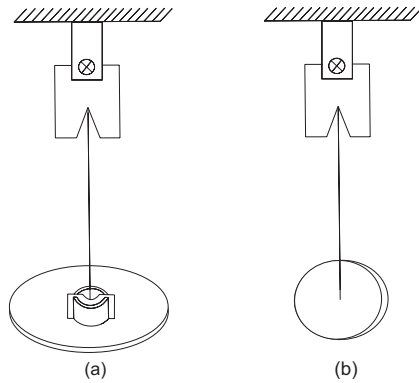


Fig. 1. Sketch of the test setups used in the experiments: (a) Horizontal; (b) Vertical disc torsion pendulum.

### III. RESULTS AND DISCUSSION

The results are arranged in two subsections. The first one deals with the measurement of fibre diameter and the second one deals with measurement of  $G_{LT}$ .

#### A. Measurement of Fibre Diameter

The diameter of fibres is determined using the method explained earlier. The optical micrographs of these three fibres are shown in Fig. 2. The average diameter is determined for each type of fibre. The details of the measured fibre diameter are given in Table I. The scatter in the measurement of glass fibre is as more compared to aramid and carbon fibres.

**Remark:** If the effect of scatter on measurement of  $G_{LT}$  is considered, then resultant scatter of four times (of scatter in measurement of fibre diameter) will be observed. This is because  $G_{LT}$  is inversely proportional to fourth power of fibre diameter (see (8)). However, the results given in the following section are calculated based on the mean diameter of each type.

TABLE I  
MEASUREMENT OF FIBRE DIAMETERS.

Diameter	Aramid	Carbon	Glass
Mean ( $\mu\text{m}$ )	12.15	6.82	15.92
Standard Deviation ( $\mu\text{m}$ )	0.07	0.129	0.996
Standard Deviation %	0.57	1.91	6.26

#### B. Measurement of $G_{LT}$

1) *Measurement of Mean Shear Modulus:* The tests are carried out for the gauge lengths as mentioned above. The summary of the results for all experiments has been reported

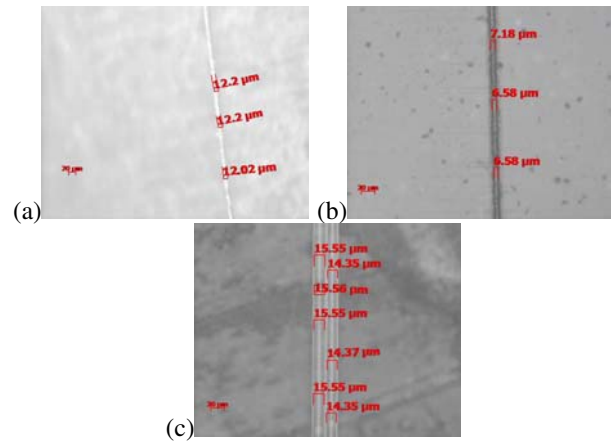


Fig. 2. Images from optical microscope at 200X magnification of (a) aramid (b) carbon and (c) glass fibres.

TABLE II  
VARIATION IN SHEAR MODULUS OBTAINED FROM TORSION PENDULUM TESTS

$G_{LT}$	Aramid Fibre		Carbon Fibre		Glass Fibre	
	Disc H	Disc V	Disc H	Disc V	Disc H	Disc V
Mean (GPa)	6.22	6.19	18.5	18.1	38.1	39.5
S D (GPa)	0.09	0.13	0.91	1.34	3.55	1.83
S D (%)	1.44	2.10	4.93	7.41	9.32	4.63

in Table II. In this table “Disc H” stands for torsion pendulum with horizontal disc setup and “Disc V” stands for torsion pendulum with vertical disc setup. It can be seen that the scatter in shear modulus of glass fibre is more as compared to the aramid and carbon fibres. Further, from Table I one can see that the scatter in shear modulus is directly related to scatter in fibre diameter. It is also observed that the measured mean values of  $G_{LT}$  for these fibres by two experimental setups differ by less than 5%. The measured values of the  $G_{LT}$  for these fibres are very close to the values reported in literature (for example, see [15]-[18], [22], [39]).

2) *Error in the Shear Modulus Calculations:* For the calculation of shear modulus various quantities, like mass of the discs, gauge lengths, time period of the oscillation and fibre diameter need to be measured. There are errors or uncertainties associated with these measurements. Together, these affect the overall error or uncertainties in the measurement of  $G_{LT}$ . This can be estimated from (8) as

$$\frac{\Delta G_{LT}}{G_{LT}} = \frac{\Delta J}{J} + \frac{\Delta L}{L} + \frac{2\Delta T}{T} + \frac{4\Delta d}{d} \quad (9)$$

For horizontal disc pendulum,  $\frac{\Delta J}{J} = \frac{\Delta m}{m} + \frac{2\Delta R}{R} = 1.5\%$ . Here,  $R$  is the radius of the disc.  $\frac{2\Delta T}{T}$  varies from 0.07% to 1.12%. Thus, for a gauge length of 10 mm of aramid fibre  $\frac{\Delta L}{L} = 1.0\%$  and  $\frac{4\Delta d}{d} = 3.3\%$  and for a gauge length of 15 mm of carbon fibre  $\frac{\Delta L}{L} = 0.7\%$  and  $\frac{4\Delta d}{d} = 5.9\%$ . Similarly, for a gauge length of 10 mm of glass fibre  $\frac{\Delta L}{L} = 1.0\%$  and  $\frac{4\Delta d}{d} = 2.5\%$ . Therefore, the maximum overall relative error in the measurement is of the order 7%, 9% and 6% for aramid, carbon and glass fibres, respectively. Similarly, for vertical disc

pendulum,  $\frac{\Delta J}{J} = \frac{\Delta m}{m} + \frac{2\Delta R}{R} + \frac{2\Delta h}{h} = 2.0\%$ . Here,  $h$  is the thickness of the disc.  $\frac{2\Delta T}{T}$  varies from 0.07% to 0.59%. Thus, the maximum overall relative error in the measurement with this pendulum is also of the order 7%, 9% and 6% for aramid, carbon and glass fibres, respectively.

The scatter in measured value of  $G_{LT}$  is attributed to scatter in diameter. Further, for same length of the fibre the measured value of  $G_{LT}$  is seen to be different. This is because the fibre diameter also varies along its length.

3) *Effect of Gauge Length on Shear Modulus:* From (8) it can be seen that the shear modulus is inversely proportional to  $d^4$ . Thus, the scatter in the measurement of fibre diameter significantly affects the shear modulus. Further, as the gauge length increases the scatter in fibre diameter increases. Therefore, in order to see the effect of gauge length on measured values of the shear modulus a set of systematic tests is conducted. The tests are conducted for gauge lengths 10, 20, 30, 40 and 50 mm on aramid fibres. The values of the shear modulus measured by both horizontal and vertical discs for these gauge lengths are shown in Fig. 3 and Fig. 4, respectively. From these figures it can be seen that as the gauge length increases the variation in the shear modulus value also increases. This is due to the increased scatter in the measured diameter of the fibre. It is to be noted that the fibre diameter measured in Table I are for a very short length as compared to the gauge lengths used in this study. The standard deviation for shear modulus varied from 0.03 (for 10 mm gauge length) to 0.08 (for 50 mm gauge length) GPa for horizontal disc and from 0.02 to 0.10 GPa (for respective gauge lengths) for vertical disc.

**Remark:** The exact specifications like precursor material, elemental constitution, manufacturer, etc. of the fibres tested are not known to the authors.

A number of individual fibres of three types have been tested. This gives more statistical information about fibre diameter and hence, the measured shear modulus. This information is very essential because some of the damage models based on micromechanical study uses this statistical data (for example see the previous work [3]-[7]).

#### IV. CONCLUSION

A simple torsion pendulum setup with horizontal and vertical discs has been developed for the measurement of longitudinal shear modulus of a single fibre. The concept of undamped free torsional oscillations has been used to calculate the longitudinal shear modulus of the fibre. Three types of fibres: aramid, carbon and glass, are tested. The effect of gauge length on shear modulus is also studied in detail for aramid fibre. The key conclusions and observations from this study are as follows:

1. The shear modulus of aramid, carbon and glass fibres is measured as  $6.22 \pm 0.09$ ,  $18.5 \pm 0.91$  and  $38.1 \pm 3.55$  GPa using horizontal disc pendulum and  $6.19 \pm 0.13$ ,  $18.1 \pm 1.34$  and  $39.5 \pm 1.83$  GPa using vertical disc pendulum, respectively. The scatter in shear modulus of glass fibre is seen more as compared to other two fibre types.
2. The values obtained by two setups differed by less than 5% only.

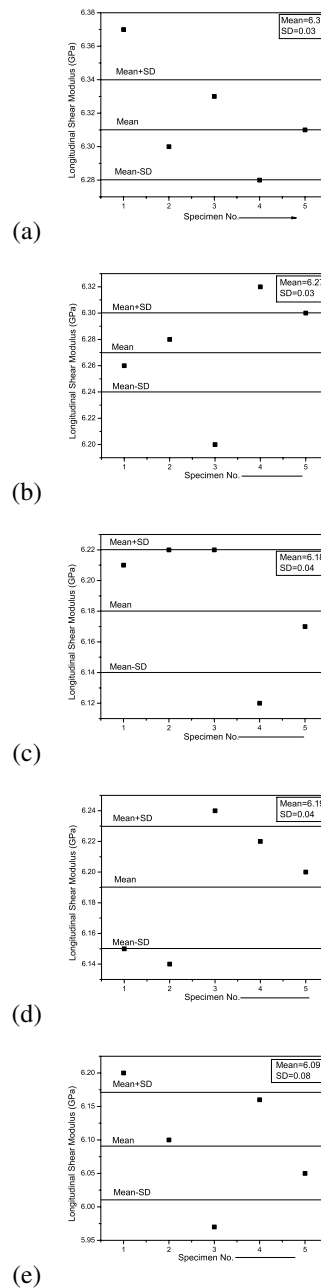


Fig. 3. Effect of gauge length on shear modulus for aramid fibre using horizontal disc pendulum for gauge length (a) 10 mm; (b) 20 mm; (c) 30 mm; (d) 40 mm and (e) 50 mm.

3. The scatter in measured values of the shear modulus is significantly affected by scatter in fibre diameters. For glass fibres the scatter in diameter is highest while for the aramid fibres is the lowest among the fibre types tested here. The same has been observed for the scatter in their longitudinal shear moduli measured.
4. The scatter in measured values of the shear modulus increases as the gauge length of the specimen is increased. In this study, this effect was studied for aramid fibres. The standard deviation varied from 0.03 to 0.08

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4. The scatter in measured values of the shear modulus increases as the gauge length of the specimen is increased. In this study, this effect was studied for aramid fibres. The standard deviation varied from 0.03 to 0.08 GPa for horizontal disc setup and from 0.02 to 0.10 GPa for vertical disc setup.
4. The scatter in the measurement of shear modulus for the three fibre types studied is less than 10%.
5. The error or uncertainty in the measurement of shear modulus for the three fibre types studied is also less than 10%.

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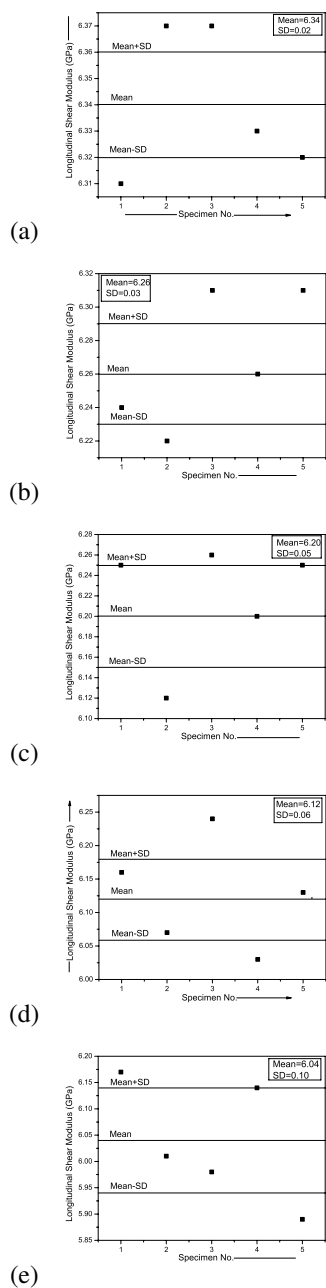


Fig. 4. Effect of gauge length on shear modulus for aramid fibre using vertical discs pendulum for gauge length (a) 10 mm; (b) 20 mm; (c) 30 mm; (d) 40 mm and (e) 50 mm.

GPa for horizontal disc setup and from 0.02 to 0.10 GPa for vertical disc setup.

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