

# A Review: Comparative Analysis of Different Categorical Data Clustering Ensemble Methods

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**Abstract**—Over the past epoch a rampant amount of work has been done in the data clustering research under the unsupervised learning technique in Data mining. Furthermore several algorithms and methods have been proposed focusing on clustering different data types, representation of cluster models, and accuracy rates of the clusters. However no single clustering algorithm proves to be the most efficient in providing best results. Accordingly in order to find the solution to this issue a new technique, called Cluster ensemble method was bloomed. This cluster ensemble is a good alternative approach for facing the cluster analysis problem. The main hope of the cluster ensemble is to merge different clustering solutions in such a way to achieve accuracy and to improve the quality of individual data clustering. Due to the substantial and unremitting development of new methods in the sphere of data mining and also the incessant interest in inventing new algorithms, makes obligatory to scrutinize a critical analysis of the existing techniques and the future novelty. This paper exposes the comparative study of different cluster ensemble methods along with their features, systematic working process and the average accuracy and error rates of each ensemble methods. Consequently this speculative and comprehensive analysis will be very useful for the community of clustering practitioners and also helps in deciding the most suitable one to rectify the problem in hand.

**Keywords**—Clustering, Cluster Ensemble methods, Co-association matrix, Consensus function, Median partition.

## I. INTRODUCTION

CLUSTERING is one of the most vital and an underpinning process in Data Mining. It also plays an imperative role in the other fields such as Machine Learning process, Pattern Recognition, Information retrieval, Spatial Data Extraction, Image Processing and World Wide Web. Data clustering mainly concerns with how to group a set of objects based on their proximity in vector space. The main objective of the cluster analysis is finding similarities between data according to the uniqueness found in the data and grouping related data objects into clusters. An excellent clustering produces a high superiority clusters with high intra class similarity and low inter class similarity. A large variety of clustering algorithms which are of well established such as K-Means, EM (Expectation Maximization) based on the spectral graph theory

[1], K-modes, GAClust [2], CobWeb [3], STIRR [4], Robust Clustering Algorithm for Categorical Attributes ROCK [5], CLICK [6], Clustering Categorical Data Using Summaries CACTUS [7], COOLCAT [8], CLOPE [9], Squeezer [10], Differential fuzzy clustering, Standard Deviation of Standard deviation Roughness algorithm, Frequency of attribute value combination algorithm and some hierarchical clustering algorithms like Divisive algorithm, LIMBO [11], single link, Fuzzy C-Means, Fuzzy C-Medoids [12]-[14] etc. are emerged over earlier periods. Conversely it is known that there is no single clustering method is capable of providing accurate and appropriate cluster results [14]. Since by applying a clustering algorithm to the data set it works on the basis of the internal criteria i.e. similarity or dissimilarity measures used in that algorithm. At the same time if two different clustering algorithms were applied to the same data set consequently it will results in very different clusters solutions. Therefore this critical concern is very difficult to evaluate the exact clustering results. In cluster analysis the evaluation of the results are associated to the use of Cluster Validity Indexes which is used to measure the quality of clustering results [14]. Nevertheless to overcome this serious issue combining multiple clustering approaches in an ensemble framework may allow one to take advantage of the strengths of individual clustering approaches. The general outlier of the cluster ensemble is done by achieving the solutions from the different base clustering which are then aggregated to form a final partition [13]. This Meta level approach involves these two major tasks of generating a cluster ensemble and then producing a final partition normally referred as the consensus function [15], [13]. Precisely the great challenge in clustering ensemble is the definition of most suitable consensus function which is capable of improving the consequences of single clustering algorithm. Accordingly the rest of this paper is followed with the methodical process of the different ensemble methods and concludes with the hope of that this comparative study will be very useful for the evaluation of future clustering ensemble methods.

## II. CLUSTER ENSEMBLE TECHNIQUE OVERVIEW

Cluster ensembles are supposed to be a robust and most perfect alternative to single clustering runs. It is the process of grouping up of multiple clustering solutions to obtain a consensus result by merging different partitions based upon well defined rules. It also provides for a visualization tool to examine cluster number, membership, and boundaries. In this sense ensemble clustering is a potential approach to generate more accurate clusters than might be possible using an

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individual clustering approach [15]. It generally involves two major tasks as Generation step in which generating several clustering solutions by applying clustering algorithm are done and the Consensus step through which final cluster partition is produced. The general basic construction of the cluster ensemble method was shown in Fig. 1 [13].

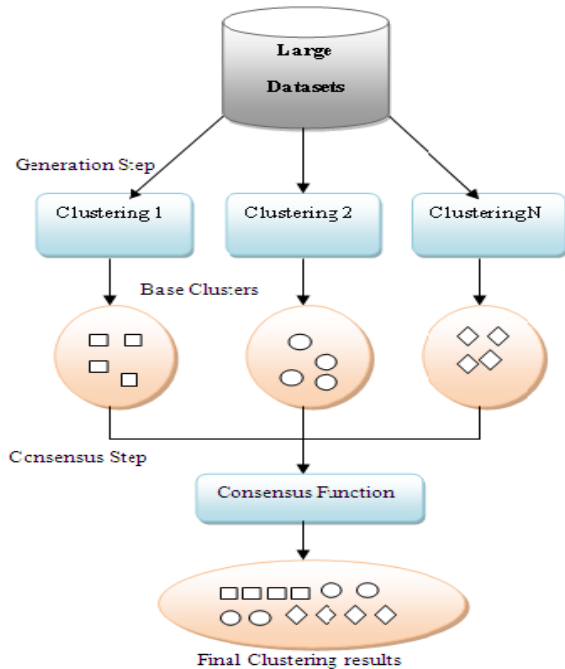


Fig. 1 Basic Process of Cluster Ensembles

#### A. Generation Steps

In this generation step there are no constraints about how the partitions must be obtained. Since during the creation process [14] different clustering algorithms, or the same algorithm with different parameters initialization, different object representations, and subsets of objects or projections of the objects on different subspaces can be used to produce the different base cluster solutions as shown in Fig. 2. In spite of this process even a weak clustering algorithms are capable of producing high quality consensus clustering in concurrence with the proper consensus function.

#### B. Consensus Steps

In this step consensus functions are developed and are made available for gaining the ultimate data partition from the different base clustering results. This consensus function has the large capability of improving the results of the single clustering algorithms. It involves two approaches such as object co-occurrence and median partition. In the first approach it deals with the measuring the number of occurrences of an object in a single cluster and also it analysis how many times two objects belongs together in the same cluster. In the second approach it deals with the partition that maximizes the similarity with all partitions in the cluster ensemble. The complexity of this median partition method is the improper analysis of the dissimilarity measures. Even though these approaches are evolved still there are several

questions raised such as, Which clustering algorithms should be used?, Which are the correct parameters?, Which are the exact dissimilarity measures?, Which is the best heuristic approach to solve the problem or to come close to the solution? [14]. Therefore a bunch of clustering ensemble methods is projected over recent years to answer those questions.

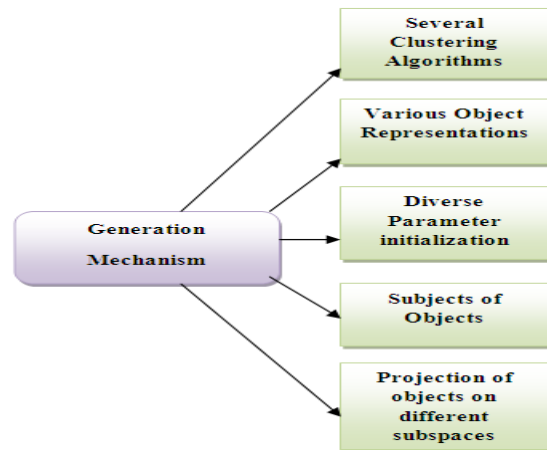


Fig. 2 Primary Cluster Ensemble Generation Steps

### III. DIFFERENT CLUSTER ENSEMBLE METHODS

The following sections will present the some diverse collection of cluster ensemble methods. And also for each method its systematic working process and features are elucidated.

#### A. Hybrid Fuzzy Cluster Ensemble (HFCE)

This Hybrid Fuzzy Ensemble method is mainly proposed for enhancing the performance and quality of the tumor clustering from bio-molecular dataset. Here fuzzy theory is implemented into the cluster ensemble paradigm in order to accurately denote the samples corresponding to different types of cancer data. Fuzzy theory is mainly used to generate the fuzzy matrices in the ensemble. This ensemble framework was emerged with four kinds of hybrid fuzzy ensembles as HFCE-I, HFCE-II, HFCE-III, and HFCE-IV.

HFCE-I method [52] uses the Affinity Propagation (AP) algorithm to extract the base clustering results on sample dimension of the dataset. This in turn exemplifies the fuzzy matrices in the ensemble which is based upon the fuzzy membership functions. Initially the base samples are randomized by the AP algorithm.

HFCE-II method uses the AP algorithm for clustering on the attribute dimension of the dataset. It mainly chooses the random attribute from each clusters then generates the subspace. The newly produced subspace of the dataset is applied with fuzzy C-means algorithm to generate the set of fuzzy matrices. Finally it obtains the consensus function for aggregating all the fuzzy matrices in order to generate the final results.

HFCE-III initially applies AP algorithm to generate base clustering on the attribute dimension and generates subspaces

of the dataset similar to HFCE-II. Then it again uses the AP algorithm to be applied on the sample subset of the dataset and extracts the cluster results. This in turn leads to applying appropriate consensus function to obtain the final clustering solutions.

HFCE-IV generally integrates the techniques used in both HFCE-I and HFCE-II to extract the base clustering results. Then to obtain the ultimate final cluster solution, fuzzy C-means or the normalized cut algorithm is used to précis the fuzzy matrices of both methods and obtain the results.

Hence this Hybrid fuzzy cluster ensemble method is well suited for performing the tumor clustering from the cancer gene expression datasets.

### B. Knowledge Based Cluster Ensemble Method (KCE)

Knowledge based Cluster Ensemble technique [53] mainly integrates the prior knowledge of the information in the dataset into the cluster ensemble process. In particular the prior knowledge about the data is illustrated in the Pairwise constrains in which it helps in enhancing the quality and the accuracy of the clustering results. Initially it adopts the spectral clustering term to generate the base clustering solutions. From the given dataset  $D^b (b \in \{1 \dots, B\})$  with  $n$  number of data objects, the spectral clustering partitions these into  $K$  classes. This clustering algorithm first constructs the an Affinity Matrix  $F$  whose entry is defined as given below,

$$f_{i,j} = \gamma(x_i, x_j) \quad (1)$$

where  $\gamma(x_i, x_j)$  denotes the Euclidean distance between the sample points  $x_i$  and  $x_j$ . Then it creates the diagonal matrix  $R$  whose matrix entry  $r_{ii} (i \in \{1 \dots n\})$  is measured as follows,

$$r_{ii} = \sum_{j=1}^n f_{ij} \quad (2)$$

Spectral Clustering also constructs the normalized matrix  $T$  which is denoted as given below,

$$T = F^{-\frac{1}{2}} R F^{-\frac{1}{2}} \quad (3)$$

Furthermore it selects the largest eigenvectors of  $T$  and then generates the matrix  $Z$ . In the next stage, KCE evaluates the confidence factor for each clustering solution. This factor will be high if the clustering solutions suits most of the Pairwise constrains or else the confidence factor will be low. Finally KCE generates a matrix by considering all the results of the membership clustering solutions to extract the ultimate cluster result through the consensus partition in which spectral clustering itself serves as a consensus function. Hence this KCE method achieves the best performance in majority of the cancer datasets, along with the Novertis multi-tissue dataset, SRBCT dataset and St. Jude dataset.

### C. Weighted Cluster Ensemble Method (WCE)

A Weighted cluster is a subset of data points together with a vector of weights such that the points in the cluster are close to each other. In this ensemble method [16] Locally Adaptive

Clustering algorithm was used and it discovers clusters in subspaces spanned by different combinations of dimensions through local weightings of features. The major benefit of this Locally Adaptive clustering was that it avoids the risk of loss of information encountered in global dimensionality reduction techniques. This ensemble method consists of two approaches as follows.

#### 1. Weighted Similarity Partitioning Algorithm (WSPA)

This technique [16], [51] starts initially by running locally adaptive clustering algorithm  $m$  times with different  $h$  values. Then for each data point  $x_i$  the weighted distance from the cluster  $c_{ls}$  is calculated by the below formula as,

$$d_{il} = \sqrt{\sum_{s=l}^D w_{ls} (x_{is} - c_{ls})^2} \quad (4)$$

where  $d_{il}$  is the larger corresponding capability credited to the cluster  $c_{ls}$  and  $w_{ls}$  is the weighted clusters. Then the probabilistic estimation for embedding the clustering result is given by,

$$P(c_l | x_i) = \frac{D_i - d_{il} + 1}{k D_i + k - \sum_l d_{il}} \quad (5)$$

After that to compute the similarity between the data points  $x_i$  and  $x_j$  both cosine similarity measure and Kullback-Leibler (KL) divergence measures were applied as given below,

$$S(x_i, x_j) = \frac{P_i^i P_j}{|P_i| |P_j|} \quad (6)$$

The above formula denotes the cosine similarity measure in which it detects the probability vectors associated to  $x_i$  and  $x_j$ . Then the distance between  $x_i$  and  $x_j$  was computed using KL divergence formula as follows,

$$d(x_i, x_j) = (\frac{1}{2} \sum_{l=1}^k P_{il} \log_2 \frac{P_{il}}{P_{jl}}) + (\frac{1}{2} \sum_{l=1}^k P_{jl} \log_2 \frac{P_{jl}}{P_{il}}) \quad (7)$$

Finally a consensus function that guides the computation of the consensus partition is defined by the formula  $\psi = \frac{1}{m} \sum_{i=1}^m S_i$ . After this complete graph  $G = (V, E)$  where  $|V| = n$  and  $V_i || x_i$  was constructed. Main aim and feature of this method is to generate robust and stable cluster solutions.

#### 2. Weighted Bipartite Partitioning Algorithm (WBPA)

This approach mainly maps the problem of finding a consensus partition to a bipartite graph partitioning problem. It overcomes the shortcomings of Weighted Similarity Partitioning Algorithm [16] in which it assigns only low similarity values to both pairs of a data set where as Weighted Bipartite Partitioning Algorithm has the ability to differentiate the two cases by modeling both instance-based and cluster-based similarities. The starting process of this approach was similar to the Weighted Similarity Partitioning algorithm. Only additional measure in this method is the formation of the matrix using the vectors of posterior probabilities. Hence based on that matrix a bipartite graph to which the consensus

partition problem maps. Thus the bipartite graph was constructed with number of vertices and each represents the cluster of the ensemble.

#### D. K-Means Cluster Ensemble based On Center Matching Scheme (KMCE)

In this method center matching scheme [17] is projected for constructing a consensus function in the K-Means cluster ensemble learning. The well known K-Means algorithm has a striking characteristic feature due to its computational simplicity. Here it was chosen for the ensemble. The working process of this method starts by extracting the output sequence of K-Means cluster centers using the K-Means clustering. Then it randomly selects the cluster sequence as a reference one and rearranges the other cluster sequences according to the reference sequence. Let  $C_r = \{c_{r1}^1, c_{r2}^2, c_{r3}^3\}$  be the reference sequence and  $C_p = \{c_{p1}^1, c_{p2}^2, c_{p3}^3\}$  be the any cluster sequence. Then a weight matrix between the two sequences is constructed as follows,

$$W_{rp} = \begin{bmatrix} 2.3 & 2.8 & 2.7 \\ 4.6 & 3.9 & 1.7 \\ 2.0 & 0.9 & 3.3 \end{bmatrix}$$

To find an efficient center matching, Hungarian algorithm is used through the formula given below,

$$\min = \sum_{i=1}^k \sum_{j=1}^k W_{ij}^{rp} B_{ij} \quad (8)$$

where  $B_{ij}$  denotes the indicator variables to determine the center matching between the two sequences. Labeling the data using these matched cluster sequences [17] is done. Hence it results in producing multiple partitions or clustering which do not need matching again. Finally these multiple clustering is combined to consensus clustering using some combinational rules such as voting rules [18].

#### E. Extended Evidence Accumulation Clustering Ensemble Method (EEAC)

This method is highly employed to select the more robust cluster in the final ensemble. It generally selects the best performing cluster results rather than choosing all the generated cluster solutions for the ensemble. Those clusters which satisfy the stability criteria can participate in the cluster ensemble which was measured using Normalized Mutual information (NMI). A stable cluster [19] is the one that has high likelihood of reoccurrence across multiple applications of the clustering method. After applying the stability threshold to the each cluster then selected clusters are used to construct the co-association matrix. The stability of the cluster  $C_i$  is measured as given below,

$$\text{Stability}(C_i) = \frac{1}{M} \sum_{l=1}^M \text{NMI}_i \quad (9)$$

where  $M$  is the number of data partitions available in reference set and  $i$  denotes the  $i^{\text{th}}$  partition in that same reference set. In

the next step for truly recognize the pair wise similarity a co-association matrix was computed by,

$$C(i, j) = \frac{n_{ij}}{\max(n_i, n_j)} \quad (10)$$

where  $n_i$  and  $n_j$  are the number present in remaining (after stability threshold) clusters for the  $i^{\text{th}}$  and  $j^{\text{th}}$  data points, respectively. Also,  $n_{ij}$  counts the number of remaining clusters which are shared by both data points indexed by  $i$  and  $j$ , respectively. Finally hierarchical method is applied over the generated matrix to mine the final partition. Hence the main outstanding aspects of this Extended Evidence Accumulation clustering Ensemble approach [19] is the stability measurement for each clusters and the accuracy in deciding the final partition.

#### F. Squared Error Adjacent Matrix Clustering Ensemble Method (SEAM)

This new method mainly focus on how to combine the multiple data partitions to get a consistent partition for a given data set using the information obtained in the different clustering results. This Squared Error Adjacent Matrix algorithm [20], [21], [51] is mainly based upon the similarity matrix which is defined as the co-association matrix. It has the high potential of finding the final data partition without predefining the number of clusters or any value of the thresholds when similarity matrix is given. This matrix is constructed by measuring the co-occurred times of the data pairs in the same cluster, the  $N$  data partitions of  $n$  data objects are mapped into an  $n \times n$  co-association matrix which is expressed below,

$$S(i, j) = \frac{n_{ij}}{N} \quad (11)$$

where  $n_{ij}$  is the number of times the pair  $(i, j)$  is located in the similar cluster among the  $N$  data partitions. The value of  $S(i, j)$  represents the similarity of the data objects  $x_i$  and  $x_j$ . Thus the Squared Error Adjacent Matrix ensemble method can find the final partition of the data set over the given similarity matrix with low complexity.

#### G. Adaptive Spectral Clustering Ensemble Selection Method (ASCE)

This method can adaptively access the number of component members which is not owned by many of the ensemble methods. In this, system spectral clustering [22], [23] is used as basic learner of the ensemble system. Spectral clustering ensemble approach is based on re-sampling technique and Population Based Incremental Learning algorithm [24]. Hence this search approach is more stable and faster to solve more complex optimization problems. It mainly denotes that random variables are independent. The distribution density was computed through the product of the random variables. Updated probability measure was given below,

$$P_{l+1}(x) = \prod_{i=1}^n P_{l+1}(x_j) \quad (12)$$

However Population Based Incremental Learning algorithm is mainly used to detect the optimum clustering ensemble for its plainness and robustness. After that re-sampling the clustering set in accordance to the probability vector is done to compute the consensus partition. Finally the clustering set which posses the probability of being selected above the threshold level is picked for ensemble. The key feature of this method is that it is highly effective when the ensemble size is large.

#### H. Link based Clustering Ensemble Method (LCE)

This link based cluster ensemble method denotes the discovery of unknown values in the cluster co-association matrix [25]. The matrix analyses the pair wise-similarity between the objects and if similarity occurs it enter the value as "1" otherwise the entries are left unknown and simply record as "0". This Link based clustering ensemble methodology [13] involves three stages as

- Creating base clustering to form a cluster ensemble.
- Generating the Refined cluster association Matrix RM using a link based similarity algorithm.
- Producing final data partition by exploring special graph partitioning technique.

Refined Matrix (RM) [13] is the enhanced variation of the co-association matrix. For each clustering  $\prod_t t = 1 \dots M$  and their corresponding clusters  $C_1^t \dots C_{k_t}^t$  where t is the number of clusters in the clustering results. The association degree  $RM(x_i, cl) \in [0,1]$  that data point  $x_i \in X$  has with each cluster  $cl \in \{C_1^t \dots C_{k_t}^t\}$  is estimated as follows,

$$RM(x_i, cl) = \begin{cases} 1, & \text{if } cl = C^t * (x_i), \\ sim(cl, C^t * (x_i)), & \text{otherwise,} \end{cases} \quad (13)$$

where  $C^t * (x_i)$  is a cluster label to which data point  $x_i$  belongs. In addition,  $sim(Cx, Cy) \in [0,1]$  denotes the similarity between any two clusters  $Cx, Cy$ , which can be discovered using the following link-based algorithm. The process of the link based algorithm entirely depends on the Weighted Triple Quality factor [13] in which it mainly denotes the construction of weighted graphs  $G = (V, W)$  where V represents the set of vertices denoting each cluster and W represents the set of weighted edges between the clusters. To determine the quality of the clusters it's mandatory to find the rarity of links connected with each cluster in a network. Hence the WTQ measure of cluster  $Cx, Cy \in V$  with respect to each triple  $C_k \in V$  is estimated by,

$$WTQ_{xy}^k = \frac{1}{w_k} \quad (14)$$

The accumulative WTQ score from all triples (1..q) between clusters  $Cx, Cy$  can be found using the below measure,

$$WTQ_{xy} = \sum_{k=1}^q WTQ_{xy}^k \quad (15)$$

Then the similarity between the clusters  $Cx, Cy$  can be estimated by,

$$Sim(Cx, Cy) = \frac{WTQ_{xy}}{WTQ_{max}} * DC \quad (16)$$

where  $WTQ_{xy}$  is the value of any two clusters and  $WTQ_{max}$  is the maximum of  $WTQ_{xy}$  and  $DC \in [0,1]$  is a constant delay factor. Finally by applying consensus function to the RM a final clustering partition can be exploited. Thus the main key feature is that it is a powerful method for decomposing an undirected graph with good performance being exhibited in diverse application areas.

#### I. Selective Spectral Clustering Ensemble Method (SELSCE)

This approach is introduced to construct the selective ensemble in order to explore the diverse and qualified final cluster partition. To generate the selective ensemble the initial step is to pick the good and efficient base clustering solution through spectral clustering technique [26] and also it produces the individual learner based on the approach given in reference to [27]. Here NMI (Normalized Mutual Information) is used to measure the diversity of the component clustering as given below,

$$NMI(\prod_a, \prod_b) = \frac{\sum_{h=1}^{k_a} \sum_{l=1}^{k_b} n_{hl} \log_2 [N * \frac{n_{hl}}{n_l * n_h}]}{\sqrt{[\sum_{h=1}^{k_a} n_h \log_2 \frac{n_h}{N}] * [\sum_{l=1}^{k_b} n_l \log_2 \frac{n_l}{N}]}} \quad (17)$$

where  $\prod_a, \prod_b$ , are the two clustering then  $k_a$  and  $k_b$  are the number of clusters in  $\prod_a$  and  $\prod_b$  respectively.  $n_{hl}$  represents the number of instances in the  $h^{th}$  cluster of  $\prod_a$  and  $l^{th}$  cluster of  $\prod_b$  concurrently. In order to find the greater diversity between the two clustering the NMI measure was slightly changed and denoted it as Div [26].

$$Div = 1 - NMI \quad (18)$$

However diversity of the cluster accuracy also an important factor to be considered. The function which takes into account both accuracy and diversity simultaneously is given below,

$$Sim = -(Div * \ln Div + (1 - Div) * \ln(1 - Div)) \quad (19)$$

After the above process the final selection of best cluster for ensemble is achieved by two steps such as,

- Computing the pair-wise distance between the component clusters thereby discarding the nearest one as determined by its distance.
- Repeated progress for the remaining clustering until all of them is either selected or discarded.

Therefore this ensemble technique achieves better performance among other traditional clustering algorithms. And an efficient feature in this method is that the computational cost of the selection process is low.

### J. Bayesian Cluster Ensemble Method (BCE)

Bayesian cluster ensemble method was emerged for being a mixed membership model for learning cluster ensembles [28]. It basically denotes the Bayesian approach which deals with Bayes' theorem with two distinct interpretations. This Bayesian Cluster Ensemble method generates a Bayesian graph model from the base clustering solutions. From the generative model it is assumed that  $\theta_i$  is sampled from Dirichlet distribution with the parameter  $\alpha$  and the consensus cluster  $h$  for each  $z_{ij}$  selected from  $\theta_i$ , separately. After this generation process in order to estimate the mixed-membership of each object to the consensus clusters Variation inference [28] is calculated as follows,

$$q(\theta_i, z_i | \gamma_i, \varphi_i) = q(\theta_i | \gamma_i) \prod_{j=1}^m q(z_{ij} | \varphi_{ij}) \quad (20)$$

where  $\gamma_i$  is the Dirichlet distribution parameter and  $\varphi_i = \{\varphi_{ij}, [j]_1^M\}$  are said to be a the discrete distribution parameters. Then Generalized Bayesian Cluster Ensemble algorithm [28] was proposed in which it deals with combining both the base clustering results and feature vectors of original data points to yield a consensus clustering. Hence the outstanding feature of this Generalized Bayesian Cluster Ensemble method is its versatile nature due to its applicability to several variants of the cluster ensemble problem including missing value cluster ensembles, row distributed and column distributed cluster ensembles.

### K. Three Staged Cluster Ensemble Method (TSCE)

This ensemble method [30] is mainly used for clustering the mixed data points in which the datasets contain both numerical and categorical attributes. The main aim of this technique is to find relatively high quality cluster and then to utilize an aggregation method to produce the final clustering result that minimizes the number of disagreements [29], [30], [51]. As the name implies this technique is composed of the following three stages of the process.

- Building BASE clusters and this process repeats until it detects that no samples are left in the data sets.
- Refining the Initial cluster is started by selecting the BASE of the second cluster obtained and calculates its similarity with all the samples in the first cluster.
- Verification is done by refining the BASE cluster to focus whether the solution can be further improved or not.

However, three staged ensemble method was mainly constructed as a core modeling method and are used for generating a series of clustering results with diverse conditions for a given dataset.

### L. Exact Method based Cluster Ensembles (EXAMCE)

This method was mainly proposed to produce the high quality ensemble solutions better than the local search methods and it also to outperform the best known technique for the Minimum Sum of Squares Clustering (MSSC) problems [31] on several benchmark data sets. Exact Method based Cluster Ensemble technique seeks to optimally recombine the partially generated solutions of different base

clustering results to extract better feasible solutions to the original problem.

This process was iteratively made through local search heuristics until it finds no more further improvement can be done. The recombination step involves the search for the globally optimal solution of a restricted Set-Covering Problem [31] with a side constraint on the number of clusters in the final solution. Solving the set covering problem (SCP<sub>R</sub>) [32] optimally is still a NP-Hard problem but practically it can be solved quite easily. The Set covering problem contains the matrix  $A_B$  (having only  $q$  columns) that only involves the groups returned as solutions by the base clusters such as given below,

$$(SCP_R) \min \sum_{i=1}^q c([A_B]_i) x_i \quad (21)$$

$$s.t \quad \begin{cases} A_{B^x} \geq e \\ \sum_{i=1}^q x_i = k, \\ x_i \in B \quad i = 1 \dots q \end{cases}$$

After this measure the duplicates are eliminated from the clusters selected by  $x$  which in turn produces a new set of clusters that are of highly feasible. Then the newly formed cluster is localized to evaluate the cost and then expanded to return the final partitioning solution. The major striking feature of this ensemble algorithm is its capability to solve the problems involving large number of clusters especially in the application area of fraud detection. It also performs well on illuminating the clustering structure as measured by the Adjacent Rand index and in other combinatorial optimization problems.

### M. Effects of Resampling Method and Adaptation on Clustering Ensemble Efficacy

In this approach, Non-adaptive and Adaptive Resampling schemes for the integration of the multiple independent and dependent clustering solutions were proposed. In this adaptive technique [33], [51] the individual partitions in the cluster ensembles are linearly produced by clustering specially selected subsamples of the given dataset. This adaptive scheme involves the process of Resampling, Relabeling, and finally as an upshot of the relabeling the consistency index of the cluster partitions are computed. In Non-adaptive Resampling scheme [34]-[36] the main goal is to obtain a reliable clustering with measurable uncertainty from a set of different  $k$ -means partitions. The key idea of the approach is to aggregate multiple partitions produced by clustering of pseudo-samples of a dataset. Furthermore the non-adaptive technique involves two methods such as Bootstrap in which sampling the subsets of data is done with replacement and Sub sampling method in which it deals with sampling of the data without replacement. To generate the similar labels of the clusters throughout the ensemble partitions a new technique called Relabeling is applied to each partition in the ensemble using some fixed reference partitions. The most inherent feature of this technique is the Resampling process of the original data.

#### N. Projective Clustering Ensembles Method (PCE)

In this respect, the Projective Clustering Ensembles (PCE) [37] is defined to deal with the high dimensionality and multiple clustering issues. PCE is formulized as an optimization problem and is designed to satisfy the desirable requirements on independence from the specific cluster ensemble algorithm and the skill to handle the hard and soft data clustering. These projective clusters [38]-[40] are mainly referred as the subsets of several input data having different subsets of features associated to them. The formal definition of the problem of projective clustering ensembles (PCE) [41] is presented here. The main aspire of this PCE is to define methods that exploit the information provided by an ensemble of projective clustering solutions (i.e., *projective ensemble*) to compute a projective consensus clustering. The information provided by any projective ensemble is two-fold which are as follows,

1. Data are grouped in clusters
2. Features assigned to clusters

After the two-fold method the techniques applied in this projective clustering approach is Multi-Objective Evolutionary algorithm [42] based Projective clustering and the Expectation Maximization based projective clustering Ensemble process. Hence the main salient features of this method are the capability of handling the high dimensionality and multi view data issues.

#### O. An Improved Method for Multi-Objective Clustering Ensemble Algorithm (IMOCLE)

In this approach, Improvement of the multi-objective cluster ensemble algorithm which is expressed as IMOCLE [43] was proposed. This method mainly shows the superiority of the other techniques and the capability of finding the optimum number of clusters and accuracy. It refers to both multi-objective methods [44] and cluster ensemble techniques in optimization process. The major systematic procedure of this algorithm is as follows

- a) Initial base cluster results are obtained by applying several different clustering algorithms on the given dataset.
- b) Several objective functions are optimized in the development process. This objective function can be obtained through the calculation of the similarity between the cluster partitions as follows,

$$Sim(\Pi_i) = \frac{1}{n} \sum_{j=1}^n S(\Pi_i, \Pi_j) \quad (22)$$

- c) In addition to the above step special crossover [45] is applied to combine two parents using cluster ensemble technique.
- d) Finally set of cluster ensembles are generated.

#### P. A Generalized Adjusted Rand Index for Cluster Ensemble (ARImp)

In this approach a new method called Adjusted Rand Index [46], [51] was proposed between similarity matrix and cluster partition to measure the consistency between the different set of clustering results and their associated consensus matrix in a

cluster ensemble. *ARI* measure is highly defined as the adjusted form of Rand Index used mainly for the purpose of grouping the elements in the dataset. From the mathematical point of view it is stated that this measure is related to the accuracy evaluation even if the class labels are not applicable. This measure is highly meaningful in analyzing the cluster performance without the underlying labels rather than with few similarity matrices between the partitions. The Adjusted Rand Index (*ARI*) measure [47], [48] is define as follows,

$$S_0 = \sum_{i=1}^{Kp} \sum_{j=1}^{Kq} \binom{N_{ij}}{2}, \quad S_1 = \sum_{i=1}^{Kp} \binom{N_i}{2}$$

$$S_2 = \sum_{j=1}^{Kq} \binom{N_j}{2}, \quad S_3 = \frac{2s_1 s_2}{N(N-1)}$$

$$ARI(P, Q) = \frac{S_0 - S_3}{0.5(S_1 + S_2) - S_3} \quad (23)$$

where  $P = \{P_1, P_2, \dots, P_{Kp}\}$  and  $Q = \{Q_1, Q_2, \dots, Q_{Kq}\}$  be the two partitions on a data set  $X$  with  $N$  objects and the  $N_{ij}$  are the number of objects in each cluster partitions. After finding the *ARI* measure in addition to preserving the desirable properties of *ARI*, filtering method to serve for identifying less effective cluster ensemble method was applied. This approach was experimented on the most popular UCI data sets.

#### Q. Fuzzy Clustering Ensemble Algorithm for Partitioning Categorical Data (FCE)

In this approach, the fuzzy clustering ensemble algorithm [49] is proposed mainly to make use of the relationship degree between different attributes for pruning a part of the features in the data set. Pruning is highly mandatory as it prevents the surplus and unwanted attributes from reducing the efficiency of the algorithm through declining accuracy rates. The systematic process of this Fuzzy clustering ensemble algorithm was as follows,

- a) By setting the initial parameters numbers of base clusters are generated.
- b) Pruning [50] the redundant attributes is done.
- c) Searching for the subsets of Descartes.
- d) Choosing one object from each of the subsets as initial cores.
- e) Compute the membership degree of the cluster and value of the objective function.
- f) Finally search for the nearest object from to the clusters from the initial core and sets the collection of cluster ensembles.

Thus the main key feature of this fuzzy clustering ensemble is to obtain the optimal number of clusters and also it establishes the relationship between the objects in the dataset under the unsupervised circumstances.

#### IV. COMPARISON OF CLUSTER ENSEMBLE METHODS

This section exemplifies [51] the comparison of the previously described different ensemble methods based on different parameters. The main thought of this contrast is not

to examine which is the best clustering ensemble method but to differentiate the methods based on its behavioral performance and its features in which it helps the users to select the appropriate cluster ensemble method for solving their problem on hand. In Table I, we summarized the previously denoted ensemble methods in relate to its highlighting features and limitations of each technique which are as follows:

#### A. Ensemble Size

Ensemble is the method of cumulating the cluster partitions together in order to improve the individual clustering algorithms thereby it produces efficient results in accuracy. This Ensemble size denotes the number of clusters obtained in the ensemble through merging of the different base clustering solutions to form the final partition. This size varies in two forms as fixed size in which the cluster length is defined previously where as in variable size the ensemble size has no limitation.

#### B. Types of Consensus Function Used

Consensus function comprises of two types such as Object Co-occurrence method and Median Partition method. First type deals with measuring the number of Co-occurrences of an object in a single cluster and the second type deals with the partition that maximizes the similarity with all partitions in the cluster ensemble.

#### C. Dimensionality

This property denotes the capacity of the datasets used for the experimental analysis of the ensemble methods. Capacity of the datasets are classified into small and large by analyzing through the number of data points, attributes values, classes, features and patterns occurring in the dataset.

#### D. Type of Datasets Used

Datasets used for the experimental setup comprised of three types such as Numerical Datasets and Categorical Datasets and Mixed numerical & categorical datasets. First type consists of only a bunch of numerical data points, the second type involves the text data points related to the particular domain whereas the third type of datasets deals with combination of the first and second type.

#### E. Algorithm Used for Base Clustering

Base clustering algorithms are selected and used in each method mainly for the repeated runs of that single clustering algorithm with several sets of parameter initializations. This base clustering is mainly used for the generation of cluster ensembles. Apart from this a different clustering algorithms can also be used as a base clustering to perform heterogeneous ensemble creation.

Thus the following Table I presents the salient features of each ensemble methods. We investigated their abilities and compare them based on the Ensemble size, types of consensus function, dimensionality, types of dataset, and the Algorithm used for base cluster generation.

TABLE I  
 SUMMARIZED CLUSTER ENSEMBLE METHODS

| Clustering Ensemble Methods | Ensemble Size | Type of Consensus Function used | Dimensionality (size of the dimensions used in the datasets) | Type of Dataset used            | Algorithm used to build Base Clustering | Features  |
|-----------------------------|---------------|---------------------------------|--|---------------------------------|---|---|
| HFCE                        | Fixed         | Median Partition                | Small and Large  | Categorical                     | Affinity Propagation                    | Performs best accuracy results and produces stable clusters |
| KCE                         | Fixed         | Median Partition                | Large  | Categorical                     | Spectral Clustering                     | Usability of Prior Knowledge of dataset                     |
| WSPA                        | Fixed         | Object Co-occurrence            | Small and Large  | Categorical                     | Locally Adaptive Clustering             | Generation of Robust and Stable Clusters                    |
| KMCE                        | Variable      | Median Partition                | Small  | Categorical                     | K-Means                                 | Computational Simplicity                                    |
| EEAC                        | Fixed         | Object Co-occurrence            | Large  | Categorical                     | K-Means                                 | Higher Stability and accuracy in clusters                   |
| SEAM                        | Fixed         | Object Co-occurrence            | Small  | Categorical                     | K-Means                                 | Low Complexity  |
| ASCE                        | Variable      | Median Partition                | Small and Large  | Categorical                     | Spectral Clustering                     | Effective for Complex optimization problems                 |
| LCE                         | Fixed         | Object Co-occurrence            | Small and Large  | Categorical                     | K-Modes                                 | Efficient in discovery of unknown values in Cluster matrix  |
| SELSCE                      | Variable      | Object Co-occurrence            | Small  | Categorical                     | Spectral Clustering                     | Computational cost of Selection process is low              |
| BCE                         | Fixed         | Object Co-occurrence            | Small and Large  | Categorical                     | K-Means                                 | Versatile Nature due to its applicability                   |
| TSCE                        | Variable      | Object Co-occurrence            | Small  | Mixed numerical and categorical | K-Means                                 | Spotting most likely number of Clusters automatically.      |
| EXAMCE                      | Variable      | Object Co-occurrence            | Small and Large  | Categorical                     | K-Means                                 | Efficient clustering in the area of fraud detection system  |
| RMACE                       | Fixed         | Object Co-occurrence            | Small  | Categorical                     | K-Means                                 | Resampling of the original data                             |
| PCE                         | Variable      | Median Partition                | Small and Large  | Categorical                     | Projective Clustering                   | Handling high dimensionality and multi view data issues     |
| IMOCLE                      | Fixed         | Object Co-occurrence            | Small and Large  | Categorical                     | K-Means                                 | Capability of finding optimal number of clusters            |
| ARImp                       | Variable      | Object Co-occurrence            | Small  | Categorical                     | K-Means                                 | Expression of consistency between the clusters.             |
| FCE                         | Fixed         | Median Partition                | Small  | Categorical                     | K-Means                                 | Maintains Relationships between objects in datasets         |



In order to examine the quality of several Cluster ensemble methods, experiments were performed on the several UCI benchmark repositories, real world datasets and artificial datasets. Table II shows the experiments performed by the above mentioned cluster ensemble methods and compared their average accuracy and error rates.

The empirical results of cluster ensemble algorithms on several real, artificial and UCI datasets clearly exemplifies that many of the ensemble techniques achieved highest accuracy on Iris, Wine, Zoo and Mushroom datasets and also many of the algorithms produced lowest accuracy rate on Glass dataset. In general most of clustering ensemble methods needs to improve their accuracy levels.

Finally Fig. 3 clearly states that the accuracy rates of individual cluster ensemble methods and finally it helps the clustering researchers as well as practitioners in deciding the further improvement of the ensemble methods.

#### V. CONCLUSION

Cluster Ensembles have been came into sight as a recent offspring for rectifying the negative aspects of the individual clustering consequences. This technique was mainly emerged as a high-flying method to enhance the stability, robustness, individuality, and accuracy of unsupervised learning solutions. This integration process of the ensemble method is really helpful and acts as bedrock for detecting and compensating the possible errors in single clustering algorithms. Consequently this proportional study reveals some of the different categorical cluster ensemble approaches including their

systematic functioning process and salient features of each method along with the average accuracy and error rates of each technique. Hence the original contribution of this paper is the methodical work flow of each techniques and the comparative table denotes differential analysis, characteristics, and limitations of the diverse ensemble methods along with the graphical representation of the accuracy levels of different ensemble methods. The comparison result proves that the many of the proposed works in cluster ensemble technique faces accuracy problem on different real world and artificial datasets. This investigation makes better understanding for the readers and also hopes to be more legible and useful for the society of clustering researchers to innovate more remarkable and efficient clustering ensemble methods. And hence most of the ensemble approach needs to improve their accuracy level; therefore, further progressing of accuracy can be an imperative research in future.

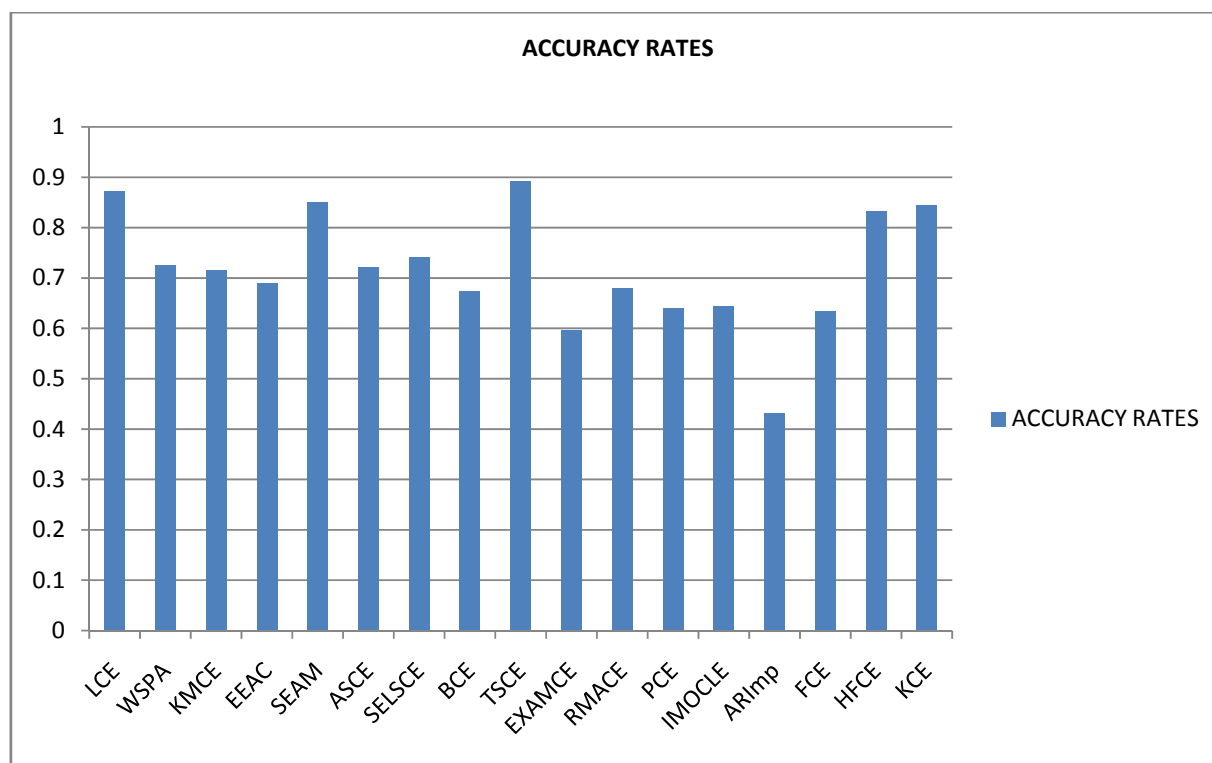


Fig. 3 Statistical Analysis of the Average Accuracy rates of several Ensemble methods

TABLE II  
COMPARISON OF ACCURACY AND ERROR RATES IN DIFFERENT CLUSTERING ENSEMBLE METHODS

| Ensemble Methods | Datasets       | Accuracy | Average Accuracy Rate | Average Error Rate | Ensemble Methods | Datasets        | Accuracy | Average Accuracy Rate | Average Error Rate |
|------------------|----------------|----------|-----------------------|--------------------|------------------|-----------------|----------|-----------------------|--------------------|
| LCE              | Zoo            | 0.94     | 0.873                 | 0.127              | TSCE             | Iris            | 0.94     | 0.893                 | 0.107              |
|                  | Lymphography   | 0.79     |                       |                    |                  | Wine            | 0.93     |                       |                    |
|                  | Soyabean       | 0.75     |                       |                    |                  | Votes           | 0.87     |                       |                    |
|                  | Vote           | 0.89     |                       |                    |                  | Cancer          | 0.96     |                       |                    |
|                  | Breast Cancer  | 0.97     |                       |                    |                  | Mushroom        | 0.89     |                       |                    |
|                  | Mushroom       | 0.89     |                       |                    |                  | Zoo             | 0.93     |                       |                    |
|                  | 20Newsgroups   | 0.78     |                       |                    |                  | Cleve           | 0.84     |                       |                    |
|                  | KDDCup99       | 0.98     |                       |                    |                  | Credit Approval | 0.79     |                       |                    |
| WSPA             | Three Gaussian | 0.98     | 0.726                 | 0.274              | EXAMCE           | Thyroid         | 0.84     | 0.596                 | 0.404              |
|                  | Iris           | 0.79     |                       |                    |                  | Wine            | 0.73     |                       |                    |
|                  | WDBC           | 0.59     |                       |                    |                  | Glass           | 0.41     |                       |                    |
|                  | Breast Cancer  | 0.80     |                       |                    |                  | Ionosphere      | 0.47     |                       |                    |
|                  | Letter(A,B)    | 0.61     |                       |                    |                  | Segmentation    | 0.51     |                       |                    |
|                  | SatImage       | 0.41     |                       |                    |                  | Iris            | 0.69     |                       |                    |
|                  | Spam2000       | 0.90     |                       |                    |                  | Wine            | 0.73     |                       |                    |
|                  | Vehicle        | 0.45     |                       |                    |                  | Wine            | 0.73     |                       |                    |
| KMCE             | Waveform       | 0.71     | 0.715                 | 0.285              | PCE              | ION             | 0.79     | 0.641                 | 0.359              |
|                  | Ionosphere     | 0.70     |                       |                    |                  | Star/Galaxy     | 0.51     |                       |                    |
|                  | Wine           | 0.95     |                       |                    |                  | Iris            | 0.96     |                       |                    |
|                  | Iris           | 0.88     |                       |                    |                  | Wine            | 0.83     |                       |                    |
|                  | WDBC           | 0.75     |                       |                    |                  | Glass           | 0.47     |                       |                    |
|                  | Liver disorder | 0.57     |                       |                    |                  | Ecoli           | 0.76     |                       |                    |
|                  | Breast Cancer  | 0.95     |                       |                    |                  | Yeast           | 0.41     |                       |                    |
|                  | Wine           | 0.97     |                       |                    |                  | Segmentation    | 0.44     |                       |                    |
| EEAC             | Yeast          | 0.47     | 0.690                 | 0.310              | IMOCLE           | Letter          | 0.33     | 0.432                 | 0.568              |
|                  | Glass          | 0.48     |                       |                    |                  | Isolet          | 0.95     |                       |                    |
|                  | Bupa           | 0.58     |                       |                    |                  | Gisette         | 0.72     |                       |                    |
|                  | Iris           | 0.89     |                       |                    |                  | Shapes          | 0.68     |                       |                    |
|                  | Wine           | 0.70     |                       |                    |                  | Waveform        | 0.51     |                       |                    |
|                  | Breast Cancer  | 0.96     |                       |                    |                  | Chowdary        | 0.92     |                       |                    |
|                  | Iris           | 0.88     |                       |                    |                  | Gordon          | 0.89     |                       |                    |
|                  | Wine           | 0.96     |                       |                    |                  | West            | 0.50     |                       |                    |
| ASCE             | Segmentation   | 0.73     | 0.721                 | 0.279              | ARImp            | Laiho           | 0.51     | 0.635                 | 0.365              |
|                  | Heart          | 0.78     |                       |                    |                  | Chen            | 0.70     |                       |                    |
|                  | Lung           | 0.65     |                       |                    |                  | Yeoh            | 0.40     |                       |                    |
|                  | WDBC           | 0.91     |                       |                    |                  | Bredel          | 0.60     |                       |                    |
|                  | SatImage       | 0.68     |                       |                    |                  | Glass           | 0.27     |                       |                    |
|                  | Ionosphere     | 0.70     |                       |                    |                  | Iris            | 0.91     |                       |                    |
|                  | Vehicle        | 0.38     |                       |                    |                  | Vehicle         | 0.15     |                       |                    |
|                  | Sonar          | 0.54     |                       |                    |                  | Wine            | 0.40     |                       |                    |
| SELSCE           | Iris           | 0.87     | 0.742                 | 0.258              | FCE              | Yellow-Small    | 0.74     | 0.832                 | 0.168              |
|                  | Lung           | 0.60     |                       |                    |                  | Zoo             | 0.53     |                       |                    |
|                  | WDBC           | 0.82     |                       |                    |                  | Wine            | 0.85     |                       |                    |
|                  | Segmentation   | 0.72     |                       |                    |                  | Dermatology     | 0.80     |                       |                    |
|                  | SatImage       | 0.70     |                       |                    |                  | Breast tissue   | 0.62     |                       |                    |
|                  | Iris           | 0.89     |                       |                    |                  | WDBC            | 0.93     |                       |                    |
|                  | WDBC           | 0.88     |                       |                    |                  | Leukemia        | 0.91     |                       |                    |
|                  | Ionosphere     | 0.71     |                       |                    |                  | Lung Cancer     | 0.87     |                       |                    |
| BCE              | Pima           | 0.66     | 0.675                 | 0.325              | HFCE             | Normal Tissues  | 0.85     | 0.845                 | 0.155              |
|                  | Glass          | 0.55     |                       |                    |                  | Wine            | 0.86     |                       |                    |
|                  | Bupa           | 0.56     |                       |                    |                  | Iris            | 0.93     |                       |                    |
|                  | Wine           | 0.71     |                       |                    |                  | Heart           | 0.60     |                       |                    |
|                  | Balance        | 0.53     |                       |                    |                  | Breast Cancer   | 0.90     |                       |                    |
|                  | Segmentation   | 0.59     |                       |                    |                  | Lymphoma        | 0.91     |                       |                    |
|                  | Zoo            | 0.53     |                       |                    |                  | Leukemia        | 0.92     |                       |                    |
|                  |                |          |                       |                    |                  |                 |          |                       |                    |

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