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Nonlinear Controller Design for Active Front Steering System

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Abstract— Active Front Steering system (AFS) provides an electronically controlled superposition of an angle to the steering wheel angle. This additional degree of freedom enables a continuous and driving-situation dependent on adaptation of the steering characteristics. In an active steering system, there needs be no fixed relationship between the steering wheel and the angle of the road wheels. Not only can the effective steering ratio be varied with speed, for example, but also the road wheel angles can be controlled by a combination of driver and computer inputs. Features like steering comfort, effort and steering dynamics are optimized and stabilizing steering interventions can be performed. In contrast to the conventional stability control, the yaw rate was fed back to AFS controller and the stability performance was optimized with Sliding Mode control (SMC) method. In addition, tire uncertainties have been taken into account in SM controller to provide the control robustness. In this paper, 3-DOF nonlinear model is used to design the AFS controller and 8-DOF nonlinear model is used to model the controlled vehicle.

Keywords— Active Front Steering (AFS), Sliding Mode Control method (SMC), Yaw rate, Vehicle Stability, Robustness

I. Introduction

ACTIVE Front Steering (AFS) systems have been introduced to improve handling stability under adverse road conditions. The law in some countries requires that there must be a direct mechanical connection between the driver's steering wheel and the road wheels. This would seem to make an active steering drive-by-wire system impossible. This is not quite the case, however. If a differential element is inserted in the steering column, then the steer angle of the road wheels can be the sum of two angular inputs. One can be the steer angle commanded by the driver through the steering wheel and the other can be the angle from a rotary actuator, such as an electric motor and gear train, commanded by a computer. In an active steering system, there needs be no fixed relationship between the steering wheel angle and the angle of the road wheels. This means that the response of the steering wheel inputs can be varied and an automobile can be made to respond to steering inputs much as a reference vehicle would. [1, 2]

At slow speed, the steering is made more responsive so less motion of the steering wheel is required for parking but at high speeds the steering ratio is made slower so that the driver does not feel that the car reacts too violently.

Huh and **Kim** (2001) devise an active steering controller that eliminates the difference in steering response between driving on slippery roads and dry roads. The controller is based on feedback of lateral tire force estimates derived from vehicle roll motion. [3]

Segawa et al (2002) apply lateral acceleration and yaw rate feedback to a steer-by-wire vehicle and demonstrate that active steering control can achieve greater driving stability than differential brake control. [4]

W.Klier et al (2003) proposed the concept and functionality of the active front steering system. They focused on a modular system concept and its respective advantages and requirements. [5]

D.Chen et al (2008) proposed the controller for an active front steering system. In this research the stability performance has been optimized with LQR (Linear Quadratic Regulator). HILS (Hardware-In-the-Loop Simulation) tests were conducted to demonstrate the performance of the designed AFS controller. [1]

This paper discusses a yaw stability control algorithm. The controller is designed based on the sliding mode method to improve the vehicle stability and maneuverability.

II. Vehicle Modeling

In this paper three-degree-freedom (3-DOF) model will be utilized for the AFS controller design and eight-degree-freedom (8-DOF) model will be used as a controlled plant of the vehicle for control system evaluations through computer simulations.

A. 8-DOF Vehicle Model

A nonlinear vehicle handling model which is used for simulation purpose is developed for this study. The vehicle is modeled based on this model to be controlled by the AFS controller. This model consists of 8-DOF which include three planar motions of the vehicle plus body roll motion relative to the chassis about the roll axis and the rotational dynamics of four wheels. Fig.1 shows the vehicle model with coordinate system, degrees of freedom and external forces. The equations of motion for the model are given as:

Longitudinal motion:

$$m(\dot{V}_x - rV_y) - m_s h(\varphi\dot{r} - 2\dot{r}\varphi) = \sum F_x \qquad (1)$$

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Lateral motion:

$$m(\dot{V}_y + V_x r) + m_s h(\ddot{\varphi} - r^2\varphi) = \sum F_y \tag{2}$$

Yaw motion:

$$I_{zz}\dot{r} + (I_{zz}\gamma - I_{xz}\ddot{\varphi}) - m_s h(\dot{V}_x - rV_y)\varphi = \sum M_z \tag{3}$$

Roll motion:

$$(I_{xx} + mh^2)\ddot{\varphi} + m_s h(\dot{V}_y + V_x r) + (I_{zz}\gamma - I_{xz})\dot{r} - (mh^2 + I_{yy} - I_{zz})r^2\varphi = \sum M_x \tag{4}$$

Wheel motion:

$$I_w\dot{\omega}_i = -R_w F_{xwi} + T_i \tag{5}$$

Where:

$$\sum F_x = F_{x1} + F_{x2} + F_{x3} + F_{x4} \tag{6}$$

$$\sum F_y = F_{y1} + F_{y2} + F_{y3} + F_{y4} \tag{7}$$

$$\sum M_z = l_f(F_{y1} + F_{y2}) - l_r(F_{y3} + F_{y4}) + M_{zc} \tag{8}$$

$$\sum M_x = [m_s gh - (K_{\varphi f} + K_{\varphi r})]\varphi - (C_{\varphi f} + C_{\varphi r})\dot{\varphi} \tag{9}$$

$$m = m_s + m_{uf} + m_{ur} \tag{10}$$

$$M_{zc} = \frac{t}{2}[(F_{x1} + F_{x3}) - (F_{x2} + F_{x4})] \tag{11}$$

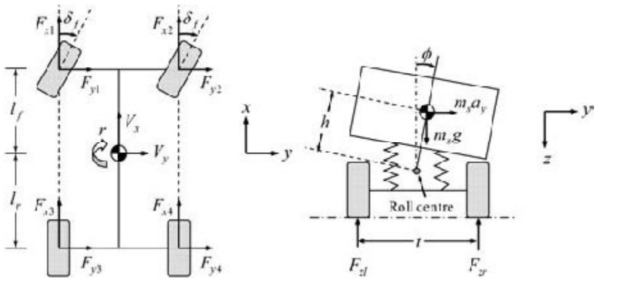

Fig.1 8-DOF vehicle model [7]

In the above equations, the resultant longitudinal and lateral forces acting on the ***i***th wheel in the vehicle fixed coordinate system, $\mathbf{F_{xi}}$ and $\mathbf{F_{yi}}$, have the following relationships with the tire forces along the wheel axes, $\mathbf{F_{xwi}}$ and $\mathbf{F_{ywi}}$, as shown in Fig.2.

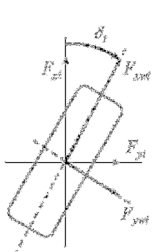

Fig.2 Wheel definition [7]

$$\begin{Bmatrix} F_{xi} \\ F_{yi} \end{Bmatrix} = \begin{bmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{bmatrix} \begin{Bmatrix} F_{xwi} \\ F_{ywi} \end{Bmatrix} \quad (i = 1, \dots, 4) \tag{12}$$

Because of the suspension system is not considered in this modeling and normal tire forces have an effect on the longitudinal, lateral forces and the self-aligning torque, the normal forces should be modeled as following equations. According to the quasi-static longitudinal and lateral load transfers, the instantaneous vertical tire load acting on each wheel $\mathbf{F_{zi}}$ during dynamic maneuvers is the sum of the static tire load plus transfer that is due to longitudinal acceleration, lateral acceleration, and body roll motion respectively. This effect can be described as:

$$F_{z1} = \frac{mgl_r}{2l} - \frac{ma_x h_{cg}}{2l} + \frac{a_y}{t}\left(\frac{m_s l_{rs} h_f}{l} + m_{uf}h_{uf}\right) + \frac{1}{t}\left(-K_{\varphi f}\varphi - C_{\varphi f}\dot{\varphi}\right) \tag{13}$$

$$F_{z2} = \frac{mgl_r}{2l} - \frac{ma_x h_{cg}}{2l} - \frac{a_y}{t}\left(\frac{m_s l_{rs} h_f}{l} + m_{uf}h_{uf}\right) - \frac{1}{t}\left(-K_{\varphi f}\varphi - C_{\varphi f}\dot{\varphi}\right) \tag{14}$$

$$F_{z3} = \frac{mgl_f}{2l} + \frac{ma_x h_{cg}}{2l} + \frac{a_y}{t}\left(\frac{m_s l_{fs} h_r}{l} + m_{ur}h_{ur}\right) + \frac{1}{t}\left(-K_{\varphi r}\varphi - C_{\varphi r}\dot{\varphi}\right) \tag{15}$$

$$F_{z4} = \frac{mgl_f}{2l} + \frac{ma_x h_{cg}}{2l} - \frac{a_y}{t}\left(\frac{m_s l_{fs} h_r}{l} + m_{ur}h_{ur}\right) - \frac{1}{t}\left(-K_{\varphi r}\varphi - C_{\varphi r}\dot{\varphi}\right) \tag{16}$$

B. 3-DOF Vehicle Model

The three degrees of freedom (3-DOF) model, which is a good representation of the lateral vehicle dynamics in the nonlinear handling region, is employed for AFS controller design. The states in this model are Lateral motion, yaw motion and roll motion. This model can be described by the following state equations with small wheel angle and constant forward speed assumptions:

$$\dot{V}_y = -rV_x - h\ddot{\varphi} + hr^2\varphi + \frac{1}{m}(F_{yf} + F_{yr}) \tag{17}$$

$$\dot{r} = \frac{1}{I_{zz}}\left[(I_{zz}\gamma - I_{xz})\ddot{\phi} + mh(-rV_y)\phi + F_{yf}L_f - F_{yr}L_r\right] \tag{18}$$

$$\ddot{\varphi} = \frac{1}{(I_{xx} + mh^2)}\left[-mh(\dot{V}_y + rV_x)(I_{zz}\gamma - I_{xz})\dot{r} + (mh^2 + I_{yy} - I_{zz})r^2\varphi - (C_{\varphi f} + C_{\varphi r})\dot{\varphi} - (K_{\varphi f} + K_{\varphi r} - mgh)\varphi\right] \tag{19}$$

In this model F_{yf} and F_{yr} is computed based on the linear tire model.

III. Tire Modeling

In order to simulate the limit handling situations where strong non-linearity is present, the nonlinear ' PACEJKA' tire model [6] with combined longitudinal and lateral slip is employed the tire forces can be illustratively express as:

$$\{F_{xwi}, F_{ywi}\} = f(\lambda_i, \alpha_i, F_{zi}) \tag{20}$$

In recent years, an empirical method for characterizing tire behavior known as the Magic Formula has been developed and used in vehicle handling simulations. The Magic Formula, in its basic form, can be used to fit experimental tire data for characterizing the relationships between the cornering force and slip angle, self-aligning torque and slip angle, or braking effort and skid. It is expressed by:

$$y(x) = D\ \sin\{C\ \tan^{-1}[Bx - E\ (Bx - \tan^{-1} Bx)]\} \tag{21}$$

$$Y(X) = y(x) + S_v$$
$$x = X + S_h$$

Where **Y(X)** represents *cornering force, self-aligning torque, or braking effort* and **X** denotes *slip angle or skid.* Coefficient **B** is called *stiffness factor*, **C** *shape factor,* **D** *peak factor,* and **E** *curvature factor* .$\mathbf{S_h}$ and $\mathbf{S_v}$ are the *horizontal shift and vertical shift*, respectively. (For further information refer to Ref [6])

The linear tire model equation is used to design the AFS controller but the 'PACEJKA' tire model is used to model the controlled plant of the vehicle.

$$F_{yi} = 2C_{\alpha i}\alpha_i \tag{22}$$

IV. AFS Controller Design

In this paper the AFS controller is designed based on the Sliding Mode Control (SMC) method to improve vehicle steerability by tracking the reference yaw rate. The reference model of this controller is based on the 3-DOF vehicle model. Model imprecision may come from actual uncertainty about the plant *(e.g.,* unknown plant parameters), or from the purposeful choice of a *simplified* representation of the system's dynamics *(e.g.,* modeling friction as linear, or neglecting structural modes in a reasonably rigid mechanical system). From a control point of view, modeling inaccuracies can be classified into two major kinds [8]:

- *Structured* (or *parametric)* uncertainties
- *Unstructured* uncertainties (or *un-modeled dynamics)*

It is believed that drivers intend to control the yaw rate when the vehicle travels around a corner; therefore the reference model indeed reflects the desired relationship between the driver steer inputs and the vehicle yaw rate. The yaw rate generated by the reference model is therefore chosen as the reference signal to be tracked by the active front steering controller. Consequently, the AFS controller is designed to force the vehicle to follow the reference yaw rate through driving the tracking error between the actual and desired yaw to zero. In this way, they make contributions to steerability improvement by assisting the driver in steering the vehicle and helping the driver to avoid extreme handling situations. The AFS acts as a steering correction system by applying an additional steer angle to that demanded by the driver.

$$\delta = \delta_c + \delta_f \tag{23}$$

The driver's input is:

$$\delta_f = \frac{\delta_{sd}}{OSR} \tag{24}$$

The ***OSR*** term is Overall Steering Ratio that is 17.4 in a conventional vehicle. Now by following equations, the AFS controller is designed based on the Sliding Mode Control method.

$$e = r - r_d \qquad the\ error \tag{25}$$

$$\dot{e} = \dot{r} - \dot{r}_d \tag{26}$$

The following sliding surface and sliding reachability condition are selected.

$$s = e \tag{27}$$

$$\dot{s} = -\lambda s \ \rightarrow\ \dot{e} = -\lambda e \quad \rightarrow \dot{e} + \lambda e = 0 \tag{28}$$

Now, the 3-DOF vehicle model equations (17-19) and equations (27 & 28) are used to derive the sliding control low:

$$\xrightarrow{eq(18),(26)} \dot{e} = \dot{r} - \dot{r}_d = \frac{1}{I_{zz}}\ [(I_{zz}\gamma - I_{xz})\ddot{\emptyset} + mh(-rV_y)\emptyset + F_{yf}L_f - F_{yr}L_r] - \dot{r}_d \tag{29}$$

$$\xrightarrow{eq(25),(28),(29)} \frac{1}{I_{zz}}\ \left[(I_{xz} - I_{zz}\gamma)\ddot{\emptyset} + mh(-rV_y)\emptyset + 2L_fC_{\alpha f}\delta_{eq} - \left(\frac{2L_fC_{\alpha f} - 2L_rC_{\alpha r}}{V_x}\right)V_y - \left(\frac{2{L_f}^2C_{\alpha f} + 2{L_r}^2C_{\alpha r}}{V_x}\right)r\right] - \dot{r}_d + \lambda(r - r_d) = 0 \tag{30}$$

$$\delta_{eq} = \frac{1}{2l_fC_{\alpha f}}\left(\dot{r}_d - \frac{1}{I_{zz}}\ \left[(I_{xz} - I_{zz}\gamma)\ddot{\emptyset} + mh(-rV_y)\emptyset - \left(\frac{2L_fC_{\alpha f} - 2L_rC_{\alpha r}}{V_x}\right)V_y - \left(\frac{2{L_f}^2C_{\alpha f} + 2{L_r}^2C_{\alpha r}}{V_x}\right)r\right] + \lambda(r_d - r)\right) \tag{31}$$

$$\delta = \delta_{eq} - k * sgn(s) \rightarrow$$
$$\delta = \delta_{eq} - k * sgn(r - r_d) \tag{32}$$

Where λ and k are positive parameters to be tuned in controller design and sgn() is the sign function.

However, the presence of the discontinuous term in equation (32) may cause chattering, which involves extremely high control effort and may also excite high-frequency unmodeled dynamics [8]. In order to eliminate this effect, the sign function in equation (32) is replaced by the saturation function, $sat(s/\emptyset)$, which is used to approximate a continuous control within a boundary layer around the sliding surface. The saturation function $sat(s/\emptyset)$ is defined as:

$$sat(s/\emptyset) = \begin{cases} s/\emptyset & if\ |s| \le \emptyset \\ sgn(s/\emptyset) & if\ |s| > \emptyset \end{cases} \qquad (33)$$

Thus the continuous approximation of the control law in equation (32) is given as:

$$\delta = \delta_{eq} - k * sat\left(\frac{r - r_d}{\phi}\right) \qquad (34)$$

Where $\emptyset$ is the boundary layer thickness.
Where $\mathbf{r_d}$ is desired yaw rate in under-steer condition.

$$r_d = \frac{V_x \delta_f}{L\,(1 + K_u V_x^2)} \qquad (35)$$

Where $\mathbf{L = L_f + L_r}$ and $\mathbf{K_u}$ is the under-steer coefficient and calculated by following equation:

$$K_u = \frac{m\,(l_r C_{\alpha r} - l_f C_{\alpha f})}{2 C_{\alpha f} C_{\alpha r} L^2} \qquad (36)$$

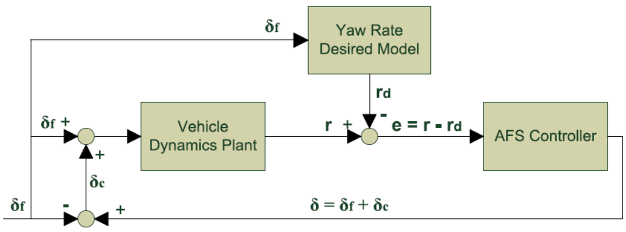

Fig.3 AFS controller block diagram

V. Results

In the processes of design, development and improvement of the vehicle, first the vehicle is evaluated with the simulation software on the computer before the designed system or subsystem is evaluated on a real vehicle and proving ground. MATLAB software is used for the simulation.

To evaluate the performance of the AFS controller, the slalom maneuver is used (Fig.4). In this maneuver the sinusoidal torque is exerted on the steering wheel by the driver. The friction coefficient between the tire and the road surface is 0.8, therefore the vehicle is moving on a dry road. The initial speed of the vehicle is about 80 Km/h. This maneuver is used to evaluate the speed of the performance and the response of the controlling system when it encounters disturbances.

Fig.6 shows the angle of the front wheels where δ_f is the conventional angle and $\delta_f + \delta_c$ is the angle, which is corrected by the AFS controller. Fig.7 shows the corrective angle, which is added to the driver's input by the AFS controller.

The deviation of the conventional vehicle from the desired track of the vehicle is larger than that of the controlled vehicle as shown in Fig.8.

As shown in Fig.9, the uncontrolled vehicle behaves badly with respect to the deriver's input while the controlled vehicle covers the desired yaw rate properly. Fig.10 demonstrates the high capability of the controller to control the lateral speed of the vehicle.

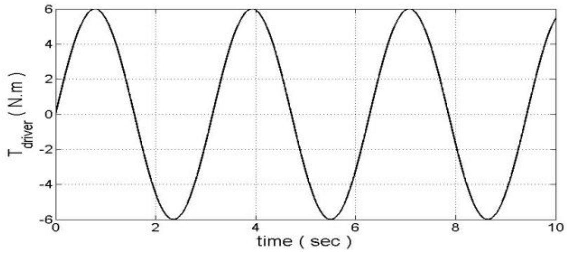

Fig.4 Driver's torque

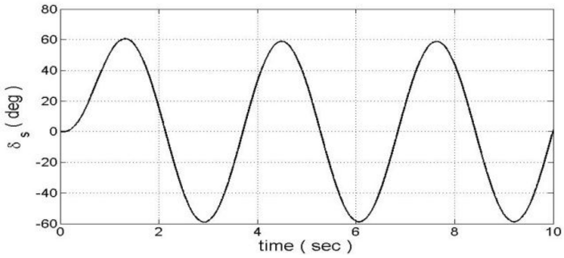

Fig.5 Steering wheel angle

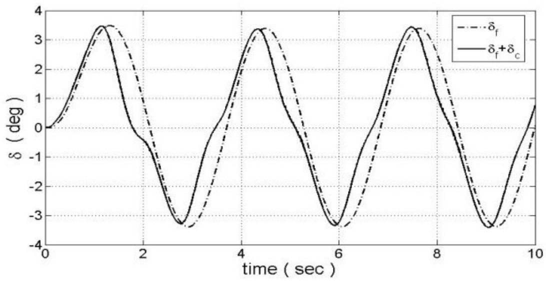

Fig.6 Front wheel angle

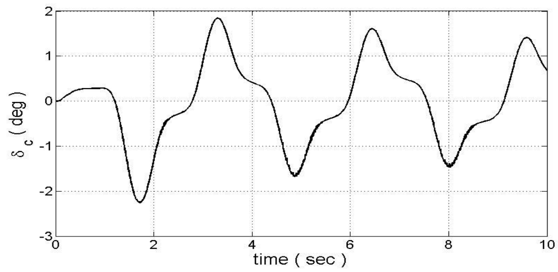

Fig.7 AFS corrective angle

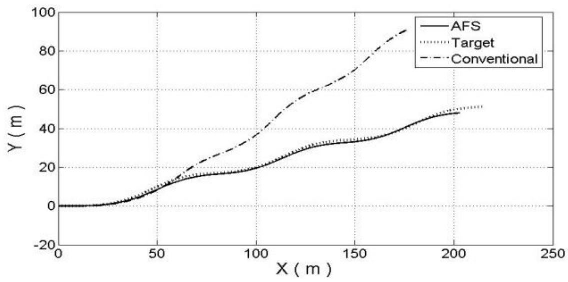

Fig.8 Vehicle track

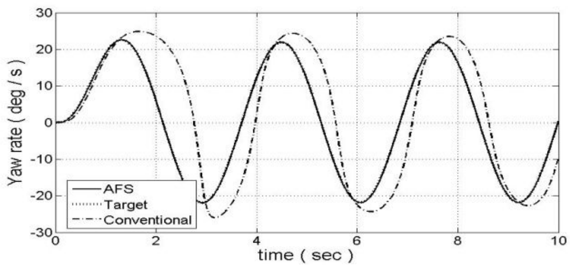

Fig.9 Vehicle yaw rate

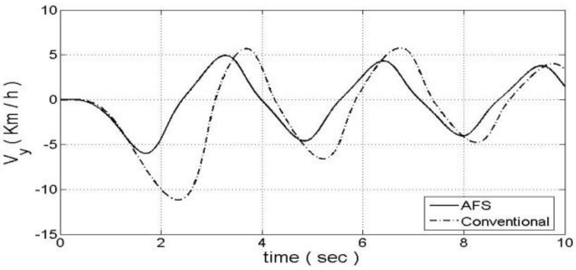

Fig.10 Lateral speed of the vehicle's center of gravity

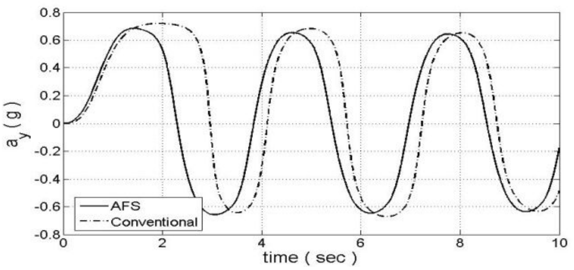

Fig.11 Lateral acceleration of the vehicle

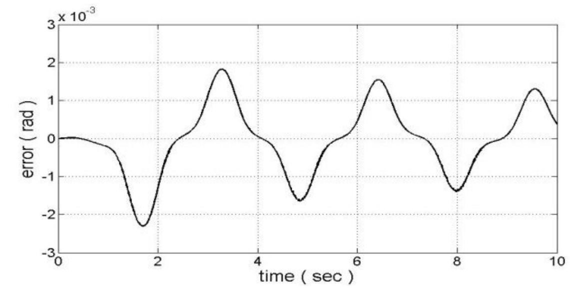

Fig.12 AFS controller error

VI. Conclusion

In this paper, a new method for the vehicle dynamics control was described. For this reason, the sliding mode controller has been used to design the active front steering controller.

- The 8-DOF model has been provided to simulate the vehicle and the assessment of the function of vehicle stability control systems.
- The PACEJKA tire model with combined longitudinal and lateral slip has been used to model the tire's nonlinear characteristics.
- The yaw stability controller as the Active Front Steering System (AFS) has been designed based on the sliding mode control method and 3-DOF nonlinear model. This controller corrects the angle of the front wheels to control the yaw rate of the vehicle. Therefore this controller improves the stability and maneuverability of the vehicle on dry roads, wet roads and snowy roads.

At the end, in order for the present research to be more complete and practical, the following future works are proposed:

- The evaluation of the differential braking system and anti-lock brake system's performance when the active front steering system is used in the vehicle.
- The simulation of the driver and the evaluation of the driver's role in the dynamics behavior of the vehicle which is equipped with these controllers.
- The usage of the other methods to design the controllers and compare these systems to other controlling systems.
- Design estimators and observers to estimate the mass, moment of inertia of the vehicle and the longitudinal and lateral forces of the tire, and evaluate the effects of these elements on the performance of the controllers.

Appendix

TABLE I
VALUE OF PARAMETERS [7]

Variable name	Variable magnitude	Variable unit
m	1300	Kg
m_s	1095.7	Kg
m_{uf}	95.5	Kg
m_{ur}	108.8	Kg
L_{fs}	1.2227	m
L_{rs}	1.4393	m
L_f	1.2247	m
L_r	1.4373	m
t	1.4376	m
h_{cg}	0.5253	m
h_{uf}	0.313	m
h_{ur}	0.313	m
h	0.445	m
h_f	0.130	m
h_r	0.110	m
I_{xx}	346.7	Kg-m^2
I_{zz}	1808.8	Kg-m^2
I_{xz}	21.09	Kg-m^2
I_w	2.11	Kg-m^2
R_w	0.285	m
$K_{\varphi f}$	66175	N-m/rad
$K_{\varphi r}$	66175	N-m/rad
$C_{\varphi f}$	3511	N-m-s/rad
$C_{\varphi r}$	3511	N-m-s/rad
g	9.81	m/s^2
γ	0.854	deg
$C_{\alpha f}$	60000	N/rad
$C_{\alpha r}$	60000	N/rad

TABLE 2
DEFINITION OF PARAMETERS [7]

Notation	Description	unit
$C_{\alpha f}$, $C_{\alpha r}$	Cornering Stiffness of Front , rear axle	N/rad
$C_{\varphi f}$, $C_{\varphi r}$	Front , rear suspension roll damping	Nm/rad s
h	Distance from sprung mass centre of gravity (CG) to the roll axis	m
h_{cg}	Height of vehicle CG	m
h_f , h_r	Height of front , rear roll centre	m
h_{uf} , h_{ur}	Height of front , rear unsprung mass CG	m
I_w	Wheel moment of inertia about the spin axis	Kg m^2
I_{xx}	Sprung mass moment of inertia about the roll axis	Kg m^2
I_{xz}	Sprung mass product of inertia about the roll and yaw axes	Kg m^2
I_{zz}	Vehicle moment of inertia about the z axis	Kg m^2
$K_{\varphi f}$, $K_{\varphi r}$	Front , rear suspension roll stiffness	N m /rad
L_f , L_r	Distance from the vehicle CG to the front , rear axle	m
L_{fs} , L_{rs}	Distance from the sprung mass CG to the front , rear axle	m
m_{uf} , m_{ur}	Front , rear un-sprung mass	Kg
m , m_s	Total mass , sprung mass of the vehicle	Kg
t	Wheel track	m
R_w	Effective wheel rolling Radius	m
γ	Inclined angle between roll axis and x axis	deg
V_x , V_y	Longitudinal , lateral speed of the vehicle's center of gravity	m/s
a_x , a_y	Longitudinal , lateral acceleration of the vehicle's body	m/s^2
φ	roll angle	rad
r	Yaw rate	rad/s
λ_i	Longitudinal slip	-
α	Lateral slip	rad
F_x , F_y , F_z	Longitudinal , Lateral , Normal force of tire	N

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