A Heuristic for the Integrated Production and Distribution Scheduling Problem
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Abstract—The integrated problem of production and distribution scheduling is relevant in many industrial applications. Thus, many heuristics to solve this integrated problem have been developed in the last decade. Most of these heuristics use a sequential working principal or a single decomposition and integration approach to separate and solve subproblems. A heuristic using a multi step decomposition and integration approach is presented in this paper and evaluated in a case study. The result show significant improved results compared with sequential scheduling heuristics.

Keywords—Production and outbound distribution, integrated planning, heuristic, decomposition and integration.

I. INTRODUCTION

Production and distribution are essential operational functions in a supply chain [1]. These two functions are operated by different companies mostly due to the ongoing trends to focus on core competencies and outsourcing [2]. In fact, companies involved in a supply chain are legally independent and represent a profit centre. Each company carries out its own process and the related order planning and scheduling. The results of this separated planning and scheduling activities aim to fulfill individual objectives of the respective company using decisions from upstream companies as an input. It becomes apparent, that this separated planning and scheduling is not optimal for the overall supply chain performance [3]-[5]. The integrated planning and scheduling of these functions is a promising approach in order to stay competitive or to improve the competitiveness of the entire supply chain. Production and distribution processes can be linked seamlessly with one another by applying the integrated planning approach. This represents the goal of integrated planning [6]. Companies are cooperating as equal partners in a truly integrated planning and scheduling. Consequently, there is no negotiation power at one partners’ side due to an unbalanced cost distribution.

Numerous papers have been published during the last decade presenting problem solving methods for the integrated problem considered. This increasing interest is due to the practical importance of the problem and its potential effects on supply chain performance [7]. Consequently, there is a number of terms in the literature dealing with the integration of production and distribution planning and scheduling describing the same problem like integrated production and distribution scheduling problem (IPODS), production distribution problem (PDP) as well as production, inventory, distribution routing problem (PIDRP) or production and transportation scheduling problem (PTSP) [1]. In this paper we use the abbreviation IPODS.

The remainder of this paper is organized as follows: The second section provides the development of the integrated problem statement including its mathematical modeling based on the respective subproblems and the given models in the literature. Solution methods and strategies to solve the IPODS are presented in the third section. A framework for a new multi step decomposition and integration (we abbreviate this as MSDI in the following) to solve the IPODS is presented in the fourth and a detailed heuristic is introduced in the fifth section. We evaluate this multi step heuristic in the sixth section.

II. PROBLEM STATEMENT

In general, an integrated scheduling problem combines at least two subproblems from different operational functions in a single problem and its related optimization model [8]. Thus, the requirements, restrictions and optimization goals of each subproblem have to be combined in one integrated problem and considered simultaneously [1], [8], [9]. Consequently, the integrated planning and scheduling problem as a special case of the general planning and scheduling problem is a multi criteria optimization problem [10], [11]. The objective of this integrated consideration is the optimization of the entire problem, whereby the objectives of the single planning domain recede into the background [12]. In general, the goal of the integrated planning and scheduling is to improve the competitiveness of the entire supply chain by improving the utilization of resources and decreasing the overall cost situation.

The integrated problem consists of the two subproblems production scheduling and outbound distribution scheduling. There is a problem formulation for each of the subproblems in the following subsections. Furthermore, we show how the submodels can be combined using a linking element and an overall objective function. It is obvious, that the integrated problem considered in this paper is NP-hard as it considers two subproblems, whereby each of this is NP-hard already [7]. A proof of this theorem can be found in [13].
A. Production Scheduling

We consider a job shop configuration for the production scheduling subproblem. The definition of the Job Shop Scheduling (JSS) problem is standardised and given by a number of authors with varying notations \([10],[14]-[16]\):

A set of \(J\) out of \(n\) Jobs with \(J = \{J\}_{i=1}^{n}\) have to be processed on a set of \(M\) out of \(m\) machines with \(M = \{M\}_{k=1}^{m}\). Each job \(J\) consists of a processing sequence of \(m\) operations \(O = \{O_1, O_2, ..., O_m\}\), which are also called tasks. All the tasks necessary to process one job have to follow the processing sequence \(O_1, O_2, ..., O_m\). Thus, there is a precedence relationship of tasks. There are \(P\) operations with \(P = \sum_{i=1}^{n} O_k\) in the entire production stage. The operation \(O_k\) belongs to job \(J\), which is processed on machine \(M_k\) having a processing time of \(p_k\). Furthermore, only one operation can be processed on a machine at the same time without interruption. The objective is to find a schedule, that outlines an operating sequence for each machine that optimises a particular function. The objective of the classical JSS problem is the minimisation of the makespan \(C_{\text{max}}\). The mathematical formulation of the JSS problem uses the following objective function:

\[
\min \left(C_{\text{max}} = \min \left(\max \left(t_{ik} + p_{ik}\right)\right) \right) \forall J_i \in J, M_k \in M
\]

Subject to:

\[
t_{ij} - t_{ij} \geq p_{ij} \quad \forall (i,j) \in R
\]

\[
t_{ij} - t_{ij} \geq p_{ij}, \text{ or } t_{ij} - t_{ij} \geq p_{ij} \quad \forall (i,j) \in (E_k, 1 \leq k \leq m)
\]

\[
t_{ij} \geq 0 \quad \forall i \in O
\]

Equation (2) ensures the precedence relationship of tasks belonging to one job. Equation (3) guarantees that each task has to be processed without interruption and that there are no re-entrant tasks. Furthermore, this constraint expresses that one task cannot be processed at the same time by two or more machines and that operation \(j\) can be processed before or after completion of operation \(j\) but not simultaneously. Thus, a set of operations \(E_k\) is defined for each machine \(M_k\) after finishing the scheduling procedure, which does not belong to one job but which has to be processed on each machine. The processing of tasks can start at \(t_{ik} \geq 0\) at the earliest (see (4)).

B. Outbound Distribution Scheduling

A limited number of identical transport vehicles with a limited capacity based at a central depot are assumed for the distribution stage of the IPODS problem \([17]\). The task of this fleet of vehicles is to deliver finished goods to a defined number of nodes in order to fulfill the individual required quantity of each single node. Closed loop routes and symmetric distances are the basic requirements for this distribution scheduling problem \([18]\), which is also known in the literature as the capacitated vehicle routing problem (CVRP). In analogy to the previously given definition of the JSS problem, the CVRP is also standardized and given by a number of authors with varying notations \([19]-[21]\):

A weighted graph \(G = (V,E)\) with a set of nodes \(V = H \cup \{0\}\) and a set of edges \(E = \{(0 \times H) \cup (H \times \{0\})\}\), whereby \(I \subset H \times H\) is the set of linking edges which is the basis for describing the standardized distribution scheduling problem. All nodes \(e,f \in H = \{1,2,...,h\}\) have to be supplied with required quantity \(d_e\) of a homogenous finished good starting and ending at a central depot. A number of \(V\) identical transport vehicles each with a maximal capacity of \(K_{\text{max}}\) are used for delivery. The costs for transportation of a finished good \(c_{ef} \geq 0\) from node \(e\) to node \(f\) with \(0 \leq e,f \leq n\) are independent from the utilization of the transport capacity. The objective is to find a schedule that minimizes the overall transportation costs. The mathematical formulation of this scheduling problem uses the following objective function:

\[
\min \sum_{(e,f) \in E} c_{ef} a_{ef}
\]

Subject to:

\[
\sum_{f \in N} a_{ef} = 1 \quad \forall e \in H
\]

\[
\sum_{f \in N} a_{f0} + \sum_{f \in N} a_{0f} = 2 \nu \quad \forall e \in H
\]

\[
\sum_{f \in N} a_{ef} + \sum_{f \in N} a_{ef} = 2 \quad \forall e \in H
\]

\[
0 \leq b_e \leq K_{\text{max}} \quad \forall e \in H
\]

\[
0 \leq d_e \leq K_{\text{max}} \quad \forall e \in H
\]

\[
a_{ef} \in \{0,1\} \quad \forall (e,f) \in E
\]

with

\[
a_{ef} = \begin{cases} 1, & \text{if } (e,f) \in E \text{ is used by one transport vehicle} \\ 0, & \text{otherwise} \end{cases}
\]

\[
b_e \text{ integer} \quad \forall e \in H
\]

\[
c_{ef} \geq 0 \quad \forall (e,f) \in H
\]

Equation (6) guarantees that each node is assigned to exactly one tour. Equation (7) ensures that each single tour starts and ends at the depot. Furthermore, each single node is served by transport vehicle exactly once (see (8)), which means, that load splitting is not allowed. Equations (9) and
(10) dealing with the capacity limitation of transport vehicles, meaning that the demand at a single node is not exceeding the transport capacity of a single vehicle. The binary variable \(a_{ef}\) is used in order to express if the edge between knot \(e\) and knot \(f\) is already used by a transport vehicle (see (11)). Furthermore, the splitting of order volume is not allowed and thus the amount of loaded finished goods is an integer value. There is a non negativity constraint for costs for the transport from node \(e\) to node \(f\) as positive as well as the total transportation costs consequently (12).

### C. Intermediate Storage as a Linking Element

The intermediate storage is a resource with a limited capacity \(\kappa_{\text{max}}\) for storing finished goods for a short time until the loading starts the distribution process. Order dependent costs \(c_{t_{\text{stor}}}\) occur when using the intermediate storage. The total intermediate storage costs \(c_{t_{\text{stor}}}\) are calculated as the sum of all order dependent costs. The objective function is about the minimization of the total intermediate storage costs:

\[
\min(c_{t_{\text{stor}}}) = \sum_{i=1}^{n} c_{t_{\text{stor}}}
\]

Subject to:

\[
s_i = s_{i-1} + s_i - s_i
\]

Equation (14) ensures that the capacity limitation is not exceeded. Again, there is the non negativity constraint (16), ensuring that all costs as well as the total costs are positive.

### D. Integrated Problem Formulation

The integrated mathematical formulation of the IPODS problem is the result of the combination of mathematical formulations of the subproblems. Nevertheless, there is a need for extensions and modifications in order to align one integrated problem formulation due to the fact that there are different objectives in each subproblem. Thus, we define the minimization of costs as the overall integrated objective and modify and transform the given mathematical formulations in the following.

There is no consideration of economic indicators like costs in the classical JSS problem. The consideration of costs or the transformation of process indicators into costs is a relevant aspect for calculation of efforts and benefits. Thus, we replace the existing objective function for the JSS problem. The new objective is to find a schedule, that minimizes the overall production costs \(c_{\text{prod}}\). The overall production costs comprise of fixed and variable costs \((c_{\text{prodfix}}\) and \(c_{\text{prodvar}}\)\) as well as setup costs \(c_{\text{prodset}}\). Furthermore, there are penalty costs \(c_{\text{prodpen}}\) included in the objective function in case of delayed finish of production.

In analogy to the JSS problem, we transform the objective function without changing the objective of minimization of the overall transportation costs. Thus, the overall transportation costs \(c_{\text{transp}}\) consist of fixed transportation costs \(c_{\text{transpfix}}\) occurring if a tour \(T_v\) is realized as well as variable transportation costs \(c_{\text{transpvar}}\) and penalty costs \(c_{\text{transppen}}\) in case of late delivery.

Now it is possible to give a mathematical formulation of the IPODS problem after modifying and extending the mathematical formulations of the subproblems. The following mathematical formulation of the IPODS includes elements from [22]-[26].

**Objective function:** Subject to:

\[
t_j - t_i \geq p_i \quad \forall (i, j) \in R
\]

\[
t_j - t_i \geq p_i \text{ or } t_i - t_j \geq p_j \quad \forall (i, j) \in E_k, 1 \leq k \leq m
\]

\[
t_i \geq 0 \quad \forall i \in \bar{O}
\]

\[
c_{\text{prodfix}}(i) \geq 0 \quad \forall i \in J \text{ and } \forall k \in M
\]

\[
c_{\text{prodvar}}(i) \geq 0 \quad \forall i \in J \text{ and } \forall k \in M
\]

\[
c_{\text{prodset}}(i) \geq 0 \quad \forall i \in J \text{ and } \forall k \in M
\]

\[
s_i = s_{i-1} + s_i - s_i
\]

\[
\kappa_{\text{max}} \geq s_i
\]

\[
c_{\text{stor}} \geq 0
\]

\[
\sum_{f \in N} a_{ef} = 1 \quad \forall e \in H
\]

\[
\sum_{f \in N} a_{f0} + \sum_{f \in N} a_{bf} = 2 \nu \quad \forall e \in H
\]

\[
\sum_{f \in N} a_{ef} + \sum_{f \in N} a_{ef} = 2 \quad \forall e \in H
\]

\[
0 \leq b_e \leq \kappa_{\text{max}} \quad \forall e \in H
\]

\[
0 \leq d_e \leq \kappa_{\text{max}} \quad \forall e \in H
\]
\[ a_{ef} \in \{0, 1\} \quad \forall (e, f) \in E \quad (33) \]

with
\[ a_{ef} = \begin{cases} 1, & \text{if } (e, f) \in E \text{ is used by one transport vehicle} \\ 0, & \text{otherwise} \end{cases} \]

\[ b_e \text{ integer} \quad \forall e \in H \quad (34) \]

with \[ b_e \text{ – Loading of a transport vehicle arriving in } e \]
\[ c_{\text{transfix}}^{\text{trans}} \geq 0 \quad \forall \nu \in T \quad (35) \]
\[ c_{\text{transvar}}^{\text{trans}} \geq 0 \quad \forall (e, f) \in H \quad (36) \]
\[ c_{\text{transper}}^{\text{trans}} \geq 0 \quad \forall i \in J \quad (37) \]
\[ A_i \in \{0, 1\} \quad \forall \nu \in T \quad (38) \]
\[ \sum A_i \geq 1 \quad \forall i \in J \text{ and } \forall e \in H \quad (39) \]
\[ c_{\text{prod}}^{\text{trans}} \leq t_v \quad \forall i \in J \text{ and } \forall \nu \in T \quad (40) \]

III. SOLUTION METHODS AND STRATEGIES

In analogy to other NP-hard problems, there are two basic approaches to solve the integrated problem stated in the previous section. The first approach is the reduction of the problem size and the examination of small problem instances. This allows the application of exact solution methods and the consideration of the entire integrated problem. This can be obtained by using e. g. a small number of machines and transport vehicles or the consideration of one planning period instead of rolling planning periods.

The second approach is the application of heuristics in order to achieve a good solution. Referring to [27], heuristics for the IPODS are separated in two groups, whereas interaction methods like collaboration and auction based approaches are one group. The second group of heuristics uses decomposition and integration, which are appropriate methods to split the integrated problem in smaller subproblems [28]. This group of heuristics is widely spread in the literature and applied in many IPODS with different scenarios (see e.g. [1] and [29] for an overview on the operational level and e.g. [9] as well as [30] considering the strategic and tactical level). After decomposition, the sub-problems can be solved separately (decoupled) or simultaneously (coupled). In case of a decoupled consideration, the results have to be integrated after solving each single sub-problem whereas the integration step is included in the coupled approach. Fig. 1 depicts a classification of solution methods and strategies referring to [27].
this, there is a calculation of relevant indicators for each integrated starting solution. Then, an iterative modification of the integrated starting solution starts in order to identify improvements of relevant indicators like costs after each modification. In case of an improvement of indicators, the modified solution is the new starting solution for the next iterative modification. The iterative modification and thereby the integration ends when there is no further improvement possible. Local search and sorting algorithms (e.g. [4]) as well as linear search (e.g. [3]) and evolutionary algorithms (e.g. [32]) are suitable methods for generating modified integrated schedules. In analogy to the decoupled approach, the improvement potential depends on the size of the neighborhood or the size of the population and the selected improvement method. Additionally, there is no guarantee for any improvement by applying improvement methods. Nevertheless, the coupled approach promises a higher potential improvement and provides a higher integration level in comparison to the decoupled approach.

An increased integration level and thus an increased performance of indicators are the motivation for the development of a new integrated planning method. This is due to the fact that already existing integrated planning methods provide an improvement potential up to 20% of operating costs in contrast to sequential planning (see e.g. [3], [4], [7] and [32]). Thus, the goal is to guarantee a highly coupled integration of the single planning problems in order to achieve an increased supply chain performance as well as an improved acceptance of the integrated planning result within the supply chain.

IV. MSDI FRAMEWORK

A generic working principle based on [2] was defined as a framework in order to guarantee a highly integrated planning result. Decomposition and Integration are applied in two different steps of the solution procedure without defining solution methods in detail. We call this MSDI framework, which is depicted in Fig. 2 and described afterwards.

The first step is the clustering of already known orders using e.g. the backward scheduling and a selection criterion like the delivery date assigned. For the backward scheduling the already known production time and the transport time of the orders are used. By using a classifier we receive a number of clusters as a first result. Each cluster contains orders with a defined range of remaining time until delivery date. This clustering can be defined as the first decomposition of the overall problem in a number of subproblems not concerning a functional decomposition but using a selection criterion. The following steps are applied for each cluster.

After this clustering we start a planning step in each cluster. This planning step is the second decomposition and comprises an isolated planning of production and distribution as described for the coupling approach. As there is no detailed definition of solution methods in a framework, already known planning methods from the respective planning domain depending on structural aspects are recommended. For example, dispatching rules or shifting-bottleneck-heuristics are applicable for production scheduling as well as the sweep-heuristics or nearest-neighbor-rule for generating a distribution schedule. The result of the planning step is an initial production and a transportation schedule considering all orders of the first cluster. These initial schedules are the input for the next step.

The goal of integration step 1 is to integrate the initial schedules of this cluster in order to achieve an initial integrated schedule. Therefore, e.g. sorting and evolutionary algorithms as well as permutation based rules and local search algorithms could be used. The aim is to put similar orders together and to reduce transportation or setup costs in this cluster. The result of the integration step 1 is an integrated cluster schedule in each cluster, which is used as an input for the next step. Afterwards, there is an integration of integrated cluster schedules into one integrated planning result in integration step 2. Therefore, unused machine or transportation time slots in an integrated cluster schedule can be checked if jobs can shift from other integrated cluster schedules. The goal of this inter schedule integration is to identify further improvements like load consolidation or machine utilization in order to prevent setup costs or low utilization transports subject to given restrictions. Different methods are applicable for the integration step 2.

Summarizing this working principle, it can be stated that the integrated planning result (obtained in integration step 2) is a combination of already integrated cluster results (obtained in integration step 1). Thus, there is a guarantee to achieve a higher integration level of subproblems considered.

V. MSDI HEURISTIC

We developed a detailed heuristic based on the MSDI framework in order to achieve further improvement of integrated solution methods and improvement potential. Thus, the definition of suitable methods is required for application in different steps.

The selection criterion for the clustering of orders is the due date. Thus, we develop a list with the most urgent due date on
the first position and a decreasing due date for other orders. Now there are two possibilities to cluster using the due date. The first option is to define a specific due date range for each cluster, e.g. one cluster comprises all orders with a due date within the next three days. The second option is to define a fixed number of orders within each cluster. We take the second option in this paper.

The goal of the planning step is to define performance benchmarks which are used as upper and lower bounds for both subproblems production and distribution scheduling. The benchmarks are applied in order to limit the number of alternative schedules to be generated in the integration step 1. Therefore, we consider all orders within one cluster and make a push and a pull scheduling. We use dispatching rules EDD and PT+WINQ+SL referring to [15] and [33] for production scheduling and set the result as a benchmark. The savings algorithm referring to [34] is used to generate a starting solution for the distribution scheduling which is improved by 2-opt and 3-opt procedure referring to [35]. The result of the push planning is a very good production schedule which is set as the lower bound for the production schedule and a very poor distribution schedule because the distribution scheduling is totally dominated by the production. This poor distribution schedule and its related performance is set as the upper bound for distribution scheduling. In contrast to this, the result of the pull planning is a very good distribution schedule which is set as the lower bound for the distribution schedule performance and a very poor production schedule which is set as the upper bound for the production schedule performance.

The integration step 1 is about generation of integrated schedules as alternative for both production and distribution. The goal of the integration step is to find an integrated cluster schedule with an improved overall performance and a higher integration level than a schedule, which is created by a sequential push or pull scheduling. The basic idea is to find an alternative schedule, whereas the performance of the respective subproblem is in the range between upper and lower bound and the overall performance has an improved value. Therefore, the sweep algorithm referring to [36] is a suitable as a methodical procedure to generate alternative schedules. The number of alternatives is equal to the number of nodes in the distribution network. One production schedule is created using a pull strategy for each of the alternative schedules generated. If there is a worse performance of the alternative schedule than the upper bound for the respective subproblem, the alternative schedule is eliminated. Thereby, the number of alternatives is limited for the further procedure. The result of the integration step 1 is an integrated cluster schedule for each cluster.

The goal of integration step 2 is the coordination of integrated cluster schedules in order to find an integrated planning result using the integrated cluster schedules. Therefore, a local search algorithm with a defined neighborhood is applicable. The task is to find swaps of jobs in the processing order or in the distribution schedule to prevent setup costs or to achieve load consolidation in the case of low utilized transport capacities.

VI. Evaluation
The evaluation of the MSDI heuristic is based on two different scenarios for the respective subproblems given in the literature linked by an intermediate storage. The objective function is to minimize total costs.

A. Order Generation
An instance generator was designed to create order data. Each order is represented by a tuple containing an individual order ID, a customer ID, a product type ID, the ordered amount of this product type, the calculated production due date, the calculated distribution due date, the calculated delivery due date and the release date. There are 17 customers and three different products in the scenario. The probability of initiating an order is equally distributed among the customers. The probability of ordering one of the three different product types by a customer is also equally distributed. The due dates and release dates are calculated based on an average utilization of 80% of the machines in the production scenario using a due date factor of 3 as referred in [33].

B. Production Scenario
The production scenario is based on a real world job shop scenario as referred in [37]. There are three different job shops whereas there are three identical machines in of them. Each machine within a job shop is able to do three different tasks with different but fixed processing times. Each job shop provides a queue for waiting jobs. Furthermore, there is an intermediate storage at the end of the production line containing finished jobs until the beginning of loading for delivery. Fig. 3 depicts the structure of the production scenario. Each job has to pass all job shops for production.

C. Distribution Scenario
The distribution scenario is also taken from the literature and comprises a distribution network with 17 nodes and one depot located in the centre (see [38]). The nodes are identical with the 17 biggest cities in Germany. The edges linking the cities are corresponding to the German motorway network. The intermediate storage is located at the depot. There is a limited fleet of identical transportation vehicles. The maximum load capacity is set to 8 units. The structure of the distribution network is given in Fig. 4.
D. Results

We used simulation experiments with 350 jobs for each of the 100 reputations for the evaluation of the MSDI heuristic and for comparison against sequential push and pull scheduling using the methods described in the previous section. Furthermore, penalties for a late end of production and thus for late delivery are calculated as a part of production costs.

As a result of the simulation experiments it turn out, that the costs of the MSDI heuristic are in the range of upper and lower bound for each subproblem (see the dotted lines in Fig. 5). Furthermore, it can be easily observed, that the best scheduling result of a subproblem is not the best starting point in case of an integrated consideration.

Additionally it became clear, that the overall costs for the integrated planning result obtained by the MSDI heuristic are on a lower level than the costs for the methods compared with. The overall costs obtained by the respective scheduling method are depicted in Fig. 6.

Summarizing the results it can be stated, that the MSDI heuristic is a suitable method for integrated scheduling. The MSDI heuristic provides an improvement potential of 17.7% in compared with Push EDD and 16.6% compared with Push PT+WinQ+SL. There is an improvement of 6.9% even in comparison with the Pull Savings heuristic.

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