

Dependent Weighted Aggregation Operators of Hesitant Fuzzy Numbers

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Abstract—In this paper, motivated by the ideas of dependent weighted aggregation operators, we develop some new hesitant fuzzy dependent weighted aggregation operators to aggregate the input arguments taking the form of hesitant fuzzy numbers rather than exact numbers, or intervals. In fact, we propose three hesitant fuzzy dependent weighted averaging(HFDWA) operators, and three hesitant fuzzy dependent weighted geometric(HFDWG) operators based on different weight vectors, and the most prominent characteristic of these operators is that the associated weights only depend on the aggregated hesitant fuzzy numbers and can relieve the influence of unfair hesitant fuzzy numbers on the aggregated results by assigning low weights to those “false” and “biased” ones. Some examples are given to illustrate the efficiency of the proposed operators.

Keywords—Hesitant fuzzy numbers, hesitant fuzzy dependent weighted averaging(HFDWA) operators, hesitant fuzzy dependent weighted geometric(HFDWG) operators.

I. INTRODUCTION

TO fuse individual experts' preference information into an overall one, many operators have been developed in the past decades. Among them, the ordered weighted averaging(OWA) operator introduced by Yager[1] is the most widely used one, and has received more and more attention since its appearance. A lot of extensions of the OWA operator have been proposed, such as the uncertain aggregation operators[2], the induced aggregation operators[3], the uncertain linguistic aggregation operator[4]. All of the above operators consist of the following three steps[2]: (1) reorder the input arguments in descending order; (2) determine the OWA weights; and (3) multiply these ordered arguments, and then aggregate all the weighted arguments. Because the order relation and the operational laws of different type input information have been solved successfully, the key of the OWA operator is to determine its associated weights. Many scholars have studied this problem, and developed some useful approaches[1-6]. Some recent achievements include: In 2005, Xu[5] proposed a technique to determine the OWA weights using the input arguments, and introduced some dependent OWA operators, which can relieve the influence of unfair arguments by assigning small weights to these far away from the mean. Then, this technique is extended to aggregated different type input information by many authors, for example, Xu[2,6] proposed some dependent uncertain ordered weighted aggregations operators, and Wei et al.[7] developed some dependent 2-tuple linguistic aggregation operators, including the

dependent 2-tuple ordered weighted averaging(D2TOWA) operator and the dependent 2-tuple ordered weighted geometric(D2TOWG) operator.

Recently, Torra and Narukawa[8] and Torra[9] proposed a new generalization of fuzzy set(FS): the hesitant fuzzy set(HFS), which can be applied in the situations where there are some difficulties in determining the membership of an element to a set caused by a doubt between a few different values. Much work has been done about HFS[10-12], and some aggregation operators for HFSs have been proposed. However, the weights used by all of these operators are argument-independent, which is derived only by the particular ordered positions of the aggregated arguments. It seems that in the literature there is no investigation on the dependent weighted aggregation operators, which are argument-dependent operators, for HFSs. Thus, to solve this issue, in this paper, based on the dependent weighted aggregation operators in [2,13], we propose some hesitant fuzzy dependent weighted aggregation operators to aggregate the input arguments taking the form of hesitant fuzzy numbers(HFNs), including the hesitant fuzzy dependent weighted averaging(HFDWA) operators, and the hesitant fuzzy dependent weighted geometric(HFDWG) operators.

The rest of this paper is organized as follows: In Section II, we briefly review some basic concept such as the dependent weighted averaging(DWA) operator, the dependent weighted geometric(DWG) operator and the hesitant fuzzy sets. In Section III, we present the HFDWA operators and the HFDWG operators, and some of their properties are studied. In Section IV, a numerical example is used to verify the proposed operators. Finally, Section V summarizes the paper.

II. PRELIMINARIES

A. The DWA operator and the DWG operator

The dependent weighted average(DWA) operator and the dependent weighted geometric(DWG) operator were introduced by Xu[2,6], which can be defined as follows:

Definition 1.[2,6] Let a_1, a_2, \dots, a_n be a collection of arguments, and let μ be the average value of these arguments, i.e., $\mu = \frac{1}{n} \sum_{i=1}^n a_i$, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i = 2, 3, \dots, n$, then we call

$$DWA(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i a_{\sigma(i)}$$

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the dependent weighted averaging(DWA) operator, and

$$DWG(a_1, a_2, \dots, a_n) = \prod_{i=1}^n a_{\sigma(i)}^{w_i}$$

the dependent weighted geometric(DWG) operator, where $w_i = \frac{s(a_{\sigma(i), \mu})}{\sum_{j=1}^n s(a_{\sigma(j), \mu})}$, and $s(a_{\sigma(i), \mu}) = 1 - \frac{|a_{\sigma(i), \mu}|}{\sum_{j=1}^n |a_{\sigma(j), \mu}|}$. In

fact, the DWA operator and the DWG operator can be rewritten as:

$$DWA(a_1, a_2, \dots, a_n) = \frac{\sum_{i=1}^n s(a_i, \mu) a_i}{\sum_{i=1}^n s(a_i, \mu)},$$

and

$$DWG(a_1, a_2, \dots, a_n) = \prod_{i=1}^n a_i^{s(a_i, \mu) / \sum_{i=1}^n s(a_i, \mu)},$$

respectively. Therefore, the above two operators are neat and dependent OWA operator or OWG operator.

B. Hesitant fuzzy set

Hesitant fuzzy set(HFS) was originally introduced by Torra and Narukawa[8] and Torra[9], and it permits the membership degree of an element to a set to be represented as several possible values between 0 and 1.

Definition 2. ([8,9]) Let X be a fixed set, a hesitant fuzzy set(HFS) on X is in terms of a function that when applied to X returns a subset of $[0,1]$, which can be represented as the following mathematical symbol:

$$E = \{ \langle x, h(x) \rangle | x \in X \}, \quad (1)$$

where $h(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set E . For convenience, we call $h(x)$ a hesitant fuzzy number(HFN) and H the set of all the HFNs.

For two HFNs h_1 and h_2 , Xu and Xia[10] proposed some distance measures for HFNs, and in this paper, we utilize the following hesitant normalized Hamming distance:

$$d(h_1, h_2) = \frac{1}{l_{\max}} \sum_{j=1}^{l_{\max}} |h_1^{\sigma(j)} - h_2^{\sigma(j)}|, \quad (2)$$

where $h_1^{\sigma(j)}$ and $h_2^{\sigma(j)}$ are the j th largest values in h_1 and h_2 , respectively, and $l_{\max} = \max\{l_1, l_2\}$, l_1 is the number of values in h_1 , and l_2 is the number of values in h_2 . In fact, in most cases, $l_1 \neq l_2$, thus, in [10], Xu and Xia have proposed a technique to extend the shorter one until both of them have the same length. They suggested to add the same value several times in the shorter one, and the value can be any one in it. In the following, we add the minimum value in the shorter one.

Definition 3.[11] For a HFN h , $s(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma$ is called the score function of h , where $\#h$ is the number of the elements in h . Moreover, for two HFNs h_1 and h_2 , if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Let h, h_1 and h_2 be three HFNs, then the operational laws on the HFNs are given as follows.

- (1) $h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\}$.
- (2) $\lambda h = \cup_{\gamma \in h} \{1 - (1 - \gamma)^\lambda\}$.
- (3) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\}$.
- (4) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 \gamma_2\}$.

III. HFDWA AND HFDWG OPERATORS

First, we define the mean, the variance and the similarity of a collection of HFNs.

Definition 4. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers with the same number of values (otherwise, we can extend the shorter ones by adding the minimum element in them until all the HFNs have the same number of values), and then we define the mean of these hesitant fuzzy numbers as \tilde{h} , where

$$\tilde{h} = \left\{ \frac{1}{n} \sum_{i=1}^n h_i^{\sigma(l)}, \sigma(l) = 1, 2, \dots, p \right\}, \quad (3)$$

in which $h_i^{\sigma(l)}$ is the l th largest values in $h_i (i = 1, 2, \dots, n)$, and p is the number of values in $h_i (i = 1, 2, \dots, n)$.

Example 1. Let $h_1 = \{0.1, 0.2, 0.3\}$, $h_2 = \{0.4, 0.5\}$, $h_3 = \{0.3\}$ be three HFNs, then we extend h_2, h_3 until they have the same length of h_1 . That is: $h_2 = \{0.4, 0.4, 0.5\}$, $h_3 = \{0.3, 0.3, 0.3\}$, thus their mean $\tilde{h} = \{0.2667, 0.3, 0.3667\}$. Based on the distance measure and mean of the HFNs, we can define the variance of HFNs as follows.

Definition 5. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers, then we define the variance of these hesitant fuzzy numbers as

$$\tilde{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^n d^2(h_i, \tilde{h})}. \quad (4)$$

Definition 6. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers, then we call

$$s(h_{\sigma(i)}, \tilde{h}) = 1 - \frac{d(h_{\sigma(i)}, \tilde{h})}{\sum_{j=1}^n d(h_{\sigma(j)}, \tilde{h})}, i = 1, 2, \dots, n, \quad (5)$$

the degree of similarity between the i th largest hesitant fuzzy argument $h_{\sigma(i)}$ and the mean \tilde{h} , where $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$ such that $h_{\sigma(i-1)} \geq h_{\sigma(i)}$ for all $i = 2, 3, \dots, n$.

A. HFDWA and HFDWG operators based on Xu's idea

In the following, to aggregate some HFNs, motivated by Xu[2], we give a weighting vector $w = (w_1, w_2, \dots, w_n)^T$, which can assign low weights to these values far away to their mean. That is to say, the closer an HFN is to the mid one(s), the more the weight. Then, we can define the weights as:

$$w_i = \frac{s(h_{\sigma(i)}, \tilde{h})}{\sum_{j=1}^n s(h_{\sigma(j)}, \tilde{h})}, i = 1, 2, \dots, n. \quad (6)$$

Obviously, $w_i \geq 0, i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. Especially, if $h_i = h_j$, for all $i, j = 1, 2, \dots, n$, then by (6), we have $w_i = 1/n$, for all $i = 1, 2, \dots, n$.

Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers. Based on the previous operational laws of HFNs, we extend the DWA operator and the DWG operator to the hesitant fuzzy environments.

Definition 7. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers, then we call

$$\begin{aligned} \text{HFDWA}(h_1, h_2, \dots, h_n) &= \sum_{i=1}^n w_i h_{\sigma(i)} \\ &= \sum_{i=1}^n s(h_{\sigma(i)}, \tilde{h}) h_{\sigma(i)} / \sum_{i=1}^n s(h_{\sigma(i)}, \tilde{h}), \end{aligned} \quad (7)$$

a hesitant fuzzy dependent weighted averaging(HFDWA) operator, where $h_{\sigma(i)}$ is the i th largest of h_1, h_2, \dots, h_n , and $w_i (i = 1, 2, \dots, n)$ is defined by Eq.(6).

Since $\sum_{i=1}^n s(h_{\sigma(i)}, \tilde{h}) h_{\sigma(i)} = \sum_{i=1}^n s(h_i, \tilde{h}) h_i$ and $\sum_{i=1}^n s(h_{\sigma(i)}, \tilde{h}) = \sum_{i=1}^n s(h_i, \tilde{h})$, then the HFDWA operator can be rewritten as:

$$\text{HFDWA}(h_1, h_2, \dots, h_n) = \sum_{i=1}^n s(h_i, \tilde{h}) h_i / \sum_{i=1}^n s(h_i, \tilde{h}). \quad (8)$$

Therefore, the HFDWA operator is independent of the ordering, thus it is a neat operator.

Similar to Xu[2], we have the following conclusion:

Theorem 1. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers, and let \tilde{h} be the mean of these HFNs, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$ such that $h_{\sigma(i-1)} \geq h_{\sigma(i)}$ for all $i = 2, 3, \dots, n$. If $s(h_{\sigma(i)}, \tilde{h}) \geq s(h_{\sigma(j)}, \tilde{h})$, then $w_i \geq w_j$.

Based on the DWG operator, we can define the HFDWG operator as follows:

Definition 8. Let $h_i (i = 1, 2, \dots, n)$ be a collection of hesitant fuzzy numbers on X , then we call

$$\begin{aligned} \text{HFDWG}(h_1, h_2, \dots, h_n) &= \prod_{i=1}^n h_{\sigma(i)}^{w_i} \\ &= \prod_{i=1}^n h_{\sigma(i)}^{s(h_{\sigma(i)}, \tilde{h}) / \sum_{j=1}^n s(h_{\sigma(j)}, \tilde{h})}, \end{aligned} \quad (9)$$

a hesitant fuzzy dependent weighted geometric(HFDWG) operator, where $h_{\sigma(i)}$ is the i th largest of h_1, h_2, \dots, h_n , and $w_i (i = 1, 2, \dots, n)$ is defined by (6).

Similarly, the HFDWG operator can be rewritten as:

$$\text{HFDWG}(h_1, h_2, \dots, h_n) = \prod_{i=1}^n h_i^{s(h_i, \tilde{h}) / \sum_{j=1}^n s(h_j, \tilde{h})}. \quad (10)$$

Example 2. Further consider the three HFNs, then from Definition 3, we have $h_2 > h_3 > h_1$. Hence, we can reorder the arguments $h_i (i = 1, 2, 3)$ in descending order:

$h_{\sigma(1)} = \{0.4, 0.5\}, h_{\sigma(2)} = \{0.3\}, h_{\sigma(3)} = \{0.1, 0.2, 0.3\}$. By (5), we have $s(h_{\sigma(1)}, \tilde{h}) = 0.5417, s(h_{\sigma(2)}, \tilde{h}) = 0.8750, s(h_{\sigma(3)}, \tilde{h}) = 0.5833$. And by (6), we have $w_1 = 0.2708, w_2 = 0.4375, w_3 = 0.2917$. Then, from (7), it follows that

$$\begin{aligned} \text{HFDWA}(h_1, h_2, h_3) &= 0.2708 \times \{0.4, 0.5\} + 0.4375 \times \{0.3\} \\ &\quad + 0.2917 \times \{0.1, 0.2, 0.3\} \\ &= \{0.1292, 0.1712\} + \{0.1445\} + \{0.0303, 0.0630, 0.0988\} \\ &= \{0.2776, 0.3020, 0.3286, 0.3124, 0.3356, 0.3610\}. \end{aligned}$$

Furthermore, by (9), we have

$$\begin{aligned} \text{HFDWG}(h_1, h_2, h_3) &= \{0.4, 0.5\}^{0.2708} \times \{0.3\}^{0.4375} \times \{0.1, 0.2, 0.3\}^{0.2917} \\ &= \{0.7802, 0.8288\} \times \{0.5909\} \times \{0.5109, 0.6254, 0.7039\} \\ &= \{0.2354, 0.2881, 0.3243, 0.2500, 0.3061, 0.3445\}. \end{aligned}$$

B. HFDWA and HFDWG operators based on the normal distribution

The normal distribution is one of the most commonly observed and is the starting point for modeling many natural process, it is usually found in events that are the aggregation of many smaller, but independent random events. In [2], utilizing the normal distribution, Xu introduced a normal distribution based method to determine the aggregated weights, which only depends on the aggregated arguments and has the similar property as the weights defined by (6). That is, the closer an argument is to the mid one(s), the more the weight. In fact, we give another weight vector whose components are defined as follows:

$$w_i = \frac{1}{\sqrt{2\pi\tilde{\sigma}}} e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}}, i = 1, 2, \dots, n, \quad (11)$$

where \tilde{h} and $\tilde{\sigma}$ are the mean and the variance of these aggregated arguments h_1, h_2, \dots, h_n , respectively. To normalize the weight vector $w = (w_1, w_2, \dots, w_n)^\top$, we have

$$w_i = \frac{\frac{1}{\sqrt{2\pi\tilde{\sigma}}} e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}}}{\sum_{j=1}^n \frac{1}{\sqrt{2\pi\tilde{\sigma}}} e^{-\frac{d^2(h_{\sigma(j)}, \tilde{h})}{2\tilde{\sigma}^2}}} = \frac{e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}}}{\sum_{i=1}^n e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}}}, i = 1, 2, \dots, n. \quad (12)$$

Then, we can present another hesitant fuzzy dependent weighted averaging operator:

$$\begin{aligned} \text{HFDWA}(h_1, h_2, \dots, h_n) &= \sum_{i=1}^n w_i h_{\sigma(i)} \\ &= \frac{\sum_{i=1}^n e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}} h_{\sigma(i)}}{\sum_{i=1}^n e^{-\frac{d^2(h_{\sigma(i)}, \tilde{h})}{2\tilde{\sigma}^2}}} = \frac{\sum_{i=1}^n e^{-\frac{d^2(h_i, \tilde{h})}{2\tilde{\sigma}^2}} h_i}{\sum_{i=1}^n e^{-\frac{d^2(h_i, \tilde{h})}{2\tilde{\sigma}^2}}}. \end{aligned} \quad (13)$$

Obviously, (13) is independent of the ordering, therefore, it is also a neat operator.

Furthermore, by (9) and (12), we have

$$\text{HFDWG}(h_1, h_2, \dots, h_n) = \prod_{i=1}^n h_i \frac{e^{-\frac{d^2(h_i, \tilde{h})}{2\tilde{\sigma}^2}}}{\sum_{i=1}^n e^{-\frac{d^2(h_i, \tilde{h})}{2\tilde{\sigma}^2}}} \quad (14)$$

C. HFDWA and HFDWG operators based on trigonometric function

Recently, Wang et al.[12] proposed an approach to obtaining the OWA operator's weights based on trigonometric function. In the next part, we will extend this approach to accommodate the situations where the input arguments are hesitant fuzzy numbers.

Step 1. For a collection of HFNs $h_i (i = 1, 2, \dots, n)$, by Eq.(3), compute their mean \tilde{h} .

Step 2. Compute $a = \max_i d(h_i, \tilde{h})$ and $b = \min_i d(h_i, \tilde{h})$ by (2), if $a = 0$, then set $w_i = 1/n, i = 1, 2, \dots, n$, otherwise, compute $u_i (i = 1, 2, \dots, n)$ by $u_i = (d(h_i, \tilde{h})) / (2a + 2b), i = 1, 2, \dots, n$.

Step 3. Compute the weights by:

$$w_i = \frac{\cos(u_{\sigma(i)}\pi)}{\sum_{j=1}^n \cos(u_{\sigma(j)}\pi)}, i = 1, 2, \dots, n. \quad (15)$$

Example 3. For the HFNs given in Example 1, we compute their weights by the above procedure.

(1). By (3), $\tilde{h} = \{0.2667, 0.3, 0.3667\}$. (2). By (2), $a = 0.1222$. Then, $u_1 = 0.3929, u_2 = 0.1071, u_3 = 0.3571$. (3). By (15), we have $w_1 = 0.1934, w_2 = 0.5526, w_3 = 0.2540$.

Based on the DWA and (15), we can define a new HFDWA operator as follows:

$$\begin{aligned} \text{HFDWA}(h_1, h_2, \dots, h_n) &= \sum_{i=1}^n w_i h_{\sigma(i)} \\ &= \frac{\sum_{i=1}^n \cos(u_{\sigma(i)}\pi) h_{\sigma(i)}}{\sum_{j=1}^n \cos(u_{\sigma(j)}\pi)} = \frac{\sum_{i=1}^n \cos(u_i\pi) h_i}{\sum_{i=1}^n \cos(u_i\pi)}. \end{aligned} \quad (16)$$

And we also can define a new HFDWG operator as follows:

$$\begin{aligned} \text{HFDWG}(h_1, h_2, \dots, h_n) &= \prod_{i=1}^n h_{\sigma(i)}^{w_i} \\ &= \prod_{i=1}^n h_{\sigma(i)}^{\frac{\cos(u_{\sigma(i)}\pi)}{\sum_{j=1}^n \cos(u_{\sigma(j)}\pi)}} = \prod_{i=1}^n h_i^{\frac{\cos(u_i\pi)}{\sum_{i=1}^n \cos(u_i\pi)}}. \end{aligned} \quad (17)$$

IV. MULTIPLE ATTRIBUTE DECISION MAKING UNDER HESITANT FUZZY ENVIRONMENT

In this section, we apply the proposed operators to multiple attribute decision making under hesitant fuzzy environment.

Example 4. Let us consider a factory which intends to select a new site for new buildings. Three alternatives $A_i (i = 1, 2, 3)$ are available, and the decision makers consider three criteria to decide which site to choose:

G_1 (income), G_2 (location), G_3 (environment). The weight vector w of the criteria $G_j (j = 1, 2, 3)$ is unknown. Assume that the characteristics of the alternatives $A_i (i = 1, 2, 3)$ with respect to the criteria $G_j (j = 1, 2, 3)$ are represented by HFNs $h_{ij} = \cup_{\gamma_{ij} \in h_{ij}} \{\gamma_{ij}\} (i, j = 1, 2, 3)$, where γ_{ij} indicates the degree that the alternative A_i satisfies the criterion G_j . All $h_{ij} (i, j = 1, 2, 3)$ are contained in the hesitant fuzzy decision matrix $H = (h_{ij})_{3 \times 3}$ (see Table I). First, we use (6)

TABLE I
 THE HESITANT FUZZY DECISION MATRIX.

	G_1	G_2	G_3
A_1	{0.6, 0.7, 0.8}	{0.25}	{0.4, 0.5}
A_2	{0.4}	{0.4, 0.5}	{0.3, 0.55, 0.6}
A_3	{0.2, 0.4}	{0.5, 0.6}	{0.5, 0.7}

to compute the weights for different alternatives. Then, we utilize the HFDWA operator (7) and the HFDWG operator (9) to aggregate all the performance values $h_{ij} (j = 1, 2, 3)$ of the i th line and get the overall performance value h_i corresponding to the alternative A_i . Then we calculate the scores of all the alternatives according to $h_i (i = 1, 2, 3)$. For HFDWA operator, we have:

$$s(h_1) = 0.4889, s(h_2) = 0.4466, s(h_3) = 0.5223.$$

And, for HFDWG operator, we have:

$$s(h_1) = 0.4253, s(h_2) = 0.4366, s(h_3) = 0.4796.$$

Since $s(h_3) > s(h_1) > s(h_2)$ for the HFDWA operator, then, by Definition 3, we get the ranking of the HFNs:

$$h_3 > h_1 > h_2,$$

and similarly, for the HFDWG operator,

$$h_3 > h_2 > h_1,$$

and thus, A_1 is the best alternative for all the two operators.

V. CONCLUSION

In this paper, to aggregated the hesitant fuzzy numbers, we have proposed three methods to generate the aggregating weights, and developed some hesitant fuzzy dependent weighted averaging(HFDWA) operators and hesitant fuzzy dependent weighted geometric(HFDWG) operators, which can relieve the influence of unfair HFNs on the aggregated results by assigning low weights to those "false" and "biased" ones. At the end, a practical numerical example has been presented to show the developed operators.

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