

Elastic Stress Analysis of Composite Cantilever Beam Loaded Uniformly

A. Kurşun, M. Tunay Çetin, E. Çetin, H. Aykul

Abstract—In this investigation an elastic stress analysis is carried out a woven steel fiber reinforced thermoplastic cantilever beam loaded uniformly at the upper surface. The composite beam material consists of low density polyethylene as a thermoplastic (LDFE, f.2.12) and woven steel fibers. Granules of the polyethylene are put into the moulds and they are heated up to 160°C by using electrical resistance. Subsequently, the material is held for 5min under 2.5 MPa at this temperature. The temperature is decreased to 30°C under 15 MPa pressure in 3min. Closed form solution is found satisfying both the governing differential equation and boundary conditions. We investigated orientation angle effect on stress distribution of composite cantilever beams. The results show that orientation angle play an important role in determining the responses of a woven steel fiber reinforced thermoplastic cantilever beams and an optimal design of these structures.

Keywords—Cantilever beam, elastic stress analysis, orientation angle, thermoplastic.

I. INTRODUCTION

THE thermoplastic composites are getting progressively important owing to many advantages such as specific strength and high specific stiffness, strengthen impact resistance, develop fracture toughness. Moreover thermoplastic composites hold the unique characteristic which they may be remelted, reformed and reprocessed [1]. As a consequence of their low material costs and potential for high production rates, woven thermoplastic composites strike a chord in a wide array of sectors involving construction, aerospace, automotive, and furniture industries [2]. Experimental investigations on the forming of thermoplastic composites can be found in [3]-[7].

Residual stresses in the matrix are especially important in composites because they can cause strengthen the composite material and premature failure [8]. Jeronimidis and Parkyn [9] studied residual stresses in carbon fiber/thermoplastic matrix laminates. A numerical model for loading of the process carried thermal composite laminates is improved by Domban Hansen [10]. The finite element technique is used to find elasto-plastic stresses in composite structures [1]. Karakuzu et al. [11] practiced an elastic-plastic stress analysis in an aluminum matrix composite cantilever beam loaded by a uniform or single force. The thermal residual stresses in

injection molded thermoplastics by removing thin layers from specimens and measuring the resultant curvature or the bending moment are investigated by Akay and Özden [12]. Sayman et al. [8] researched residual stresses for a thermoplastic composite cantilever beam loaded by a bending moment.

In the present study, an elastic stress analysis is investigated in thermoplastic composite cantilever beam reinforced by woven steel fibers. In this study, an analytical solution is practiced for small plastic deformations. A closed-form solution is carried out by both boundary conditions and differential equation for an anisotropic system. In addition to the composite beam is studied with various orientation angles analytically. The composite beam can be supported to by using the residual stresses.

II. ELASTIC SOLUTION

An orthotropic uniformly loaded cantilever beam which is written by Lekhnitskii [13] can be seen in Fig. 1.

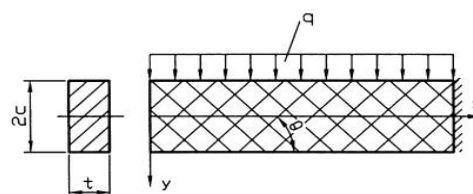


Fig. 1 Thermoplastic composite beam

The differential equation for an orthotropic cantilever beam in the plane stress case is written as,

$$a_{22} \frac{\partial^4 F}{\partial x^4} - 2a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{11} \frac{\partial^4 F}{\partial y^4} = 0 \quad (1)$$

where F is a stress function. The constants in (1) are given by Jones [14] as

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (2)$$

where a_{ij} are the components of the compliance matrix. The elements of compliance matrix are written as

$$\begin{aligned} a_{11} &= S_{11}m^4 + (2S_{12} + S_{66})m^2n^2 + S_{22}n^4 \\ a_{12} &= S_{12}(m^4 + n^4) + (S_{11} + S_{22} - S_{66})m^2n^2 \\ a_{22} &= S_{11}n^4 + (2S_{12} + S_{66})m^2n^2 + S_{22}m^4 \end{aligned} \quad (3)$$

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$$a_{16} = (2S_{11} - 2S_{12} - S_{66})nm^3 - (2S_{22} - 2S_{12} - S_{66})n^3m$$

$$a_{26} = (2S_{11} - 2S_{12} - S_{66})n^3m - (2S_{22} - 2S_{12} - S_{66})nm^3$$

$$a_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})m^2n^2 + S_{66}(m^4 + n^4)$$

where $S_{11} = 1/E_1$, $S_{22} = 1/E_2$, $S_{12} = \nu_{12}/E_1$, $S_{66} = 1/G_{12}$,
 $m = \cos \theta$, $n = \sin \theta$.

A stress function solution for the differential equation is chosen as [11],

$$F = \frac{d}{6}x^2y^3 + \frac{e}{12}xy^4 + \frac{f}{20}y^5 + \frac{k}{2}xy^2 + \frac{g}{6}y^3 + \frac{b}{2}x^2y + \frac{a}{2}x^2 \quad (4)$$

Substituting (4) into (1), gives

$$x(-4a_{16}d + 2ea_{11}) + y(4a_{12}d + 2a_{66}d - 4a_{16}e + 6fa_{11}) = 0 \quad (5)$$

So as to satisfy the equation, each term of x and y must be equal to zero as e

$$e = sd \quad (6)$$

wheres = $2a_{16}/a_{11}$ and $f = rd$

$$r = \frac{2a_{16}s - 2a_{12} - a_{66}}{3a_{11}} \quad (7)$$

The stress component become,

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = dx^2y + sdy^2 + rdy^3 + kx + gy \quad (8)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \frac{d}{3}y^3 + by + a \quad (9)$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -dxy^2 - \frac{sd}{3}y^3 - ky - bx \quad (10)$$

The boundary conditions are,

$$\sigma_y(-c) = \frac{-q}{t}$$

$$\sigma_y(+c) = 0$$

$$\tau_{xy}(\pm c) = 0$$

σ_x is not zero at the free end.

$$\int_{-c}^c \sigma_x t dy = 0 \text{ at the free end.}$$

$$\int_{-c}^c \sigma_x t y dy = 0 \text{ at the free end.}$$

where t and 2c are the thickness and height of the beam, respectively. When solved the above equations, the stress components become,

$$\sigma_x = -\frac{q}{2I} \left(x^2y + ry^3 + sxy^2 - \frac{1}{3}c^2sx - \frac{3}{5}rc^2y \right) \quad (11)$$

$$\sigma_y = -\frac{q}{2I} \left(\frac{1}{3}y^3 - c^2y \right) - \frac{q}{2t} \quad (12)$$

$$\tau_{xy} = -\frac{q}{2I} \left(-xy^2 - \frac{1}{3}sy^3 + c^2x + \frac{1}{3}sc^2y \right) \quad (13)$$

where I is inertia of the cross section of the beam.

III. PRODUCTION OF THE COMPOSITE BEAM

The composite beam material arises from woven steel fibers and low density polyethylene such as a thermoplastic (LDFE, f.2.12). Granules of the polyethylene placed in die and their temperature is increased to 160° by using electrical resistance. Afterwards, the material is kept under 2.5 MPa pressure in 5 min at the same temperature. The temperature is reduced to 30° 3min under 15 MPa. Hence a polyethylene layer is acquired. The woven steel fibers beam are settled between two plastic layers and manipulated to in the same way. Mechanical properties and yield points of the beam material are determined by using the Instron test machine and strain gauges, as given in Table I.

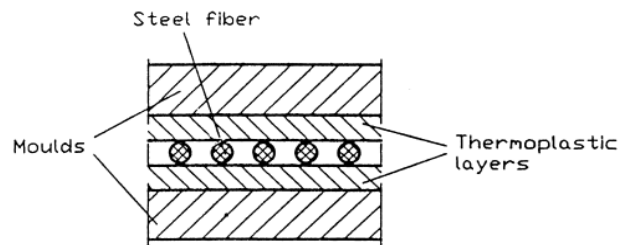


Fig. 2 Production of the composite beam

TABLE I
 MECHANICAL PROPERTIES OF THE COMPOSITE BEAM

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν
38000	1300	360	0.25

IV. RESULTS AND DISCUSSIONS

In this study, elastic stress analyses are carried out on a cantilever beam made of woven steel fiber reinforced thermoplastic by using an analytical solution. The thermoplastic composite beam is loaded uniformly. The density of the uniform force is selected at a small value 0.2 N/mm. In Table II, the variations of stress for thermoplastic composite beam subject to uniformly load is presented for different angle. As it can be seen from this table that, the stress is tensile at upper side while it is compressive at lower side, and it is linearly distributions along to thickness of composite beam. The stress is the highest value for orientation angle equals to zero, where upper side of composite beam and it is the lowest value for orientation angle equals to 30° lower side of composite beam. The stress is equals to zero at neutral axis of the composite beam for all orientation angles. As a result of the bending moment, there is a compression of the lower while there is elongation of the upper part as if can be also seen from Table II.

TABLE II
STRESS DISTRIBUTION FOR DIFFERENT ORIENTATION ANGLE

h(mm)	Stress (MPa)			
	0°	15°	30°	45°
-6.0	21.499375	21.254905	15.593922	15.611352
-5.5	19.622836	19.488222	14.305804	14.315402
-5.0	17.768452	17.720350	13.014719	13.018148
-4.5	15.934207	15.951397	11.720935	11.719710
-4.0	14.118089	14.181470	10.424724	10.420205
-3.5	12.318083	12.410678	9.126354	9.119753
-3.0	10.532174	10.639130	7.826096	7.818471
-2.5	8.758350	8.866933	6.524220	6.516478
-2.0	6.994596	7.094195	5.220995	5.213893
-1.5	5.238898	5.321025	3.916690	3.910835
0.0	0.000000	0.000000	0.000000	0.000000
0.5	-1.743614	-1.773819	-1.305923	-1.303770
1.0	-3.489242	-3.547530	-2.611576	-2.607421
1.5	-5.238898	-5.321025	-3.916690	-3.910835
2.0	-6.994596	-7.094195	-5.220995	-5.213893
2.5	-8.758350	-8.866933	-6.524220	-6.516478
3.0	-10.532174	-10.639130	-7.826096	-7.818471
3.5	-12.318083	-12.410678	-9.126354	-9.119753
4.0	-14.118089	-14.181470	-10.424724	-10.420205
4.5	-15.934207	-15.951397	-11.720935	-11.719710
5.0	-17.768452	-17.720350	-13.014719	-13.018148
5.5	-19.622836	-19.488222	-14.305804	-14.315402
6.0	-21.499375	-21.254905	-15.593922	-15.611352

V. CONCLUSIONS

In this investigation, an elastic stress analysis is applied in a thermoplastic cantilever beam loaded uniformly. The following conclusions can be derived from this study:

1. The maximum stress values are seen on both upper and bottom surfaces of the thermoplastic cantilever composite beam.
2. The stress is the highest value for orientation angle equals to zero at upper side of composite beam.
3. The stress is tensile at upper side while it is compressive at lower side, and it is linearly distributions along to thickness of composite beam.

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