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A source point distribution scheme for wave-body interaction problem

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Abstract—A two-dimensional linear wave-body interaction problem can be solved using a desingularized integral method by placing free surface Rankine sources over calm water surface and satisfying boundary conditions at prescribed collocation points on the calm water surface. A new free-surface Rankine source distribution scheme, determined by the intersection points of free surface and body surface, is developed to reduce numerical computation cost. Associated with this, a new treatment is given to the intersection point. The present scheme results are in good agreement with traditional numerical results and measurements.

Keywords—Source point distribution, panel method, Rankine source, desingularized algorithm

I. Introduction

WAVE-BODY interaction flow can be represented as a fluid boundary integral of a simple Green function. The discretisation of the flow is obtained by a desingularized boundary integral method allowing free surface source points to be placed above the calm water surface. The free surface integration domain is infinite, giving to an inherent limitation in the wave radiation condition, while the numerical free surface integration domain covered by Rankine sources is bounded. Therefore truncation of the free surface integration domain might cause reflection which can disturb the wave propagation progress. The accuracy of numerical results lies on the density of source points and the truncation of the free surface. The increase of source point density will accordingly increase computation cost. Traditionally, an auxiliary boundary over the free surface is divided into inner and outer domains. Collocation points are evenly distributed in the inner domain but increase exponentially in the outer domain. The numerical computation can be finished before the generated wave reaches the truncated boundary. With the use of this technique, satisfactory results can be obtained and the wave reflection due to the truncation can be postponed. This method was originally introduced by Lee [1] and has been widely developed [2], [3], [5]–[8]. If the simple Green function in the fluid boundary integral is replaced by a free surface Green function, the fluid boundary integral reduces to the body surface integral and the free surface integral is canceled. Numerical simulation based on the free surface Green function method is immediately obtained.

In the present paper, a new source point distribution scheme is proposed by employing a special treatment of intersection points. Instead of an evenly spaced distribution, the source point separation varies in the inner domain. Since the water wave is generated from the intersection points, the source point separation in the vicinity of the intersection points must be carefully selected. The present scheme, which utilizes body surface panel to determine source points separation, aims to increase numerical efficiency and result accuracy as well as to delay wave reflection. Second order Adam-Bashforth scheme is applied in time stepping iteration. Comparisons with existing data are provided to validate computational accuracy.

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II. Problem Formulation

We consider a two-dimensional floating body undergoing forced and linear oscillation in a fluid of infinite depth. This fluid motion is in a coordinate frame Oxy centred at the middle point of the water line of the body. The plane $y = 0$ coincides with the calm water surface which is denoted by S_f. $\hat{S}_f$ denotes an auxiliary free surface boundary over the calm water surface S_f. D denotes the fluid domain. S_b denotes the body surface. n denotes the unit normal vector field of $S_b \cup S_f$ pointing away from the fluid domain. The fluid flow is assumed to be irrotational. Thus the velocity potential ϕ of the fluid motion problem is subject to the Laplace equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \text{ in } D, \tag{1}$$

the kinematic body surface boundary condition

$$\frac{\partial\phi}{\partial n} = V_b \cdot n \text{ on } S_b, \tag{2}$$

the linearized kinematic free surface condition

$$\frac{\partial\eta}{\partial t} - \frac{\partial\phi}{\partial y} = 0 \text{ on } S_f, \tag{3}$$

and the linearized dynamic free surface condition

$$\frac{\partial\phi}{\partial t} + g\eta = 0 \text{ on } S_f. \tag{4}$$

Here V_b denotes the body surface velocity, $\eta = \eta(x, y, t)$ is the free surface wave elevation and g is gravitational acceleration. The velocity of the fluid motion vanishes at infinity, that is

$$\nabla\phi \to 0. \tag{5}$$

The conditions (1)-(5) and desingularized discretization gives rise to the boundary value equations

$$\phi(\mathbf{p}_i, t) = \sum_{j=1}^{N_F} \sigma^F_{i,j} \ln r_{i,j} + \sum_{j=1}^{N_B} \sigma^B_{i,j} \int_{\triangle_{S_j}} \ln r_i ds \tag{6}$$

and

$$\frac{\partial \phi(\mathbf{p}_i,t)}{\partial n} = \sum_{j=1}^{N_F} \sigma_{i,j}^F \frac{\partial \ln r_{i,j}}{\partial n} + \sum_{j=1}^{N_B} \sigma_{i,j}^B \int_{\triangle_{S_j}} \frac{\partial \ln r_i}{\partial n} ds \quad (7)$$

for $j = 1, ..., N_f + N_b$, where N_F and N_b are panel numbers of S_b and S_f, $\{\mathbf{p}_i\}_{i=1}^{N_b+N_f}$ are source points distributed on $S_b \cup S_f$, $\triangle_{S_j}$ is the jth panel of the body surface, $r_{i,j} = |\mathbf{p}_i - \mathbf{q}_j|$ is the distance between the collocation point $\mathbf{p}_i$ and the isolated source point $\mathbf{q}_j \in \hat{S}_f$, $r_i = |\mathbf{p}_i - \mathbf{q}|$ denotes the distance between the collocation point $\mathbf{p}_i$ and the source point $\mathbf{q}$ continuously distributed in $\triangle_{S_j}$, $\sigma_{i,j}^F$ denotes the source strength with respect to the source point $\mathbf{q}_j$ and $\sigma_{i,j}^B$ represents for source strength with respect to the source panel $\triangle_{S_j}$.

The iteration of the velocity potential and velocity starts from the trivial initial condition $\phi = 0$ and $\nabla\phi = 0$. In every time step, the velocity potential and its derivative on $S_f \cup S_b$ can be obtained from solving the equations (2)-(7).

Similar to [10], the auxiliary free surface boundary $\hat{S}_f$ is divided into an inner domain containing 50 source points and an outer domain containing 30 source points, but the distance between neighboring isolated source points in the inner domain is defined as

$$\text{dis}_{\text{in}}(\mathbf{q}_{i+1}, \mathbf{q}_i) = \frac{L_b \alpha_i}{\omega^2}, \quad i = 1, ..., 49, \quad (8)$$

where L_b is the length of the body surface panel $\triangle_{S_1}$ ending at an intersection point, ω is the body oscillation frequency, and the parameter α_i is defined by Table I. The 30 source points in the outer domain are distributed in the following format:

$$\text{dis}_{\text{out}}(\mathbf{q}_{i+1}, \mathbf{q}_i) = \text{dis}_{\text{in}}(\mathbf{q}_{50}, \mathbf{q}_{49}) \times 1.05^{\frac{i(i+1)}{2}}, \quad i = 1, ..., 29. \quad (9)$$

The distribution of the source points in the inner domain are determined by the body surface panel length L_b, since the panel $\triangle_{S_1}$ are critical for the numerical convergence and result accuracy. Equation (8) is frequency-dependent and allows a longer inner regain compared to the traditional distribution. This distribution scheme can produce surface wave covering nearly 5 wave lengths in the inner domain and over 100 wave lengths in the outer range. Generally, satisfactory results can be produced before the wave reaches the truncated boundary and long-time simulation results can be obtained by simply adding more source points in the outer range. The source distribution scheme (8) using a nearly linear increment is fundamentally different from the constant distance source distribution scheme. In total, this method only needs 160 points which is a significant reduction compared to the 260 points in traditional method.

Additionally, different to the double collocation points treatment [4] with respect to an intersection point, only a single collocation point is allocated just at the intersection point position. That is, the intersection point is simply treated as a body collocation point.

TABLE I
THE VALUE OF α_i FOR LINEAR PROBLEM

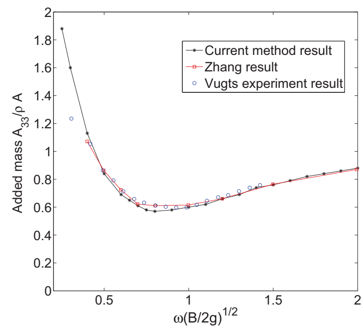
i	1	2	3-5	6-8	9-10	11-12	13-14	15-16	17-18	19-20	21-50
α_i	1.2	2.8	4	8	12	16	20	24	32	40	48

III. RESULTS

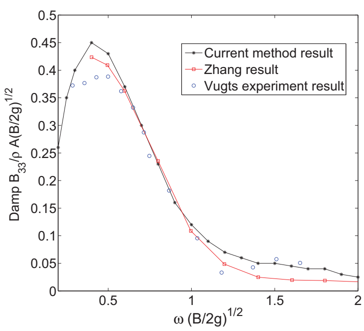
In the numerical simulation, the floating body is respectively selected to be a circular cylinder of the radius to draft ratio $R/T = 1.0$, a box with the beam to draft ratio $B/T = 2.0$. The body surface is equally divided into N_b panels. The numerical scheme converges rapidly as the panel number N_b increases. Specially there is little difference of the numerical results between $N_b = 36$ and $N_b > 36$. Therefore the body panel number $N_b = 40$ is selected throughout the numerical test in this paper.

The motion amplitude is 10% of the draft for all cases. The numerical simulation (2)-(7) produces the velocity potential ϕ and then hydrodynamics force applied on the oscillating body. The added mass and damping coefficients are obtained by applying the Fourier analysis on the force. The mass and damping coefficients are compared with the experimental data [9] and numerical results based on traditional source distribution method [10]. Figs. 1-4 show that results produced from the proposed method are in good agreement with experimental data and numerical results from traditional source distribution scheme. The damping coefficient results with respect to the cylinder and box in sway motion are even better than those produced from the traditional method(Figs. 2 and 4).

Furthermore, we examine the local response around the floating circular cylinder with the radius $R = 10m$. Fig. 5 presents the dynamic pressure distribution around the cylinder when $\omega(B/2g)^{1/2} = 1.0$ and $t = 10s$. The pressure is symmetrical around the body and smoothly varies between the two intersection points. The present results also show the efficiency of the intersection point treatment in the present investigation.

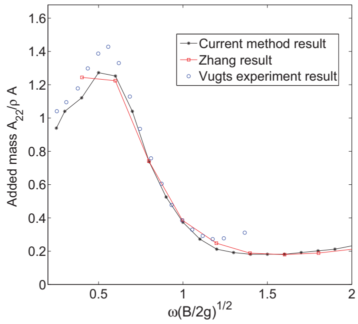


(a)

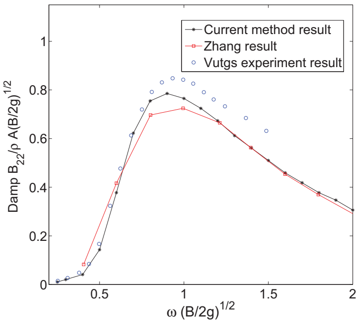


(b)

Fig. 1. Added mass (a) and damping coefficient (b) of cylinder in heave

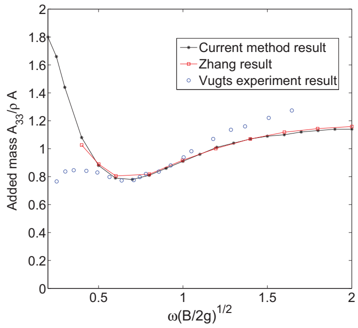


(a)

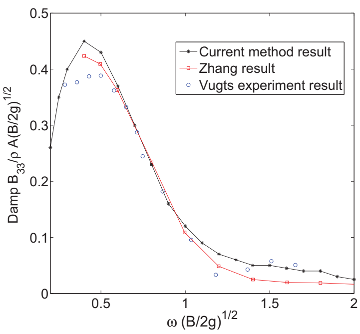


(b)

Fig. 2. Added mass (a)and damping coefficient (b) of cylinder in sway



(a)

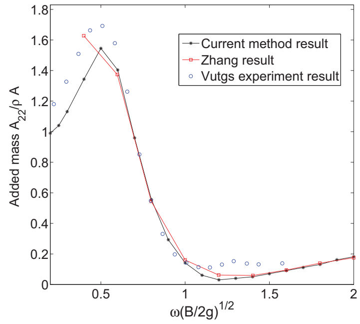


(b)

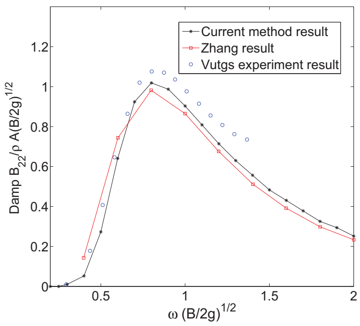
Fig. 3. Added mass (a) and damping coefficient (b) of box in heave

IV. Conclusions

A source distribution scheme over calm water surface together with a new intersection point treatment are developed



(a)



(b)

Fig. 4. Added mass (a) and damping coefficient (b) of box in sway

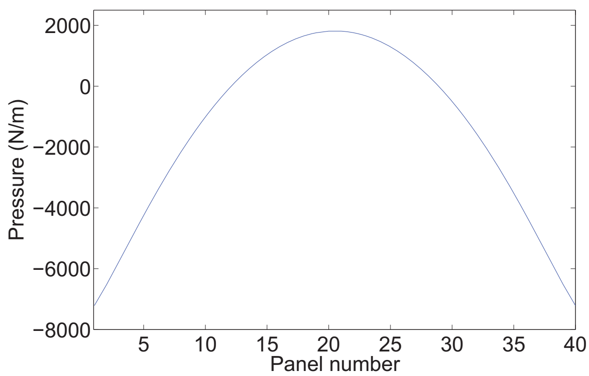


Fig. 5. Pressure distribution around circular cylinder when $t = 10s$,

to improve the desingularized boundary integral method for two-dimensional wave-body interaction problem. The neighboring sources distances distributed above the calm water surface are carefully selected to reduce the source number within comparable accuracy with the traditional source distribution method. In order to comply with the new distribution method, each of the two intersection points are treated as a single body collocation point. Comparisons between the present results and existing data are provided to illustrate the accuracy of the present schemes. Further work will be involved with nonlinear wave-body intersection problem.

REFERENCES

[1] T. Lee, *Nonlinear radiation problems for a surface-piercing body*, Phd thesis, University of Michigan, 1992.

[2] R. Beck, Y. Cao, and T. Lee, *Fully nonlinear water wave computations using the desingularized method*, Proceeding of the Sixth International Conference on Numerical Ship Hydrodynamics, 1994.

[3] S. Scorpio, *Fully nonlinear ship-wave computations using a multipole accelerated, desingularized method*, PhD thesis, University of Michigan, 1997.

[4] R. Beck, *Time-domain computations for floating bodies*, Applied ocean research 16(5), 267-282,1994.

[5] P. Finn, *Large amplitude nonlinear seakeeping using a desingularized method*, PhD thesis, University of Michigan, 2003.

[6] T. Lee, *fully nonlinear wave computations for arbitrary oating bodies using the delta method*, Journal of Hydrodynamics, 15(002), 24-31, 2003.

[7] X. Zhang, and R. Beck, *2-d body-exact computations in the time domain*, Proceeding of 21st International Workshop on Water Waves and Floating Bodies, 197-200, 2006.

[8] P. Bandy, *A body-exact strip theory approach to ship motion computations*, Phd thesis, University of Michigan, 2009.

[9] J. Vugts, *The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface*, International Shipbuilding progress, 251-275, 1968.

[10] X. Zhang, *Large amplitude ship motion computations using a time dependent body geometry*, PhD thesis, University of Michigan, 2007.