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Effect of Delay on Supply Side on Market Behavior: A System Dynamic Approach

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Abstract**—**Dynamic systems, which in mathematical point of view are those governed by differential equations, are much more difficult to study and to predict their behavior in comparison with static systems which are governed by algebraic equations. Economical systems such as market are among complicated dynamic systems. This paper tries to adopt a very simple mathematical model for market and to study effect of supply and demand function on behavior of the market while the supply side experiences a lag due to production restrictions.

Keywords**—**Dynamic System, Lag on Supply Demand, Market Stability, Supply Demand Model.

I. Introduction

NOWADAYS, with the developments of economies and its capabilities for being overlapped by many other fields of science like mathematics and probabilities; financial modeling and controlling economical systems are increasingly recognized. Recent incidents have gained the attention of many scholars to the study of nonlinear dynamic phenomenon in economies and monetary theories, while many economists are working on making these phenomena modeled [1].

Mathematical modeling is a method for translating issues from real systems into mathematical phrases. The recognition of these phrases, itself, helps us to find technical ways for promoting our systems [2]. According to studies have been done, mathematical modeling method is a strong way for predicting and making reliable decisions in economic markets [3].

Microeconomic systems, like other known dynamic systems, have its inputs, outputs, and state variables, sometimes called internal and external parameters. Given right analyzes and designing suitable control schemes; microeconomics could be improved like any other dynamic systems [4]-[8].

Supply-demand model is a typical example of mathematical models, which describes behavior of market. In 1870, Fleeming Jenkin wrote an essay "on the graphical representation of supply and demand "in the course of "introduc[ing] the diagrammatic Method in to the English economic literature". In that essay, he published the first graph of Supply-Demand curves that included comparative statistics from a change in supply or demand and application to the labor market [9]. That model was further improved and popularized by Alfred Marshall in the 1980 textbook "Principles of Economics" [9]. After that, microeconomics stated how decisions on supply changes prices and how change in prices, in turn, affects quantity of supplied and demanded goods [10], [11].

The simplest version of such model is static model for supply-demand which considers supply and demand quantity linear functions of price as follows:

$$D_p = a - bP$$
$$S_p = -m + sP$$

in which D_P and S_P stand for the demand and supplied quantities and P represents the price. In above relations a, b, m, and s are some positive constant parameters which are to be find based on statistical study of the market. In order for economic equilibrium condition, the quantities of supply and demand are required to be equal. This results into a solution which solves our problem of finding equilibrium price as:

$$\overline{P} = \frac{a+m}{b+s}$$

Not having been made references to time; we call the problem, static. Although these kinds of models have their benefits, they give us no idea of how the actual price changes from an existing equilibrium value to a new equilibrium point as a result of any change in supply or demand. Moreover, it cannot predict how long it might take to achieve the new equilibrium condition.

This paper aims to show how a dynamic model can be used to study dynamic behavior of different markets, to sudden change in demand while there is a lag in supply side. Different markets are characterized by different values of parameters a, b, m and s appearing in static supply-demand model. To this end we first develop a dynamic model to incorporate effect of excess demand and study effect of this problem in different markets. The model is further developed to account for the effect of time lag in supply side. The model then is used to study price change in dairy industry in Iran.

This paper consists of four sections. After this introduction a discreet time dynamic model is presented in Section II. A comprehensive study of dynamic behavior of the system is also given in this section. Third section is devoted to study of milk industry of Iran using the presented model. Some concluding remarks in forth section bring the paper to its end.

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II. Dynamic Model

In the previous section the simplest static model for price has been described in this section we determine a discreet time dynamic model to predict effect of change in demand on the market, then, a modified version of the model, is presented which can be used to study the effect of time lag on the market.

A. Effect of Excess Demand

Let us consider a market in which there is an excess demand. In this case the static model is not good enough to predict behavior of the system and we need to consider that, the price increases in proportion to the excess demand over supply as follows:

$$P_{n+1} = P_n + \theta[D(P_n) - S(P_n)] \quad (1)$$

in which P_n donates the price of the good at the end of n-th time interval, h is ascribed to equal intervals of time and θ is taken as a measure of the speed of price response to excess demand. After using demand and supply functions and defining $\alpha \hat{=} (a+m)\theta$ and $\beta \hat{=} 1-\theta(b+s)$, (1) can be rewritten as:

$$P_{n+1} - \beta P_n = \alpha \quad (2)$$

General solution to this first order difference equation with constant right hand side has the following form:

$$P_n = k + C\beta^n \quad (3)$$

In which C and k are two constants. To obtain k we substitute (3) in (2) which yields:

$$k - \beta k = \alpha \rightarrow k = \frac{\alpha}{1-\beta} = \frac{a+m}{b+s} = \overline{P}$$

and to find C we assume initial condition to be $P(0) = P_0$, which gives:

$$P_n = \overline{P} + (P_0 - \overline{P})\beta^n$$

Letting n to take integer values 1, 2, ... we generated a *history of prices* and consequently, a history of quantities demanded and supplied at various prices.

For studying the price, it is helpful to notice that, at each instant of time, the value of P is equal to the sum of the equilibrium value and the initial disequilibrium $(P_0 - \overline{P})$, which is amplified or dampened by the factor β^n.

$|\beta| > 1$ Makes the price amplified by passage of the time, the equilibrium solution is unstable and price tends to infinity.

$|\beta| < 1$ Makes the equilibrium solution stable and the price grows to the equilibrium price.

Solution to the first order discrete time equation can also fluctuate, if β takes negative values. In such cases, which present improper oscillation, the response is dampened if $\beta > -1$, and explosive if $\beta < -1$.

It is obvious that the two parameters b and s, which are donated respectively to the slopes of the demand and supply curves, are the most influential on oscillatory behavior of price. The convergence to equilibrium is guaranteed for $-1 < \beta < 1$ which is equivalent to satisfying the $0 < b+s < 2$ condition if θ is considered to be equal to one. Recalling that bands are chosen to be positive in real situations, one may show the type of response in parameter space as shown in Fig. 1.

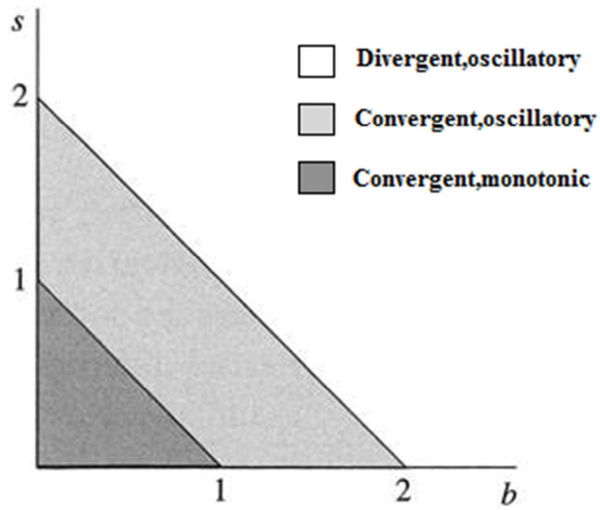

Fig. 1 Parameter space for the discrete-time partial equilibrium model

In cases where $\beta < -1$, instability is due to sort of overshooting phenomena. To understand behavior of system in such cases assume that the price is too low and there is positive excess demand, so the mechanism will make the price rises, but the change is too large. Having negative excess demand, which is caused by the new high price, a second adjustment follows, leading to another price which is far too low and so on. By these steps in the price, the discrepancy from equilibrium is getting larger and larger.

B. Effect of Lag on the Supply Side

When there is an asymmetric reaction to a price change on the part of consumer and the producer, the former adjusts demand to price changes immediately, whereas the latter needs time to adjust supply [6]. This adjustment might take time equal to several time interval of discrete time equation (1). Here for simplicity we assume this lag to be equivalent to one time interval. In this case, we may write:

$$S_n = -m + sP_{n-1} \quad (4)$$

Substituting (4) into (1) we get:

$$P_{n+1} = (1-\theta b)P_n - \theta s P_{n-1} + \theta(a+m) \tag{5}$$

Defining:

$$\tilde{z}_n = \tilde{P}_{n-1} = P_{n-1} - \frac{a+m}{b+s}$$

We may rewrite (5) in state-space form as follows:

$$\tilde{z}_{n+1} = \tilde{P}_n$$
$$\tilde{P}_{n+1} = (1-\theta b)\tilde{P}_n - \theta s \tilde{z}_n + \theta(a+m)$$

Dynamic of this system is then characterized by the eigenvalues of the constant matrix *B*.

$$B = \begin{pmatrix} 0 & 1 \\ -\theta s & 1-\theta b \end{pmatrix}$$

which are the roots of the following characteristic equation:

$$\lambda^2 + (\theta b - 1)\lambda + \theta s = 0$$

Namely,

$$\lambda_{1,2} = \frac{1}{2}[(1-\theta b) \pm \sqrt{(1-\theta b)^2 - 4\theta s}]$$

Recalling that the response is stable if $\| \lambda \| < 1$ and it is oscillatory if $real(\lambda) < 0$ we try to study the response in terms of system parameters.

Depending on the sign of discriminate ($\Delta = \sqrt{(1-\theta b)^2 - 4\theta s}$) λ might be real or complex. For $\Delta < 0$, λ is complex and $\| \lambda \| = \sqrt{\theta s}$. So in such cases, stability of the system depends on the value of θs . It means that for cases where $(1-\theta b)^2 > 4\theta s$ the response is

1) Stable where $\theta s < 1$.
2) Unstable where $\theta s > 1$.

On the other hand if $\Delta > 0$, λ is real. In this case it can be simply shown that $\lambda < 1$ for all θ, b and s which satisfy $\theta b + \theta s > 0$ and $\lambda > -1$ for all θ, b and s which satisfy $2 - \theta b + \theta s > 0$ one might obtain these conditions by solving equations $\lambda = 1$ and $\lambda = -1$ respectively. These results can be summarized as follows:

All values of *b,s,* and θ which makes $(1-\theta b)^2 < 4\theta s$ and $2 - \theta b + \theta s > 0$ makes the system stable. One should note that the condition $\theta b + \theta s > 0$ is imposes no restriction due to the fact that *b,s,* and θ are all positive in real solutions. Characteristic equation has, at least, one negative root if $\theta b > 1$ which corresponds to improper oscillations. These results are also shown in Fig. 2.

For $\sqrt{\theta s} = 1$ a special persistent condition happens in the market's price. In this case we will have groups of prices which are moving related to the frequency of the oscillation (there are two subcases which depends on this frequency).These groups can provide the market with periodic or unperiodic values of prices.

These regions are shown in Fig. 2 for the case of $\theta = 1$.

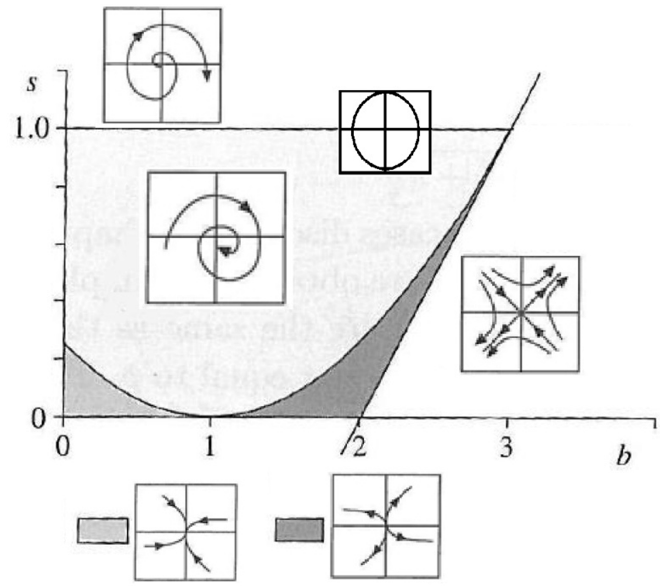

Fig. 2 Parameter space for the partial equilibrium model with a lag

III. Effect of Model Parameters on Dynamic Behavior

To examine the effect of parameters of price mathematical model on dynamic behavior of market, a real example is considered in this section. Then effect of variation in b, s, and θ, which corresponds to different economic conditions are investigated. Here we study price of milk in Iran. The market is considered as such that both supply and demand sides are acting uniformly, which means that we have single supplier and the behavior of consumers are also uniform, The price at equilibrium point is 20000Rls and average consumption of each person at this price is 0.7kgs/day. The amount of parameters of price equations at such conditions is:

$$a = 2.41089, b = 0.00008, s = 0.00013, m = 1.78897$$

This condition is depicted in Fig. 3.

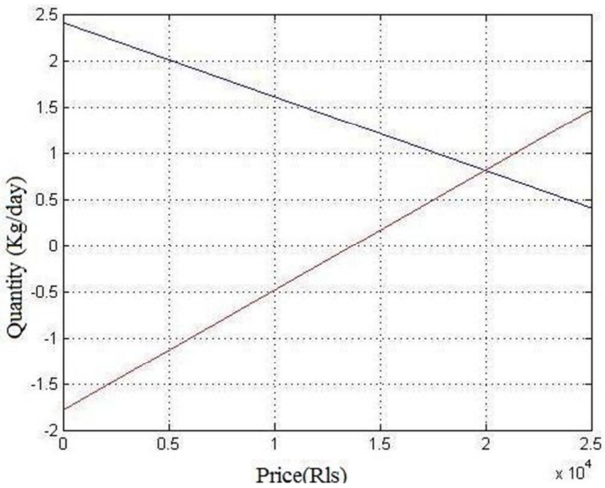

Fig. 3 Supply and Demand functions; the first case

It is also known that θ is equal to 4762 Rls/kgs. According to these parameters, the difference equation governing the market is of the following form.

$$P_{n+2} = 0.61905P_{n+1} - 0.61905P_n + 24761.23798$$

Now let us assume that demand experiences a sudden jump of one kg per month, and consider that takes one period (one week) for supply side to adjust to the market need. Fig. 4 shows the position of this situation in *(b,s)* plane which shows that the equilibrium point is stable focus.

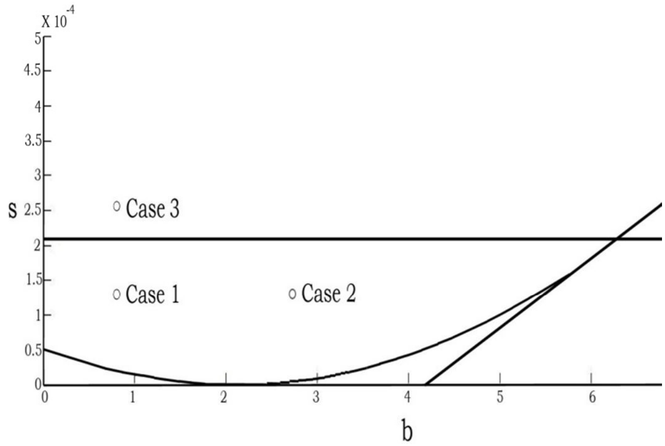

Fig. 4 Parameter space for the partial equilibrium model with a lag, regarded to our cases being studied

Time history of the market to such sudden jump is given in Fig. 5 and the phase plane trajectory of response is shown in Fig. 6.

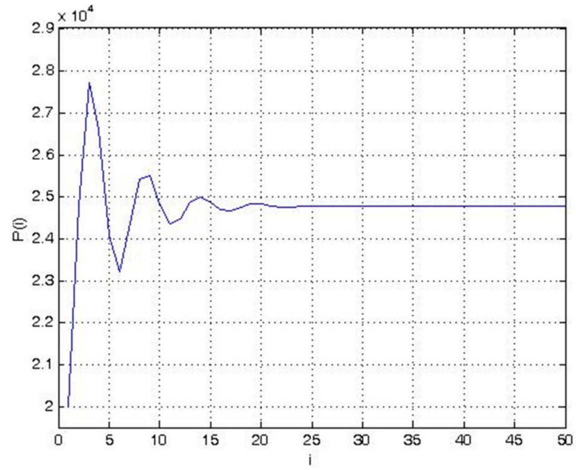

Fig. 5 Time History of the market to a sudden jump; the first case

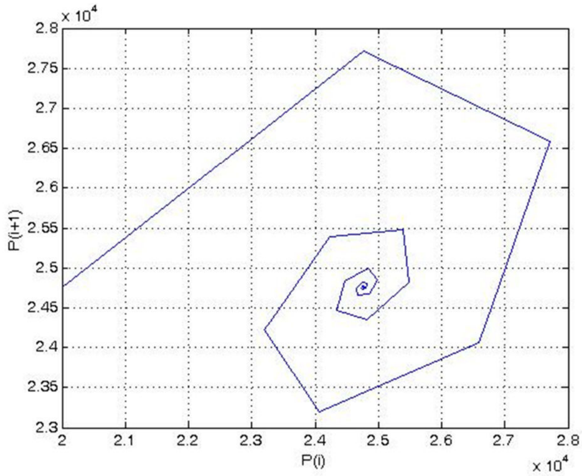

Fig. 6 Phase plane trajectory of response, the first case

Fig. 5 shows that it takes almost 4periods (almost 20weeks) before market gets stable at new equilibrium price of 24800Rls/kg. It is also worth mentioning that during this time price experiences a flip-flop behavior around the equilibrium with a period of 5.5 weeks. Also it can be predicted that as a result of the initial jump in demand, price jumps to maximum of 27500Rls/kg in 3weeks and it decreases to a minimum value of 23200Rls/kg after 6 weeks.

The second case which is studied here shows the effect of change in elasticity; i.e. the value of parameter *b*. Let us assume that in this case elasticity is increased by a factor of 240%to $b = 2.7276 \times 10^{-4}$. Such value is in accordance with a market in which consumption need is far from saturated. Such a demand behavior is what we expect to see in poor countries in which people consume 6kg/month of milk. Fig. 7 shows supply and demand functions for such cases.

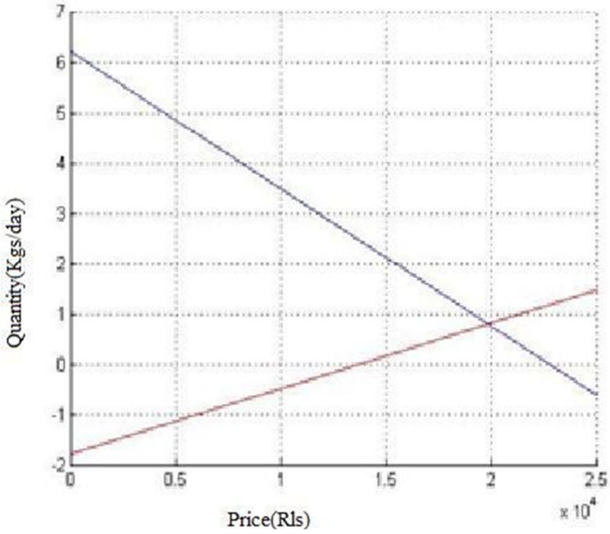

Fig. 7 Supply and Demand functions; the second case

Position of this situation in *(b-s)* plane is depicted in Fig. 4 which predicts that equilibrium point is focus in such case. Fig. 8 shows the response of market to a sudden jump on demand side.

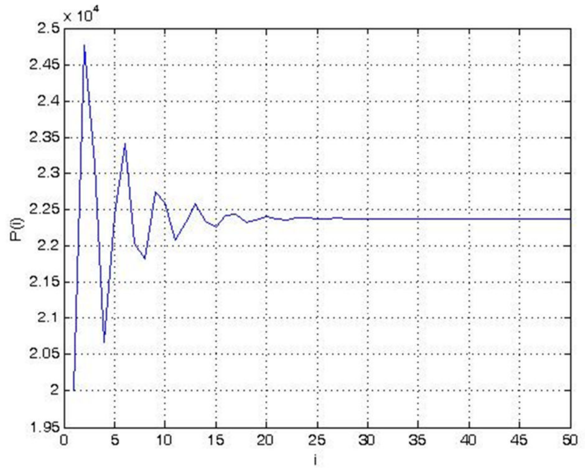

Fig. 8 Time history of response of market to a sudden jump on demand side; the second case

In this case parameters of supply and demand functions are:

$$a = 6.222, b = 2.7276 \times 10^{-4}, s = 0.00013, m = 1.78897$$

and the second order difference equation governing the market is:

$$P_{n+2} = -0.2988 P_{n+1} - 0.61904 P_n + 42909.38039$$

Comparison of Fig. 8 with5 reveals that a 240% increase in the value of b caused the period of fluctuation to be decreased by 42.8% to 4 weeks and a decrease of 10% of maximum overshoot of price to 24750Rls, while leaving the stabling time almost unaltered.

In the third case we consider that slop of supply function is raised to the value of $s = 2.5556 \times 10^{-4}$. This can show a market in which profit margin is very low and a reduction in price may have drastic effect on supply. Such a case might happen when the price of a product is damped by external factors such as political decisions, while expenses of producers are increased and has reduced profit margin.

Position of this case in *(b-s)* plane is shown in Fig. 4 which shows that equilibrium point is an unstable node. Fig. 9 shows supply and demand functions around equilibrium point, and Fig. 10 shows time history of response of market to a sudden jump on demand side.

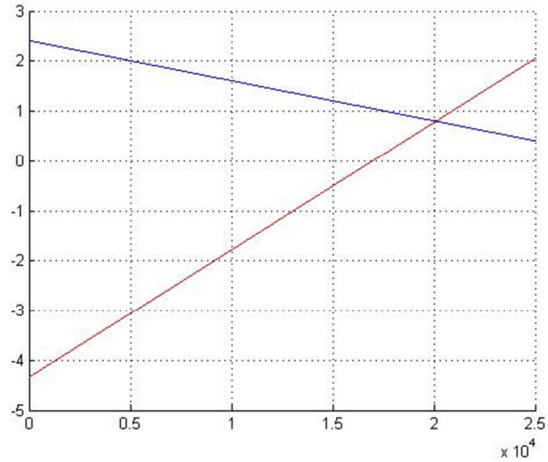

Fig. 9Supply and Demand functions; the third case

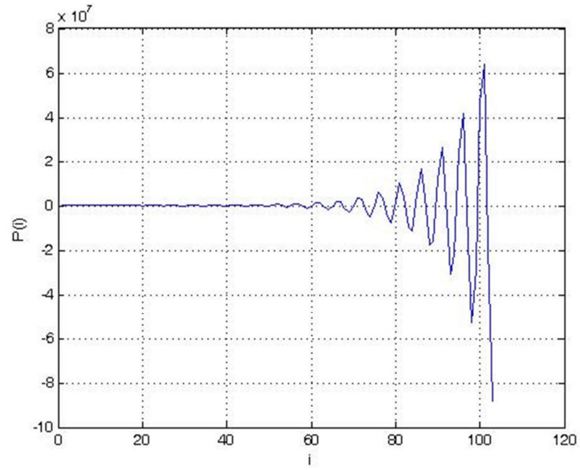

Fig. 10Time history of response of market to a sudden jump on demand side; the third case

The values of parameters of supply and demand function in this case are:

$$a = 2.41089, b = 0.00008, s = 2.5556 \times 10^{-4}, m = 4.34466.222$$

And the second order difference equation governing the market is:

$$P_{n+2} = 0.61904 P_{n+1} - 1.2169 P_n + 36930.90459$$

IV. Conclusion

In this paper, Dynamic behavior of a typical market with a lag in supply response to market is studied. To this end a mathematical model was developed and the quality of response in terms of parameters of supply-demand functions was presented. The model, then, was used to study the milk market in Iran. Effect of changes in elasticity and profit margin on the behavior of system was also studied. It was shown that decreasing profit margin beyond a certain value may result in instability of the market.

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