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New Approaches on Exponential Stability Analysis for Neural Networks with Time-Varying Delays

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Abstract—In this paper, utilizing the Lyapunov functional method and combining linear matrix inequality (LMI) techniques and integral inequality approach (IIA) to study the exponential stability problem for neural networks with discrete and distributed time-varying delays.By constructing new Lyapunov-Krasovskii functional and dividing the discrete delay interval into multiple segments,some new delay-dependent exponential stability criteria are established in terms of LMIs and can be easily checked.In order to show the stability condition in this paper gives much less conservative results than those in the literature,numerical examples are considered.

Keywords—Neural networks,Exponential stability, LMI approach, Time-varying delays.

I. INTRODUCTION

NEURAL networks have attracted many researchers attention during the past decades and have found successful applications in many areas,such as automatic control,signal processing,model identification,combinatorial optimization,and so on[1,2]. However,the occurrence of time delays is unavoidable in some of these applications,and it may cause instability of neural networks.Therefore,stability analysis of delayed neural networks has been extensively investigated by many researchers. Now, many sufficient conditions ensuring global asymptotic stability and global exponential stability for delayed neural networks have been derived [3-30].The main concern in delayed-dependent stability analysis for delayed neural networks is to enlarge the feasibility region of stability criteria to get the maximum allowable bound of time delays for guaranteed the stability. some researchers found many new approaches on stability analysis for neural networks with time-varying delay, such as introducing new Lyapunov functional,dividing delay interval and so on.

Motivated by this mentioned above,in this paper, the exponential stability problem for neural networks with both time-varying and distributed delays is considered,two new delay-dependent stability criterion for neural networks with time-varying delays will be proposed by dividing the delay interval $[0,\varsigma]$ into $[0,\frac{\varsigma(t)}{2}]$,$[\frac{\varsigma(t)}{2},\varsigma(t)]$,$[\varsigma(t),\frac{\varsigma+\varsigma(t)}{2}]$,$[\frac{\varsigma+\varsigma(t)}{2},\varsigma]$, constructing new Lyapunov-Krasovskii functional which contains some new integrals,and introducing $f(y(t-\frac{\varsigma}{2}))$ in vector $\xi(t)$,which are rarely considered in other literature .The obtained criterion are less conservative because LMI approach has been developed to deal with the problem of globally exponential stability for neural networks with time-varying delays. Finally, numerical examples are presented to illustrate the effectiveness of our results.

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II. PROBLEM STATEMENT

Consider the following neural networks with discrete and distributed time-varying delays:

$$\dot{z}(t)=-Cz(t)+Ag(z(t))+Bg(z(t-\varsigma(t)))+D\int_{t-\rho}^{t}g(z(s))ds+I_0$$
$$z(t)=\Phi(t),t\in[-h,0] \tag{1}$$

where $z(t)=[z_1(t),z_2(t),\ldots,z_n(t)]^T\in R^n$ is the neuron state vector,$g(z(t))=[g_1(z_1(t)),g_2(z_2(t)),\ldots,g_n(z_n(t))]^T$ denotes the neuron activation function ,and $I_0=[I_1,I_2,\ldots,I_n]^T\in R^n$ is a constant input vector,$C=diag\{c_i\}\in R^n$ is a positive diagonal matrix , $A=(a_{ij})_{n\times n}\in R^n$ is the connection weight matrix,$B=(b_{ij})_{n\times n}\in R^n$,and $D=(d_{ij})_{n\times n}\in R^n$ are the delayed connection weight matrices,the initial vector $\Phi(t)$ is bounded and continuous on $[-h,0]$,where $h=\max\{\rho,\varsigma\}$.

The following assumptions are adopted throughout the paper.

Assumption 1: The delay $\varsigma(t)$ is time-varying continuous function and satisfies:

$$0\le\varsigma(t)\le\varsigma,\dot{\varsigma}(t)\le\mu\le 1 \tag{2}$$

Assumption 2: Each neuron activation function $g_i(\cdot),i=1,2,\ldots,n$,in (1) satisfies the following condition:

$$\gamma_i^-\le\frac{g_i(\alpha)-g_i(\beta)}{\alpha-\beta}\le\gamma_i^+,\forall\alpha,\beta\in R,\alpha\neq\beta \tag{3}$$

where $\gamma_i^-,\gamma_i^+,i=1,2,\ldots,n$ are constants,and matrices $\Gamma_1=diag\{\gamma_1^-,\gamma_2^-,\ldots,\gamma_n^-\},\Gamma_2=diag\{\gamma_1^+,\gamma_2^+,\ldots,\gamma_n^+\}$.

Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer‘s fixed-point theorem. Assuming that $z^*=[z_1^*,z_2^*,\ldots,z_n^*]^T$is the equilibrium point of (1) and using the transformation $y(\cdot)=z(\cdot)-z^*$,system (1) can be converted to the following system :

$$\dot{y}(t)=-Cy(t)+Af(y(t))+Bf(y(t-\varsigma(t)))+D\int_{t-\rho}^{t}f(y(s))ds \tag{4}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$, $f(y(t)) = [f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))]^T$, $f_i(y_i(\cdot)) = g_i(z_i(\cdot)+z_i^*) - g_i(z_i^*)$, $i = 1, 2, \ldots, n$.
From $Eq.(4)$,$f_i(\cdot)$ satisfies the following condition:

$$\gamma_i^- \le \frac{f_i(\alpha)}{\alpha} \le \gamma_i^+, \forall \alpha \ne 0, i = 1, 2, \ldots, n. \tag{5}$$

Due to the disturbance frequent occurs in many various applications,and ρ may be distributed time-varying delays ,so by translating matrices A, B, C, D and constant ρ to function $A(t), B(t), C(t), D(t)$ and $\rho(t)$, respectively,we have

$$\begin{aligned}\dot{y}(t) = &-C(t)y(t) + A(t)f(y(t)) + B(t)f(y(t-\varsigma(t))) \\ &+ D(t)\int_{t-\rho(t)}^{t} f(y(s))ds\end{aligned} \tag{6}$$

Assumption 3: $\rho(t)$ is the time-varying continuous function and satifies: $0 \le \rho(t) \le \rho$.
Assumption 4: Setting function $A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t), D(t) = D + \Delta D(t)$, where $\Delta A(t), \Delta B(t), \Delta C(t), \Delta D(t)$ are unknown constant matrices respresenting time-varying parametric uncertainties, and are of linear fractional forms:

$$[\Delta C(t), \Delta A(t), \Delta B(t), \Delta D(t)] = GF(t)[E_c, E_a, E_b, E_d] \tag{7}$$

with

$$F^T(t)F(t) \le I \tag{8}$$

Definition 1 The equilibrium point 0 of system (7) is said to be globally exponentially stable if there exist $k > 0$ and $\gamma > 0$ such that

$$\|y(t)\| \le \gamma e^{-kt} \sup_{-h \le s \le 0} \|y(t)\|, \forall t > 0 \tag{9}$$

Lemma 1 [9].The following inequalities are true :

$$\begin{aligned}&0 \le \int_0^{y_i(t)} (f_i(s) - \gamma_i^- s)ds \le (f_i(y_i(t)) - \gamma_i^- y_i(t))y_i(t), \\ &0 \le \int_0^{y_i(t)} (\gamma_i^+ s - f_i(s))ds \le (\gamma_i^+ y_i(t) - f_i(y_i(t)))y_i(t),\end{aligned} \tag{10}$$

Lemma 2 [10]. For any constant matrix $Q, S \in R^{n\times n}, Q = Q^T > 0, S = S^T$, the following inequality hold:

$$\begin{aligned}&-\rho \int_{t-\rho}^{t} y^T(s)Qy(s)ds \\ &\le -\begin{bmatrix}\int_{t-\rho(t)}^{t} y(s)ds \\ \int_{t-\rho}^{t-\rho(t)} y(s)ds\end{bmatrix}^T \begin{bmatrix} Q & S \\ * & Q\end{bmatrix} \begin{bmatrix}\int_{t-\rho(t)}^{t} y(s)ds \\ \int_{t-\rho}^{t-\rho(t)} y(s)ds\end{bmatrix}\end{aligned} \tag{11}$$

III. Main results

In this section,a new Lyapunov functional is constructed and two less conservative delay-dependent stability criterion are obtained. First, we take up the case where $\Delta A(t) = 0, \Delta B(t) = 0, \Delta C(t) = 0, \Delta D(t) = 0$ in system (7) as follows:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\varsigma(t))) + D\int_{t-\rho(t)}^{t} f(y(s))ds \tag{12}$$

Denote

$$\begin{aligned}\xi^T(t) = &[y^T(t), y^T(t-\frac{\varsigma}{2}), y^T(t-\varsigma), y^T(t-\varsigma(t)), f^T(y(t)), \\ &f^T(y(t-\frac{\varsigma}{2})), f^T(y(t-\varsigma)), f^T(y(t-\varsigma(t))), \\ &\int_{t-\rho(t)}^{t} f^T(y(s))ds, \int_{t-\rho}^{t-\rho(t)} f^T(y(s))ds]\end{aligned}$$

Theorem 1 Given that the Assumption 1-3 hold, the system (12) is globally exponentially stable with the exponential convergence rate index k if there exist symmetric positive definite matrices $\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ * & G_{22} & G_{23} & G_{24} \\ * & * & G_{33} & G_{34} \\ * & * & * & G_{44}\end{bmatrix}, P, Q_i, i = 1, 2, 3, 4, R_i, i = 1, 2, \ldots, 5$,positive diagonal matrices $W_1, W_2, W_3, W_4, \Lambda = diag\{\lambda_1, \lambda_2, \ldots, \lambda_n\}, \Delta = diag\{\delta_1, \delta_2, \ldots, \delta_n\}$,and symmetric matrix $S_i, i = 1, 2, \ldots, 7$ such that the following LMIs hold:

$$\begin{bmatrix} R_1 & S_i \\ * & R_2 \end{bmatrix} > 0, i = 1, 4, 5. \tag{13}$$

$$\begin{bmatrix} R_3 & S_i \\ * & R_4 \end{bmatrix} > 0, i = 2, 6, 7. \tag{14}$$

$$\begin{bmatrix} E & \aleph^T Z \\ * & -Z \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} F & \aleph^T Z \\ * & -Z \end{bmatrix} < 0 \tag{16}$$

where

$$\aleph = \begin{bmatrix} -C & 0 & 0 & 0 & A & 0 & 0 & B & D & 0 \end{bmatrix}$$

$$Z = \frac{\varsigma}{2}(R_2 + R_4)$$

$$E = \begin{bmatrix}
E_{11} & E_{12} & 0 & 0 & E_{15} & E_{16} & 0 & E_{18} & E_{19} & 0 \\
* & E_{22} & E_{23} & 0 & E_{25} & E_{26} & E_{27} & 0 & 0 & 0 \\
* & * & E_{33} & 0 & 0 & E_{36} & E_{37} & 0 & 0 & 0 \\
* & * & * & E_{44} & 0 & 0 & 0 & E_{48} & 0 & 0 \\
* & * & * & * & E_{55} & E_{56} & 0 & E_{58} & E_{59} & 0 \\
* & * & * & * & * & E_{66} & E_{67} & 0 & 0 & 0 \\
* & * & * & * & * & * & E_{77} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & E_{88} & 0 & 0 \\
* & * & * & * & * & * & * & * & E_{99} & E_{9,10} \\
* & * & * & * & * & * & * & * & * & E_{10,10}
\end{bmatrix}$$

$$F = \begin{bmatrix}
F_{11} & F_{12} & 0 & 0 & F_{15} & F_{16} & 0 & F_{18} & F_{19} & 0 \\
* & F_{22} & F_{23} & 0 & F_{25} & F_{26} & F_{27} & 0 & 0 & 0 \\
* & * & F_{33} & 0 & 0 & F_{36} & F_{37} & 0 & 0 & 0 \\
* & * & * & F_{44} & 0 & 0 & 0 & F_{48} & 0 & 0 \\
* & * & * & * & F_{55} & F_{56} & 0 & F_{58} & F_{59} & 0 \\
* & * & * & * & * & F_{66} & F_{67} & 0 & 0 & 0 \\
* & * & * & * & * & * & F_{77} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & F_{88} & 0 & 0 \\
* & * & * & * & * & * & * & * & F_{99} & F_{9,10} \\
* & * & * & * & * & * & * & * & * & F_{10,10}
\end{bmatrix}$$

$$E_{11} = 2kP - 2PC - 4k\Gamma_1\Lambda + 4k\Gamma_2\Delta + 2\Gamma_1\Lambda C - 2\Gamma_2\Delta C + G_{11} + Q_1 + Q_2 + Q_3 + \frac{\varsigma}{2}(R_1 + R_3) + e^{-k\varsigma}S_1 - 2\Gamma_1 W_1 \Gamma_2$$

$$E_{12} = G_{1,2}$$

$$E_{15} = PA + 2k\Lambda - 2k\Delta - \Gamma_1\Lambda A + \Gamma_2\Delta A - C\Lambda + C\Delta + G_{13} + W_1(\Gamma_1 + \Gamma_2)$$

$$E_{16} = G_{14}, E_{1,8} = PB - \Gamma_1\Lambda B + \Gamma_2\Delta B$$

$$E_{19} = PD - \Gamma_1\Lambda D + \Gamma_2\Delta D$$

$$E_{22} = G_{22} - e^{-k\varsigma}G_{11} - e^{-k\varsigma}S_4 + e^{-2k\varsigma}S_2 - 2\Gamma_1 W_3\Gamma_2$$

$$E_{23} = -e^{-k\varsigma}G_{12}, E_{25} = G_{23}$$

$$E_{26} = G_{24} - e^{-k\varsigma}G_{13} + W_3(\Gamma_1 + \Gamma_2), E_{27} = -e^{-k\varsigma}G_{14}$$

$$E_{33} = -e^{-k\varsigma}G_{22} - e^{-2k\varsigma}S_2 - 2\Gamma_1 W_4\Gamma_2, E_{36} = -e^{-k\varsigma}G_{23}$$

$$E_{37} = -e^{-k\varsigma}G_{24} + W_4(\Gamma_1 + \Gamma_2)$$

$$E_{44} = -(1-\mu)e^{-2k\varsigma}Q_2 - 2\Gamma_1 W_2\Gamma_2 - e^{-k\varsigma}(S_1 - S_4)$$

$$E_{48} = W_2(\Gamma_1 + \Gamma_2)$$

$$E_{55} = 2\Lambda A - 2\Delta A + G_{33} + Q_4 + \rho^2 R_5 - 2W_1, E_{56} = G_{34}$$

$$E_{58} = \Lambda B - \Delta B, E_{59} = \Lambda D - \Delta D$$

$$E_{66} = G_{44} - e^{-k\varsigma}G_{33} - 2W_3, E_{67} = -e^{-k\varsigma}G_{34}$$

$$E_{77} = -e^{-k\varsigma}G_{44} - 2W_4, E_{88} = -(1-\mu)e^{-2k\varsigma}Q_4 - 2W_2$$

$$E_{99} = -e^{-2k\rho}R_5, E_{9,10} = -e^{-2k\rho}S_3, E_{10,10} = -e^{-2k\rho}R_5$$

$$F_{11} = 2kP - 2PC - 4k\Gamma_1\Lambda + 4k\Gamma_2\Delta + 2\Gamma_1\Lambda C - 2\Gamma_2\Delta C + G_{11} + Q_1 + Q_2 + Q_3 + \frac{\varsigma}{2}(R_1 + R_3) + e^{-k\varsigma}S_5 - 2\Gamma_1 W_1\Gamma_2$$

$$F_{22} = G_{22} - e^{-k\varsigma}G_{11} - e^{-k\varsigma}S_5 + e^{-2k\varsigma}S_6 - 2\Gamma_1 W_3\Gamma_2$$

$$F_{33} = -e^{-k\varsigma}G_{22} - e^{-2k\varsigma}S_7 - 2\Gamma_1 W_4\Gamma_2$$

$$F_{44} = -(1-\mu)e^{-2k\varsigma}Q_2 - 2\Gamma_1 W_2\Gamma_2 - e^{-2k\varsigma}(S_7 - S_6)$$

All the other items in matrix F satisfies $F_{ij} \neq 0$,we can get $F_{ij} = E_{ij}, i, j = 1, 2, \ldots, 10$.

Proof: Construct a new class of Lyapunov functional candidate as follow:

$$V(y_t) = \sum_{i=1}^{7} V_i(y_t)$$

$$V_1(y_t) = e^{2kt}y^T(t)Py(t)$$

$$V_2(y_t) = 2e^{2kt}\sum_{i=1}^{n}\int_0^{y_i(t)}[\lambda_i(f_i(s) - \gamma_i^- s) + \delta_i(\gamma_i^+ s - f_i(s))]ds$$

$$V_3(y_t) = \int_{t-\frac{\varsigma}{2}}^{t} e^{2ks}\eta^T(s)\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ * & G_{22} & G_{23} & G_{24} \\ * & * & G_{33} & G_{34} \\ * & * & * & G_{44} \end{bmatrix}\eta(s)ds$$

where

$$\eta^T(s) = \begin{bmatrix} y^T(s) & y^T(s - \frac{\varsigma}{2}) & f^T(y(s)) & f^T(y(s - \frac{\varsigma}{2})) \end{bmatrix}$$

$$\begin{aligned} V_4(y_t) = &\int_{t-\frac{\varsigma(t)}{2}}^{t} e^{2ks}y^T(s)Q_1 y(s)ds + \int_{t-\varsigma(t)}^{t} e^{2ks}y^T(s)Q_2 y(s)ds \\ &+ \int_{t-\frac{\varsigma(t)+\varsigma}{2}}^{t} e^{2ks}y^T(s)Q_3 y(s)ds \\ &+ \int_{t-\varsigma(t)}^{t} e^{2ks}f^T(y(s))Q_4 f(y(s))ds \end{aligned}$$

$$V_5(y_t) = \int_{-\frac{\varsigma}{2}}^{0}\int_{t+\theta}^{t} e^{2ks}(y^T(s)R_1 y(s) + \dot{y}^T(s)R_2\dot{y}(s))dsd\theta$$

$$V_6(y_t) = \int_{-\varsigma}^{-\frac{\varsigma}{2}}\int_{t+\theta}^{t} e^{2ks}(y^T(s)R_3 y(s) + \dot{y}^T(s)R_4\dot{y}(s))dsd\theta$$

$$V_7(y_t) = \rho\int_{-\rho}^{0}\int_{t+\theta}^{t} e^{2ks}f^T(y(s))R_5 f(y(s))dsd\theta$$

Then, taking the time derivative of V(t) with respect to t along the system (12) yield

$$\dot{V}(y_t) = \sum_{i=1}^{7}\dot{V}_i(y_t)$$

$$\dot{V}_1(y_t) = 2ke^{2kt}y^T(t)Py(t) + 2e^{2kt}y^T(t)P\dot{y}(t) \tag{17}$$

$$\begin{aligned} \dot{V}_2(y_t) = &4ke^{2kt}\sum_{i=1}^{n}\int_0^{y_i(t)}[\lambda_i(f_i(s) - \gamma_i^- s) + \delta_i(\gamma_i^+ s - f_i(s))]ds \\ &+ 2e^{2kt}[(f^T(y(t)) - y^T(t)\Gamma_1)\Lambda\dot{y}(t) \\ &+ (y^T(t)\Gamma_2 - f^T(y(t)))\Delta\dot{y}(t)] \\ &\leq 4ke^{2kt}[(f^T(y(t)) - y^T(t)\Gamma_1)\Lambda y(t) \\ &+ (y^T(t)\Gamma_2 - f^T(y(t)))\Delta y(t)] \\ &+ 2e^{2kt}[f^T(y(t))(\Lambda - \Delta) + y^T(t)(\Gamma_2\Delta - \Gamma_1\Lambda)]\dot{y}(t) \end{aligned} \tag{18}$$

$$\dot V_3(y_t)=e^{2kt}\eta^T(t)\begin{bmatrix}G_{11}&G_{12}&G_{13}&G_{14}\\ *&G_{22}&G_{23}&G_{24}\\ *&*&G_{33}&G_{34}\\ *&*&*&G_{44}\end{bmatrix}\eta(t) -e^{2k(t-\frac{\varsigma}{2})}\eta^T(t-\frac{\varsigma}{2})\begin{bmatrix}G_{11}&G_{12}&G_{13}&G_{14}\\ *&G_{22}&G_{23}&G_{24}\\ *&*&G_{33}&G_{34}\\ *&*&*&G_{44}\end{bmatrix}\eta(t-\frac{\varsigma}{2}) \tag{19}$$

$$\begin{aligned}\dot V_4(y_t)\le{}& e^{2kt}[y^T(t)(Q_1+Q_2+Q_3)y(t)+f^T(y(t))Q_4f(y(t))\\ &-(1-\frac{\mu}{2})e^{-k\varsigma}y^T(t-\frac{\varsigma(t)}{2})Q_1y(t-\frac{\varsigma(t)}{2})\\ &-(1-\frac{\mu}{2})e^{-2k\varsigma}y^T(t-\frac{\varsigma(t)+\varsigma}{2})Q_3y(t-\frac{\varsigma(t)+\varsigma}{2})\\ &-(1-\mu)e^{-2k\varsigma}y^T(t-\varsigma(t))Q_2y(t-\varsigma(t))\\ &-(1-\mu)e^{-2k\varsigma}f^T(y(t-\varsigma(t)))Q_4f(y(t-\varsigma(t)))]\end{aligned} \tag{20}$$

$$\begin{aligned}\dot V_5(y_t)\le{}&\frac{\varsigma}{2}e^{2kt}(y^T(t)R_1y(t)+\dot y^T(t)R_2\dot y(t))\\ &-e^{2k(t-\frac{\varsigma}{2})}\int_{t-\frac{\varsigma}{2}}^{t}(y^T(s)R_1y(s)+\dot y^T(s)R_2\dot y(s))ds\end{aligned} \tag{21}$$

$$\begin{aligned}\dot V_6(y_t)\le{}&\frac{\varsigma}{2}e^{2kt}(y^T(t)R_3y(t)+\dot y^T(t)R_4\dot y(t))\\ &-e^{2k(t-\varsigma)}\int_{t-\varsigma}^{t-\frac{\varsigma}{2}}(y^T(s)R_3y(s)+\dot y^T(s)R_4\dot y(s))ds\end{aligned} \tag{22}$$

$$\begin{aligned}\dot V_7(y_t)\le{}&\rho^2e^{2kt}f^T(y(t))R_5f(y(t))\\ &-\rho e^{2k(t-\rho)}\int_{t-\rho}^{t}f^T(y(s))R_5f(y(s))ds\\ \le{}& e^{2kt}\Big\{\rho^2f^T(y(t))R_5f(y(t))\\ &-e^{-2k\rho}\begin{bmatrix}\int_{t-\rho(t)}^{t}f(y(s))ds\\ \int_{t-\rho}^{t-\rho(t)}f(y(s))ds\end{bmatrix}^T\begin{bmatrix}R_5&S_5\\ *&R_5\end{bmatrix}\begin{bmatrix}\int_{t-\rho(t)}^{t}f(y(s))ds\\ \int_{t-\rho}^{t-\rho(t)}f(y(s))ds\end{bmatrix}\Big\}\end{aligned} \tag{23}$$

From (5), we can get that there exist positive diagonal matrices W_1, W_2, W_3, W_4 such that the following inequalities holds:

$$\begin{aligned}&e^{2kt}[-2f^T(y(t))W_1f(y(t))+2y^T(t)W_1(\Gamma_1+\Gamma_2)f(y(t))\\ &-2y^T(t)\Gamma_1W_1\Gamma_2y(t)]\ge 0\end{aligned} \tag{24}$$

$$\begin{aligned}&e^{2kt}[-2f^T(y(t-\varsigma(t)))W_2f(y(t-\varsigma(t)))+2y^T(t-\varsigma(t))W_2(\Gamma_1\\ &+\Gamma_2)f(y(t-\varsigma(t)))-2y^T(t-\varsigma(t))\Gamma_1W_2\Gamma_2y(t-\varsigma(t))]\ge 0\end{aligned} \tag{25}$$

$$\begin{aligned}&e^{2kt}[-2f^T(y(t-\frac{\varsigma}{2}))W_3f(y(t-\frac{\varsigma}{2}))+2y^T(t-\frac{\varsigma}{2})W_3(\Gamma_1\\ &+\Gamma_2)f(y(t-\frac{\varsigma}{2}))-2y^T(t-\frac{\varsigma}{2})\Gamma_1W_3\Gamma_2y(t-\frac{\varsigma}{2})]\ge 0\end{aligned} \tag{26}$$

$$\begin{aligned}&e^{2kt}[-2f^T(y(t-\varsigma))W_4f(y(t-\varsigma))+2y^T(t-\varsigma)W_4(\Gamma_1\\ &+\Gamma_2)f(y(t-\varsigma))-2y^T(t-\varsigma)\Gamma_1W_4\Gamma_2y(t-\varsigma)]\ge 0\end{aligned} \tag{27}$$

(1) When $0\le\varsigma(t)\le\frac{\varsigma}{2}$,we consider the following three zero equalities with any symmetric matrix S_1, S_2, S_4 :

$$\begin{aligned}&e^{2k(t-\frac{\varsigma}{2})}[y^T(t)S_1y(t)-y^T(t-\varsigma(t))S_1y(t-\varsigma(t))\\ &-2\int_{t-\varsigma(t)}^{t}y^T(s)S_1\dot y(s)ds]=0\end{aligned} \tag{28}$$

$$\begin{aligned}&e^{2k(t-\frac{\varsigma}{2})}[y^T(t-\varsigma(t))S_4y(t-\varsigma(t))-y^T(t-\frac{\varsigma}{2})S_4y(t-\frac{\varsigma}{2})\\ &-2\int_{t-\frac{\varsigma}{2}}^{t-\varsigma(t)}y^T(s)S_4\dot y(s)ds]=0\end{aligned} \tag{29}$$

$$\begin{aligned}&e^{2k(t-\varsigma)}[y^T(t-\frac{\varsigma}{2})S_2y(t-\frac{\varsigma}{2})-y^T(t-\varsigma)S_2y(t-\varsigma)\\ &-2\int_{t-\varsigma}^{t-\frac{\varsigma}{2}}y^T(s)S_2\dot y(s)ds]=0\end{aligned} \tag{30}$$

From (17)-(30),we can get

$$\begin{aligned}\dot V(y_t)\le{}& e^{2kt}[\xi^T(t)(E+\aleph^TZ\aleph)\xi(t)\\ &-(1-\frac{\mu}{2})(e^{-k\varsigma}y^T(t-\frac{\varsigma(t)}{2})Q_1y(t-\frac{\varsigma(t)}{2})\\ &-e^{-2k\varsigma}y^T(t-\frac{\varsigma(t)+\varsigma}{2})Q_3y(t-\frac{\varsigma(t)+\varsigma}{2}))]\\ &-e^{2k(t-\frac{\varsigma}{2})}\int_{t-\varsigma(t)}^{t}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_1&S_1\\ *&R_2\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\\ &-e^{2k(t-\frac{\varsigma}{2})}\int_{t-\frac{\varsigma}{2}}^{t-\varsigma(t)}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_1&S_4\\ *&R_2\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\\ &-e^{2k(t-\varsigma)}\int_{t-\varsigma}^{t-\frac{\varsigma}{2}}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_3&S_2\\ *&R_4\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\end{aligned}$$

(2) When $\frac{\varsigma}{2}\le\varsigma(t)\le\varsigma$,we consider the following three zero equalities with any symmetric matrix S_5, S_6, S_7 :

$$\begin{aligned}&e^{2k(t-\frac{\varsigma}{2})}[y^T(t)S_5y(t)-y^T(t-\frac{\varsigma}{2})S_5y(t-\frac{\varsigma}{2})\\ &-2\int_{t-\frac{\varsigma}{2}}^{t}y^T(s)S_5\dot y(s)ds]=0\end{aligned} \tag{31}$$

$$\begin{aligned}&e^{2k(t-\varsigma)}[y^T(t-\frac{\varsigma}{2})S_6y(t-\frac{\varsigma}{2})-y^T(t-\varsigma(t))S_6y(t-\varsigma(t))\\ &-2\int_{t-\varsigma(t)}^{t-\frac{\varsigma}{2}}y^T(s)S_6\dot y(s)ds]=0\end{aligned} \tag{32}$$

$$\begin{aligned}&e^{2k(t-\varsigma)}[y^T(t-\varsigma(t))S_7y(t-\varsigma(t))-y^T(t-\varsigma)S_7y(t-\varsigma)\\ &-2\int_{t-\varsigma}^{t-\varsigma(t)}y^T(s)S_7\dot y(s)ds]=0\end{aligned} \tag{33}$$

From (17)-(27),and (31)-(33),we can get

$$\begin{aligned}\dot V(y_t) \le\, & e^{2kt}[\xi^T(t)(F+\aleph^T Z\aleph)\xi(t)\\ & -(1-\frac{\mu}{2})(e^{-k\varsigma}y^T(t-\frac{\varsigma(t)}{2})Q_1 y(t-\frac{\varsigma(t)}{2})\\ & -e^{-2k\varsigma}y^T(t-\frac{\varsigma(t)+\varsigma}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma}{2}))]\\ & -e^{2k(t-\frac{\varsigma}{2})}\int_{t-\frac{\varsigma}{2}}^{t}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_1 & S_5\\ * & R_2\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\\ & -e^{2k(t-\varsigma)}\int_{t-\varsigma(t)}^{t-\frac{\varsigma}{2}}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_3 & S_6\\ * & R_4\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\\ & -e^{2k(t-\varsigma)}\int_{t-\varsigma}^{t-\varsigma(t)}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}^T\begin{bmatrix}R_3 & S_7\\ * & R_4\end{bmatrix}\begin{bmatrix}y(s)\\ \dot y(s)\end{bmatrix}ds\end{aligned}$$

Hence,combined with the Schur complement and (13)-(16), we can obtain $\dot V(y_t)\le 0$,this means the system (12) is guaranted to be asymptotically stable for $0\le\varsigma(t)\le\varsigma, 0\le\rho(t)\le\rho$,on the other hand,we have the followings:

$$V_1(y_0)\le\lambda_{max}(P)\|y(0)\|^2\le\lambda_{max}(P)\sup_{-h\le s\le 0}\|y(s)\|^2 \tag{34}$$

$$\begin{aligned}V_2(y_0)\le\,& 2\left\{[f(y(0))-\Gamma_1 y(0)]^T\Lambda+[\Gamma_2 y(0)-f(y(0))]^T\Delta\right\}y(0)\\ \le\,& 2\lambda_{max}(\Gamma_1-\Gamma_2)(\lambda_{max}(\Lambda)+\lambda_{max}(\Delta))\sup_{-h\le s\le 0}\|y(s)\|^2\end{aligned} \tag{35}$$

$$\begin{aligned}V_3(y_0)\le\,&\int_{-\frac{\varsigma}{2}}^{0}\eta^T(s)\begin{bmatrix}G_{11} & G_{12} & G_{13} & G_{14}\\ * & G_{22} & G_{23} & G_{24}\\ * & * & G_{33} & G_{34}\\ * & * & * & G_{44}\end{bmatrix}\eta(s)ds\\ \le\,&\varsigma(1+\gamma^2)\lambda_{max}\begin{pmatrix}G_{11} & G_{12} & G_{13} & G_{14}\\ * & G_{22} & G_{23} & G_{24}\\ * & * & G_{33} & G_{34}\\ * & * & * & G_{44}\end{pmatrix}\sup_{-h\le s\le 0}\|y(s)\|^2\end{aligned} \tag{36}$$

where

$$\gamma=\max_{1\le i\le n}\{|\gamma_i^-||\gamma_i^+|\}$$

$$\begin{aligned}V_4(y_0)\le\,&(\frac{\varsigma}{2}\lambda_{max}(Q_1)+\varsigma\lambda_{max}(Q_2)+\varsigma\lambda_{max}(Q_3)\\ &+\varsigma\gamma^2\lambda_{max}(Q_4))\sup_{-h\le s\le 0}\|y(s)\|^2\end{aligned} \tag{37}$$

$$\begin{aligned}V_5(y_0)\le\,&\frac{\varsigma^2}{8}\lambda_{max}(R_1)\sup_{-h\le s\le 0}\|y(s)\|^2\\ &+\lambda_{max}(R_2)\int_{-\frac{\varsigma}{2}}^{0}\int_{\theta}^{0}\dot y^T(s)\dot y(s)dsd\theta\end{aligned} \tag{38}$$

$$\begin{aligned}V_6(y_0)\le\,&\frac{3\varsigma^2}{8}\lambda_{max}(R_3)\sup_{-h\le s\le 0}\|y(s)\|^2\\ &+\lambda_{max}(R_4)\int_{-\varsigma}^{-\frac{\varsigma}{2}}\int_{\theta}^{0}\dot y^T(s)\dot y(s)dsd\theta\end{aligned} \tag{39}$$

According to $2x^Ty\le x^TYx+y^TYy$ with $Y>0$

$$\begin{aligned}\dot y^T(s)\dot y(s)\le\,&4[\lambda_{max}(C^TC)+\gamma^2\lambda_{max}(A^TA)+\gamma^2\lambda_{max}(B^TB)\\ &+\rho_m^2\gamma^2\lambda_{max}(D^TD)]\sup_{-h\le s\le 0}\|y(s)\|^2\end{aligned} \tag{40}$$

$$\begin{aligned}V_7(y_0)\le\,&\rho\lambda_{max}(R_5)\int_{-\rho}^{0}\int_{\theta}^{0}f^T(y(s))f(y(s))dsd\theta\\ \le\,&\frac{\rho^3\gamma^2}{2}\lambda_{max}(R_5)\sup_{-h\le s\le 0}\|y(s)\|^2\end{aligned} \tag{41}$$

According to (34)-(41),there exist a positive constant α,such that

$$V(y_0)\le\alpha\sup_{-h\le s\le 0}\|y(s)\|^2$$

where

$$\begin{aligned}\alpha=\,&\lambda_{max}(P)+2\lambda_{max}(\Gamma_1-\Gamma_2)(\lambda_{max}(\Lambda)+\lambda_{max}(\Delta))\\ &+(\frac{\varsigma}{2}\lambda_{max}(Q_1)+\varsigma\lambda_{max}(Q_2)+\varsigma\lambda_{max}(Q_3)+\varsigma\gamma^2\lambda_{max}(Q_4))\\ &+\frac{\varsigma^2}{8}\lambda_{max}(R_1)+\frac{3\varsigma^2}{8}\lambda_{max}(R_3)+2\varsigma^2(\lambda_{max}(C^TC)\\ &+\gamma^2\lambda_{max}(A^TA)+\gamma^2\lambda_{max}(B^TB)+\rho_m^2\gamma^2\lambda_{max}(D^TD))\\ &+\frac{\rho^3\gamma^2}{2}\lambda_{max}(R_5)+\varsigma(1+\gamma^2)\lambda_{max}\begin{pmatrix}G_{11} & G_{12} & G_{13} & G_{14}\\ * & G_{22} & G_{23} & G_{24}\\ * & * & G_{33} & G_{34}\\ * & * & * & G_{44}\end{pmatrix}\end{aligned}$$

Furthermore , we have

$$V(y_t)\ge e^{2kt}\lambda_{min}(P)\|y(t)\|^2$$

Then we can easily obtain

$$e^{2kt}\lambda_{min}(P)\|y(t)\|^2\le\alpha\sup_{-h\le s\le 0}\|y(s)\|^2$$

Which leads to

$$\|y(t)\|\le\sqrt{\frac{\alpha}{\lambda_{min}(P)}}e^{-kt}\sup_{-h\le s\le 0}\|y(s)\|^2$$

Thus by Definition 1,when the system (7) satisfies $\Delta A(t)=\Delta B(t)=\Delta C(t)=\Delta D(t)=0$ is exponentially stable with convergence rate k,and the proof is completed. ■

Based on Theorem 1,we have the following result for neural networks with time-varying.

Theorem 2 Given that the Assumption 1-4 hold, the system (6) is globally exponentially stable with the exponential convergence rate index k if there exist symmetric positive definite matrices $P,H_1,H_2,Q_i,i=1,2,3,4,R_i,i=1,2,\ldots,5,\begin{bmatrix}G_{11} & G_{12} & G_{13} & G_{14}\\ * & G_{22} & G_{23} & G_{24}\\ * & * & G_{33} & G_{34}\\ * & * & * & G_{44}\end{bmatrix}$,positive diagonal matrices $W_1,W_2,W_3,W_4,\Lambda=diag\{\lambda_1,\lambda_2,\ldots,\lambda_n\},\Delta=diag\{\delta_1,\delta_2,\ldots,\delta_n\}$,,and any symmetric matrix $S_i,i=1,2,\ldots,7$,such that the following LMIs hold:

$$\begin{bmatrix}R_1 & S_i\\ * & R_2\end{bmatrix}>0, i=1,4,5. \tag{42}$$

$$\begin{bmatrix} R_3 & S_i \\ * & R_4 \end{bmatrix} > 0, i = 2, 6, 7. \quad (43)$$

$$\begin{bmatrix} E+\aleph^T Z\aleph & \Im G & \Im G & \Psi_{11}^T H_1 & \Psi_{22}^T H_2 \\ * & -H_1+G^T ZG & G^T ZG & 0 & 0 \\ * & * & -H_2+G^T ZG & 0 & 0 \\ * & * & * & -H_1 & 0 \\ * & * & * & * & -H_2 \end{bmatrix} < 0 \quad (44)$$

$$\begin{bmatrix} F+\aleph^T Z\aleph & \Im G & \Im G & \Psi_{11}^T H_1 & \Psi_{22}^T H_2 \\ * & -H_1+G^T ZG & G^T ZG & 0 & 0 \\ * & * & -H_2+G^T ZG & 0 & 0 \\ * & * & * & -H_1 & 0 \\ * & * & * & * & -H_2 \end{bmatrix} < 0 \quad (45)$$

where

$$\Psi_{11} = \begin{bmatrix} \frac{E_c}{2} & 0 & 0 & 0 & \frac{E_a}{2} & 0 & 0 & E_b & 0 & 0 \end{bmatrix}$$

$$\Psi_1 = \begin{bmatrix} \Psi_{11} & 0 & 0 \end{bmatrix}$$

$$\Psi_{22} = \begin{bmatrix} \frac{E_c}{2} & 0 & 0 & 0 & \frac{E_a}{2} & 0 & 0 & 0 & E_d & 0 \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} \Psi_{22} & 0 & 0 \end{bmatrix}$$

$$\Im^T = [P+\Gamma_2\Delta-\Gamma_1\Lambda-CZ, 0, 0, 0, \Lambda-\Delta+ZA, 0, 0, ZB, ZD, 0]$$

Proof: System (6) can be written as

$$\begin{cases} \dot{y}(t) = -Cy(t)+Af(y(t))+Bf(y(t-\varsigma(t))) \\ \qquad +D\int_{t-\rho(t)}^{t} f(y(s))ds + G(p_1(t)+q_1(t)) \\ p_1(t) = F(t)p_2(t) \\ q_1(t) = F(t)q_2(t) \\ p_2(t) = \frac{E_c}{2}y(t) + \frac{E_a}{2}f(y(t)) + E_b f(y(t-\varsigma(t))) \\ q_2(t) = \frac{E_c}{2}y(t) + \frac{E_a}{2}f(y(t)) + E_d\int_{t-\rho(t)}^{t} f(y(s))ds \end{cases} \quad (46)$$

Based on Assumption 4, we can get that

$$p_1^T(t)p_1(t) \le p_2^T(t)p_2(t) = \varphi^T(t)\Psi_1^T\Psi_1\varphi(t)$$

$$q_1^T(t)q_1(t) \le q_2^T(t)q_2(t) = \varphi^T(t)\Psi_2^T\Psi_2\varphi(t)$$

where

$$\varphi^T(t) = [\xi^T(t), p_1^T(t), q_1^T(t)]$$

There exist two positive matrices H_1, H_2,satisfying the following inequality

$$\varphi^T(t)\Psi_1^T H_1\Psi_1\varphi(t) - p_1^T(t)H_1p_1(t) \ge 0$$
$$\varphi^T(t)\Psi_2^T H_2\Psi_2\varphi(t) - q_1^T(t)H_2q_1(t) \ge 0$$

Similarly,we can obtain that,when $0 \le \varsigma(t) \le \frac{\varsigma}{2}$,one can obtain that

$$\begin{aligned} \dot{V}(y_t) \le\; & e^{2kt}[\varphi^T(t)(\Omega_1 + \Psi_1^T H_1\Psi_1 + \Psi_2^T H_2\Psi_2)\varphi(t) \\ & - (1-\frac{\mu}{2})(e^{-k\varsigma}y^T(t-\frac{\varsigma(t)}{2})Q_1 y(t-\frac{\varsigma(t)}{2}) \\ & - e^{-2k\varsigma}y^T(t-\frac{\varsigma(t)+\varsigma}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma}{2}))] \\ & - e^{2k(t-\frac{\varsigma}{2})}\int_{t-\varsigma(t)}^{t}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_1 & S_1 \\ * & R_2\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\ & - e^{2k(t-\frac{\varsigma}{2})}\int_{t-\frac{\varsigma}{2}}^{t-\varsigma(t)}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_1 & S_4 \\ * & R_2\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\ & - e^{2k(t-\varsigma)}\int_{t-\varsigma}^{t-\frac{\varsigma}{2}}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_3 & S_2 \\ * & R_4\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \end{aligned}$$

where

$$\Omega_1 = \begin{bmatrix} E+\aleph^T Z\aleph & \Im G & \Im G \\ * & -H_1+G^T ZG & G^T ZG \\ * & * & -H_2+G^T ZG \end{bmatrix}$$

when $\frac{\varsigma}{2} \le \varsigma(t) \le \varsigma$,one can obtain that

$$\begin{aligned} \dot{V}(y_t) \le\; & e^{2kt}[\varphi^T(t)(\Omega_2 + \Psi_1^T H_1\Psi_1 + \Psi_2^T H_2\Psi_2)\varphi(t) \\ & - (1-\frac{\mu}{2})(e^{-k\varsigma}y^T(t-\frac{\varsigma(t)}{2})Q_1 y(t-\frac{\varsigma(t)}{2}) \\ & - e^{-2k\varsigma}y^T(t-\frac{\varsigma(t)+\varsigma}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma}{2}))] \\ & - e^{2k(t-\frac{\varsigma}{2})}\int_{t-\frac{\varsigma}{2}}^{t}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_1 & S_5 \\ * & R_2\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\ & - e^{2k(t-\varsigma)}\int_{t-\varsigma(t)}^{t-\frac{\varsigma}{2}}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_3 & S_6 \\ * & R_4\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\ & - e^{2k(t-\varsigma)}\int_{t-\varsigma}^{t-\varsigma(t)}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_3 & S_7 \\ * & R_4\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \end{aligned}$$

where

$$\Omega_2 = \begin{bmatrix} F+\aleph^T Z\aleph & \Im G & \Im G \\ * & -H_1+G^T ZG & G^T ZG \\ * & * & -H_2+G^T ZG \end{bmatrix}$$

According to (42)-(45),then we can obtain $\dot{V}(y_t) \le 0$.On the other hand

$$\begin{aligned} \dot{y}^T(s)\dot{y}(s) \le\; & 6\{\lambda_{max}(C^TC) + \gamma^2\lambda_{max}(A^TA) + \gamma^2\lambda_{max}(B^TB) \\ & + \rho_m^2\gamma^2\lambda_{max}(D^TD) + 3\lambda_{max}(G^TG)[\frac{1}{2}\lambda_{max}(E_c^TE_c) \\ & + \frac{\gamma_2}{2}\lambda_{max}(E_a^TE_a) + \gamma^2\lambda_{max}(E_b^TE_b) \\ & + \gamma^2\rho_m^2\lambda_{max}(E_d^TE_d)]\}\sup_{-h\le s\le 0}\|y(s)\|^2 \end{aligned} \quad (47)$$

Similarly, from (34)-(41) and (47),there exist a positive constant β, such that

$$V(y_0) \le \beta\sup_{-h\le s\le 0}\|y(s)\|^2$$

Furthermore , we have

$$\|y(t)\| \le \sqrt{\frac{\beta}{\lambda_{min}(P)}}e^{-kt}\sup_{-h\le s\le 0}\|y(s)\|^2$$

Then , Based on Theorem 1 and Definition 1, the system (6) is exponentially stable with convergence rate k, and the proof is completed. ■

Remark 1 Theorem 1 and 2 proposes an improved exponential stability condition for neural networks with discrete and distribute time-varying delays.This paper not only divide the delay interval $[0,\varsigma]$ into two ones $[0,\frac{\varsigma}{2}]$ and $[\frac{\varsigma}{2},\varsigma]$,but also divides the interval $[0,\varsigma]$ into four intervals $[0,\frac{\varsigma(t)}{2}]$,$[\frac{\varsigma(t)}{2},\varsigma(t)]$,$[\varsigma(t),\frac{\varsigma+\varsigma(t)}{2}]$,$[\frac{\varsigma+\varsigma(t)}{2},\varsigma]$, each segments has a different Lyapunov matrix in function V.In [18,19],they did not discuss by dividing interval of $\frac{\varsigma(t)}{2}$,and in [20],they didn't discuss by dividing interval of $\frac{\varsigma+\varsigma(t)}{2}$,which have potential to yield less conservative results.

Remark 2 Through model transformation,system (6) can be written as (46),this transformation can make us easy to understand to many complex problems,and two vectors $f(y(t-\varsigma)), f(y(t-\frac{\varsigma}{2}))$ are introduced in $\xi(t)$,which are rarely considered in other literature.this may lead to obtain an improved feasible region for delay-dependent stability criteria.

Remark 3 In this paper,Theorem 1 and 2 require the upper bound μ of the time-varying delay $\varsigma(t)$ to be known.However,in many cases μ is unknown,considering this situation ,we can set $Q_i = 0, i = 1,2,3,4$ in $V(y_t)$,and employ the same methods in Theorem 1 and 2,we can derive the delay-dependent and delay-derivative-independent stability criteria.

IV. Numerical examples

In this section,we provide three numerical examples to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

Example 1 Consider the system (12) with the following parameters:

$$C=\begin{bmatrix}2.3 & 0 & 0\\ 0 & 3.4 & 0\\ 0 & 0 & 2.5\end{bmatrix}, A=\begin{bmatrix}0.9 & -1.5 & 0.1\\ -1.2 & 0.1 & 0.2\\ 0.2 & 0.3 & 0.8\end{bmatrix},$$
$$B=\begin{bmatrix}0.8 & 0.6 & 0.2\\ 0.5 & 0.7 & 0.1\\ 0.2 & 0.1 & 0.5\end{bmatrix}, D=\begin{bmatrix}0.3 & 0.2 & 0.1\\ 0.1 & 0.2 & 0.1\\ 0.1 & 0.1 & 0.2\end{bmatrix}$$
$$\Gamma_1 = diag\{0,0,0\}, \Gamma_2 = diag\{0.2,0.2,0.2\}.$$

In Table I,we consider the case of $\varsigma = \rho, k = 0$,the upper bound of ς for unknown μ is derived by Theorem 1 with $Q_i = 0, i = 1,2,3,4$ in the Lyapunov-Krasovskii functional V.According to this Table,we can see this example shows that the stability condition in this paper gives much less conservative results than those in the literature.

Example 2 Consider the system (12) with the following parameters:

$$C=\begin{bmatrix}6 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 7\end{bmatrix}, A=\begin{bmatrix}1.2 & -0.8 & 0.6\\ 0.5 & -1.5 & 0.7\\ -0.8 & -1.2 & -1.4\end{bmatrix},$$
$$B=\begin{bmatrix}-1.4 & 0.9 & 0.5\\ -0.6 & 1.2 & 0.8\\ 0.5 & -0.7 & 1.1\end{bmatrix}, D=\begin{bmatrix}1.8 & 0.7 & -0.8\\ 0.6 & 0.4 & 1.0\\ -0.4 & -0.6 & 1.2\end{bmatrix}$$

TABLE I
Allowable upper bound of ς for unknown μ in Example 1

Method	Maximum of allowable ς
[16]	1.833
[17]	3.597
[18]	6.938
[19]	9.338
[20]	11.588
Theorem 1	13.914

TABLE II
Allowable upper bound of k for Example 2

Method	[18]	[19]	[20]	Theorem 1
$\varsigma = 0.5, \rho = 0.2, \mu = 0$	0.46	0.58	0.56	0.86
$\varsigma = 0.5, \rho = 0.2, \mu = 0.5$	0.21	0.35	0.35	0.73
$\varsigma = 0.6, \rho = 0.2, \mu = 0.5$	0.06	0.20	0.33	0.55
$\varsigma = 0.8, \rho = 0.2, \mu = 0.5$	0.00	0.05	0.10	0.30

Let $\Gamma_1 = diag\{-1.2, 0, -2.4\}, \Gamma_2 = diag\{0, 1.4, 0\}$.
For various ς, ρ, μ,the maximum of the exponential convergence rate index k calculated by Theorem 1.According to Table II,this example shows that the stability criterion in this paper can lead to less conservative results.

Example 3 Consider the system (6) with the following parameters:

$$C=\begin{bmatrix}6.5618 & 0 & 0\\ 0 & 5.5784 & 0\\ 0 & 0 & 7.3269\end{bmatrix}, A=\begin{bmatrix}0.3256 & -0.1904 & 0.3322\\ -0.1564 & 0.2446 & 0.3674\\ -0.1753 & 0.2956 & -0.3115\end{bmatrix},$$
$$B=\begin{bmatrix}0.1981 & -0.1313 & 0.1158\\ 0.1645 & 0.0901 & 0.1013\\ 0.0274 & -0.1518 & 0.0742\end{bmatrix}$$

$$G = 0.8I, E_a = E_b = E_c = I, \Gamma_1 = diag\{0,0,0\},$$
$$\Gamma_2 = diag\{2,2,2\}.$$

Case (1) $D = diag\{0,0,0\}$, and $E_d = 0$. First,consider the condition with $k = 0$,and unknown μ.For this case ,in [11,12],the upper bound of ς for guaranteeing stability were 0.4074 and 0.7245,respectively.However,in Theorem 2,we can get the upper bound of ς with the same condition as 2.970.
Second,consider the case of $k \neq 0$,and various μ,the upper bound of ς is derived by Theorem 2 in Table III.

Case (2) $D = \begin{bmatrix}-0.1981 & 0.1313 & -0.1158\\ -0.1645 & -0.0901 & -0.1013\\ -0.0274 & 0.1518 & -0.0742\end{bmatrix}, E_d = I$, the correspond upper bounds of ς for various k, μ derived by Theorem 2 (letting $k = 0.5, \rho = 0.1$) in Table IV.

V. Conclusion

In this paper, a new delay-dependent exponential stability criterion for neural networks with time-delaying has been

TABLE III
Allowable upper bound of ς for case (1) of Example 3

Method	Theorem 3.2
$k = 0.1, \mu = 0.5$	4.196
$k = 0.1, \mu = 0.6$	3.308
$k = 0.3, \mu = 0.6$	1.629
$k = 0.4, \mu = 0.7$	1.414

TABLE IV
ALLOWABLE UPPER BOUND OF ς FOR CASE (2) OF EXAMPLE 3

Method	Theorem 3.2
$\mu = 0$	1.395
$\mu = 0.4$	1.234
$\mu = 0.8$	1.218
Unknown μ	1.213

investigated.By dividing the delay interval and constructing new Lyapunov-Krasovskii functional which contains some new integral terms ,and fully uses the information about the bounding technique of integral terms with different free-weighting matrices in different delay intervals to reduce the conservatism of stability criteria. Finally, numerical examples have presented to illustrate the benefits and less conservativeness of the proposed method.

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