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Fractional Masks Based On Generalized Fractional Differential Operator for Image Denoising

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***Abstract*—**This paper introduces an image denoising algorithm based on generalized Srivastava-Owa fractional differential operator for removing Gaussian noise in digital images. The structures of $n \times n$ fractional masks are constructed by this algorithm. Experiments show that, the capability of the denoising algorithm by fractional differential-based approach appears efficient to smooth the Gaussian noisy images for different noisy levels. The denoising performance is measured by using peak signal to noise ratio (PSNR) for the denoising images. The results showed an improved performance (higher PSNR values) when compared with standard Gaussian smoothing filter.

***Keywords*—**Fractional calculus, fractional differential operator, fractional mask, fractional filter.

I. Introduction

FRACTIONAL integration and fractional differentiation are generalizations of the notions of integer-order integration and differentiation and they include n^{th} derivatives and n-fold integrals in particular cases. Many applications of fractional calculus in physics amount to replace the time derivative in an evolution equation with a derivative of fractional order. Fractional calculus has been applied to a variety of physical phenomena, including anomalous diffusion, transmission line theory, problems involving oscillations, nanoplasmonics, solid mechanics, astrophysics and viscoelasticity [1]-[6]. Currently, fractional calculus (integral and differential operators) is employed in signal processing and image possessing. The fractional calculus enhances the quality of images, with interesting possibilities in various image enhancement applications such as image denoising and texture enhancement [7]-[9].

In the past three decades a lot of studies have been conducted on fractional calculus and fractional differential equations, involving different operators such as Riemann-Liouville operators, Erdélyi-Kober operators, Weyl-Riesz operators, Caputo operators and Grünwald-Letnikov operators, with their applications in other fields. Moreover, the existence and uniqueness of holomorphic solutions for nonlinear fractional differential equations such as Cauchy problems and diffusion problems in complex domain are established and posed [10]-[17].

Noise is any undesired signal that contaminates an image. The digital image acquisition is the primary process by which noise appears in digital images as this process converts an optical image into a continuous electrical signal. *Noise*, arising from a variety of sources, is inherent to all electronic image sensors and the electronic components in the image environment. In Gaussian noise, all the image pixels deviate from their original values following the bell-shaped curve distribution (Gaussian curve).

Denoising is one of the most fundamental image restoration problems in computer vision and image processing. Image denoising refers to the process of recovering a digital image that has been contaminated by additive white Gaussian noise. There have been many attempts to construct digital filters to remove noise from digital images. Vijaykumar et al. in [18], presented a new fast and efficient algorithm capable in removing Gaussian noise with less computational complexity. R. H. Chan, in [19], proposed two-phase scheme for removing salt-and-pepper impulse noise. The concept of image fusion of filtered noisy images for impulse noise reduction is used in [20]. S. D. Ruikar and D. D. Doye [21] proposed different approaches of wavelet based image denoising methods. In [22] G. Vijaya and V. Vasudevan presented an interactive algorithm for image denoising and segmentation using soft computing techniques. Furthermore, A novel approach based on fractional calculus were employed in multi-scale texture segmentation [23]; design problems of variables and image denoising [24]; digital fractional order for different filters [25]. Finally, fractional differential masks based on the definitions of fractional differential operators due to Grümwald-Letnikov and Riemann-Liouvillewere introduced. Therefore, the fractional calculus in the field of image processing and signal prosecuting that has broad application prospect.

In this paper, we have aimed introduce an image denoising algorithm named generalized fractional differential image denoising algorithm based on Srivastava-Owa fractional differential operator. Initially the structures of $n \times n$ fractional masks of this algorithm are constructed. The denoising performance is measured by conducting experiments according to subjective and objective standards of visual perception values. Compared to other methods the proposed method performs well with very less computational time.

The outline of the paper is as follows: Section II explains

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the generalized fractional operator, Section III discusses the construction of our fractional differential masks scheme followed by experimental results and conclusion in Sections IV and V, respectively.

II. Generalized Fractional Operator

Srivastava and Owa, in [16], have defined fractional operators(derivative and integral) in the complex z-plane $\mathbb{C}$ as follows:

The fractional derivative of order α is defined, for a function $f(z)$ by

$$D_z^{\alpha} f(z) := \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{\alpha}} d\zeta;\ 0 < \alpha \le 1,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

The fractional integral of order α is defined, for a function $f(z),$ by

$$I_z^{\alpha} f(z) := \frac{1}{\Gamma(\alpha)} \int_0^z f(\zeta)(z-\zeta)^{\alpha-1} d\zeta;\ \alpha > 0,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{\alpha-1}$ is removed by requiring $log(z-\zeta)$ to be real when $(z-\zeta) > 0.$

Ibrahim in [17], has derived a formula for the generalized fractional integral. The n-fold integralfor natural $n \in \mathbb{N} = \{1,2,...\}$ and real $\mu,$ is defined by

$$I_z^{\alpha,\mu} f(z) = \int_0^z \zeta_1^{\mu} d\zeta_1 \int_0^{\zeta_1} \zeta_2^{\mu} d\zeta_2 ... \int_0^{\zeta_{n-1}} \zeta_n^{\mu} f(\zeta_n) d\zeta_n. \quad (1)$$

By employing the Cauchy formula for iterated integrals yields

$$\begin{aligned} &\int_0^z \zeta_1^{\mu} d\zeta_1 \int_0^{\zeta_1} \zeta^{\mu} f(\zeta) d\zeta \\ &= \int_0^z \zeta^{\mu} f(\zeta) d\zeta \int_{\zeta}^z \zeta_1^{\mu} d\zeta_1 \\ &= \frac{1}{\mu+1} \int_0^z (z^{\mu+1} - \zeta^{\mu+1}) \zeta^{\mu} f(\zeta) d\zeta. \end{aligned}$$

Repeating the above step $n-1$ times we have

$$\begin{aligned} &\int_0^z \zeta_1^{\mu} d\zeta_1 \int_0^{\zeta_1} \zeta_2^{\mu} d\zeta_2 ... \int_0^{\zeta_{n-1}} \zeta_n^{\mu} f(\zeta_n) d\zeta_n \\ &= \frac{(\mu+1)^{1-n}}{(n-1)!} \int_0^z (z^{\mu+1} - \zeta^{\mu+1})^{n-1} \zeta^{\mu} f(\zeta) d\zeta, \end{aligned}$$

which implies the fractional operator type

$$I_z^{\alpha,\mu} f(z) = \frac{(\mu+1)^{1-\alpha}}{\Gamma(\alpha)} \int_0^z (z^{\mu+1} - \zeta^{\mu+1})^{\alpha-1} \zeta^{\mu} f(\zeta) d\zeta. \quad (2)$$

where α and $\mu \neq -1$ are real numbers and the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z^{\mu+1} - \zeta^{\mu+1})^{-\alpha}$ is removed by requiring $log(z^{\mu+1} - \zeta^{\mu+1})$ to be real when $(z^{\mu+1} - \zeta^{\mu+1}) > 0.$ When $\mu = 0,$ we arrive at the standard Srivastava-Owa fractional integral, which is used to define the Srivastava-Owa fractional derivatives.

Corresponding to the generalized fractional integrals (2), we have defined the generalized differential operator of order α by

$$D_z^{\alpha,\mu} f(z) := \frac{(\mu+1)^{\alpha}}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{\zeta^{\mu} f(\zeta)}{(z^{\mu+1} - \zeta^{\mu+1})^{\alpha}} d\zeta;\ 0 < \alpha \le 1, \quad (3)$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z^{\mu+1} - \zeta^{\mu+1})^{-\alpha}$ is removed by requiring $log(z^{\mu+1} - \zeta^{\mu+1})$ to be real when $(z^{\mu+1} - \zeta^{\mu+1}) > 0.$

Proposition 1: The generalized derivative of the function $f(z) = z^{\nu}, \nu \in \mathbb{R}$ is given by

$$D_z^{\alpha,\mu} f(z) = \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{\nu}{\mu+1}+1)}{\Gamma(\frac{\nu}{\mu+1}+1-\alpha)} z^{(1-\alpha)(\mu+1)+\nu-1}.$$

Proof. We let $\eta := (\frac{\zeta}{z})^{\mu+1}$ then we have

$$\begin{aligned} D_z^{\alpha,\mu} z^{\nu} &= \frac{(\mu+1)^{\alpha}}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{\zeta^{\mu+\nu}}{(z^{\mu+1} - \zeta^{\mu+1})^{\alpha}} d\zeta \\ &= \frac{(\mu+1)^{\alpha-1}}{\Gamma(1-\alpha)} \frac{d}{dz} z^{(1-\alpha)(\mu+1)+\nu} \int_0^1 \eta^{\frac{\nu+\mu+1}{\mu+1}-1} \\ &\quad (1-\eta)^{(1-\alpha)-1} d\eta \\ &= \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{\nu}{\mu+1}+1)}{\Gamma(\frac{\nu}{\mu+1}+1-\alpha)} z^{(1-\alpha)(\mu+1)+\nu-1}. \end{aligned}$$

III. Construction of Fractional Differential Masks

Assume that the uniformly sampled signal satisfying the $n-$ degree polynomial

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$$s_n(i) = \sum_{k=0}^{n} a_k i^k \tag{4}$$

which is used to fit the given signal $i = 1,2,...,I$. Now in view of Proposition 1, we have the following condiments

$$\begin{aligned}
\phi_0 &= \frac{(\mu+1)^{\alpha-1}}{\Gamma(1-\alpha)} \\
\phi_1 &= \frac{(\mu+1)^{\alpha-1}\Gamma(\frac{1}{\mu+1}+1)}{\Gamma(\frac{1}{\mu+1}+1-\alpha)} \\
&\vdots \\
\phi_{n-1} &= \frac{(\mu+1)^{\alpha-1}\Gamma(\frac{n-1}{\mu+1}+1)}{\Gamma(\frac{n-1}{\mu+1}+1-\alpha)}.
\end{aligned} \tag{5}$$

Note that when $\mu = 0$, (5) reduces to the case of the Riemann-Liouville fractional operator. Now for two variables function like images, the negative direction of z and w coordinates, can be exprecced as

$$\begin{aligned}
D_z^{v,\mu} s(z,w) &= \phi_0 s(z,w) \\
&\quad + \sum_{k=1}^{n} \phi_k s(z-k,w)
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
D_z^{v,\mu} s(z,w) &= \phi_0 s(z,w) \\
&\quad + \sum_{k=1}^{n} \phi_k s(z,w-k).
\end{aligned} \tag{7}$$

While for two variables on the positive direction of z and w coordinates, we have

$$\begin{aligned}
D_z^{v,\mu} s(z,w) &= \phi_0 s(z,w) \\
&\quad - \sum_{k=1}^{n} \phi_k s(z+k,w)
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
D_z^{v,\mu} s(z,w) &= \phi_0 s(z,w) \\
&\quad - \sum_{k=1}^{n} \phi_k s(z,w+k).
\end{aligned} \tag{9}$$

The 2D fractional differential filtering window coefficients based on Srivastava-Owa fractional differential operator with eight directions of 180°, 90°, 0°, 270°, 45°, 135°, 315°, 225°, are shown in Fig. 1, which are respectively implemented on the directions of negative y-coordinate, negative x-coordinate, positive y-coordinate, positive x-coordinate, left upward diagonal, right upward diagonal and left downward diagonal, and right downward diagonal. In general, the mask size is taken as odd number.

The steps for the proposed fractional differential (FD) image denoising algorithm can be given as follows:

i. Set the mask window size and the values of the fractional powers *v* and μ;
ii. Add Gaussian noise with different variance values (0.01, 0.03 and 0.05);
iii. Apply the proposed algorithm, in which each pixel of the noisy image is convolved with the fractional differential mask on eight directions;
iv. Show the smoothed image by FD algorithm;
v. Calculate the PSNR between the noisy image and the output image after using FD algorithm;
vi. Apply the standard Gaussian smoothing filter;
vii. Show the smoothed image by standard Gaussian filter;
viii. Calculate the PSNR between the noisy image and the output image after standard Gaussian smoothing filter.

IV. Experimental Results

This section demonstrates that the proposed image denoising algorithm using the fractional differential masks performs better than the traditional image denoising approaches.

Performance tests for the system proposed by this paper were implemented using Matlab 2012a on Intel(R) Core i7 at 2.2GHz, 4GB DDR3 memory, system type 64-bit and Window 7. The test images employed here are the grayscale images "Woman" and "Taj" with 512×512 pixels.

The Gaussian noise is added into the image with different variance values (0.01, 0.03 and 0.05). All filters are considered to operate using 3×3 processing window mask. The values of the fractional power of the proposed mask are taken with the range $v \in (0.1, 0.9)$ and $\mu \in (1, 2)$.

The performance of filters was evaluated by computing the peak signal to noise ratio (PSNR). PSNR is an engineering term that represents the ratio between the maximum possible power of a signal and the power of corrupting noisethat affects the fidelity of its representation. It is defined via the mean squared error (MSE) for two images namely I and *K*, where one of the images is considered the original noisy image and the other is the filtered image:

$$MSE = \frac{1}{MN}\sum_{i-0}^{M-1}\sum_{j=0}^{N-1}[I(i,j) - K(i,j)]^2 \tag{10}$$

$$PSNR = \frac{10\log_{10} max(I,K)^2}{MSE} \tag{11}$$

where *max* is the maximum possible pixel value of the image. In grayscale image, this is equal to 255. Table I shows the results of PSNR values of our proposed method, comparing Gaussian smoothing filter with different Gaussian noise values. The proposed algorithm for image denoising provides satisfactory results. The higher PSNR value of the proposed

algorithm, acts as one of the important parameters to judge its performance.

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
φ_{n-1}	…	…	…	φ_2	φ_1	φ_0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(a)

0	0	0	φ_{n-1}	0	0	0
0	0	0	:	0	0	0
0	0	0	:	0	0	0
0	0	0	:	0	0	0
0	0	0	φ_2	0	0	0
0	0	0	φ_1	0	0	0
0	0	0	φ_0	0	0	0

(b)

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
φ_0	φ_1	φ_2	…	…	…	φ_{n-1}
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(c)

0	0	0	φ_0	0	0	0
0	0	0	φ_1	0	0	0
0	0	0	φ_2	0	0	0
0	0	0	:	0	0	0
0	0	0	:	0	0	0
0	0	0	:	0	0	0
0	0	0	φ_{n-1}	0	0	0

(d)

0	0	0	0	0	0	φ_{n-1}
0	0	0	0	0	…	0
0	0	0	0	…	0	0
0	0	0	…	0	0	0
0	0	φ_2	0	0	0	0
0	φ_1	0	0	0	0	0
φ_0	0	0	0	0	0	0

(e)

φ_{n-1}	0	0	0	0	0	0
0	…	0	0	0	0	0
0	0	…	0	0	0	0
0	0	0	…	0	0	0
0	0	0	0	φ_2	0	0
0	0	0	0	0	φ_1	0
0	0	0	0	0	0	φ_0

(f)

φ_0	0	0	0	0	0	0
0	φ_1	0	0	0	0	0
0	0	φ_2	0	0	0	0
0	0	0	…	0	0	0
0	0	0	0	…	0	0
0	0	0	0	0	…	0
0	0	0	0	0	0	φ_{n-1}

(g)

0	0	0	0	0	0	φ_0
0	0	0	0	0	φ_1	0
0	0	0	0	φ_2	0	0
0	0	0	…	0	0	0
0	0	…	0	0	0	0
0	…	0	0	0	0	0
φ_{n-1}	0	0	0	0	0	0

(h)

Fig. 1 Fractional differential masks on directions of 180°, 90°, 0°, 270°, 45°, 135°, 315°, 225°

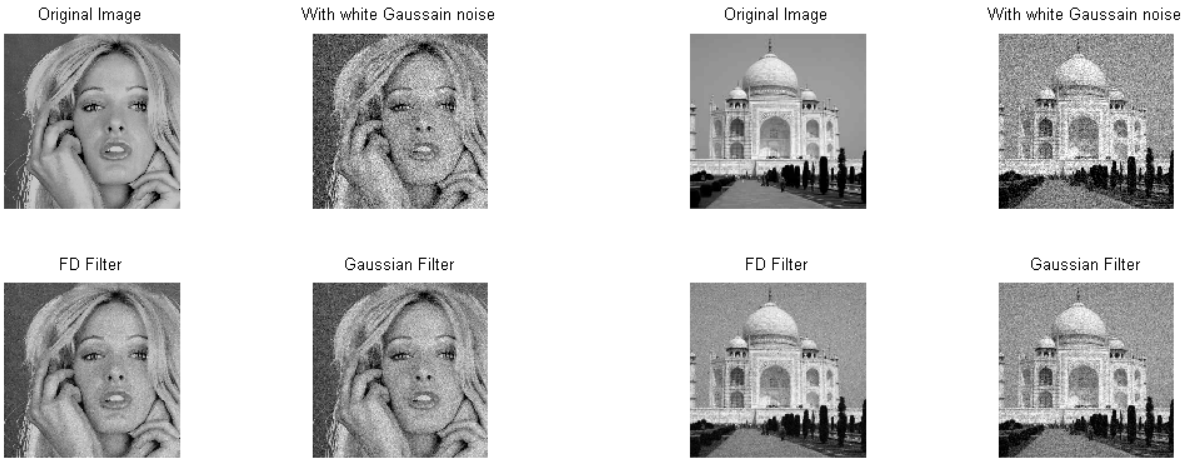


Fig. 2 Image denoising using fractional deferential and Gaussian smoothing filter with Gaussian noise variance=0.01

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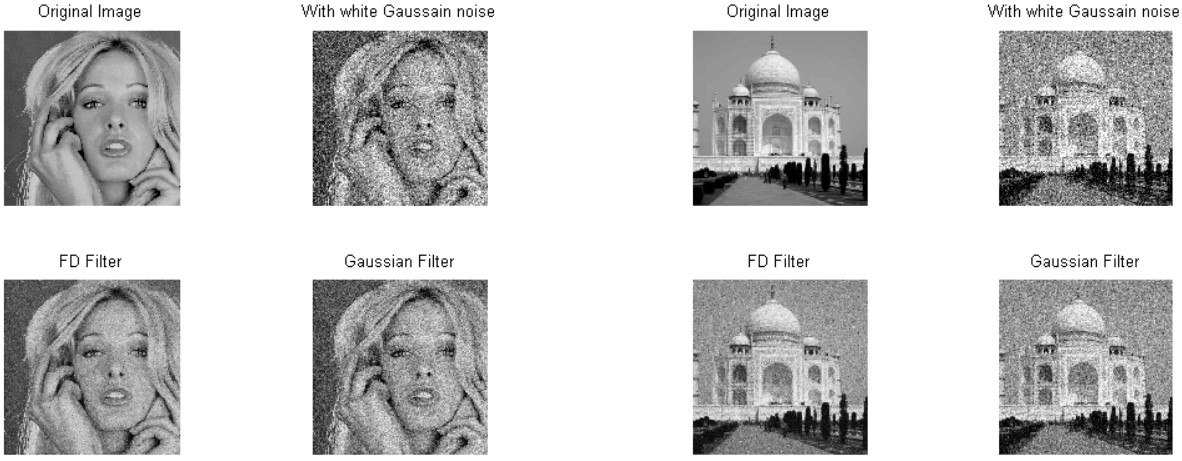


Fig. 3 Image denoisingusing Fractional deferential and Gaussian smoothing filter with Gaussian noise variance=0.03

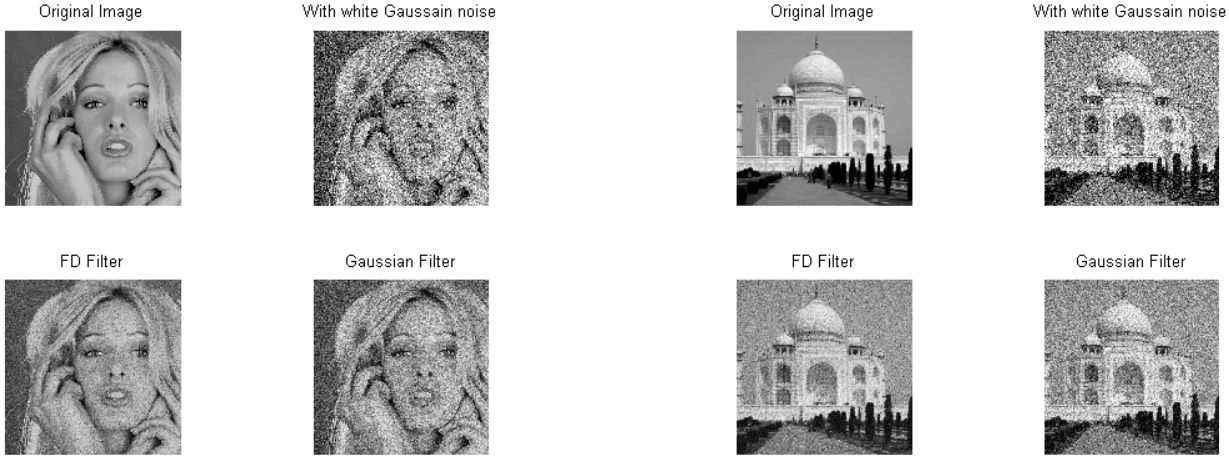


Fig. 4 Image denoisingusing Fractional deferential and Gaussian smoothing filter with Gaussian noise variance=0.05

TABLE I
RESULTS OF D DENOISING BY FRACTIONAL DIFFERENTIAL AND GAUSSIAN SMOOTHING FILTER

Variance	PSNR(dB)			
	Woman.tif(512x512)		Taj.tif(512x512)	
	Gaussian smoothing filter	Fractional differential filter (FD)	Gaussian smoothing filter	Fractional differential filter (FD)
0.01	23.8478	27.8760	24.1230	28.1127
0.03	19.4035	23.7035	19.5627	23.7447
0.05	17.4846	21.8566	17.6093	21.8224

TABLE II
COMPARISON OF THE EXPERIMENTAL RESULTS WITH OTHER STANDARD METHODS

Boats(512x512)				
σ of a noise image	Input PSNR of a noise image	Proposed algorithm	[24]	[26]
10	28.49	31.53	15.74	31.2
15	25.63	29.73	14.10	29.2

From the human visual system effect, Figs. 2-4, illustrate that the proposed denoising algorithm using fractional differential masks has good denoising performance for both testing images by different degrees of noise.

Table II shows the comparison of experimental results of the proposed algorithm with other denoising algorithms with the variance of noise (σ) of 10 and 15. The proposed algorithm for image denoising algorithm provides satisfactory results. The good PSNR of the proposed algorithm acts as one of the important parameters to judge its performance.

V. CONCLUSION

This paper had proposed an image denoising technique named as image denoising algorithm using generalized

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Srivastava-Owa fractional differential operator for smoothing the Gaussian noise in digital images. The image enhancement of the proposed fractional differential mask was increased by smoothing the intensity factors of the noisy images. Changing the values of the fractional powers ν and μ will allow adjusting the fractional differential filter coefficients to each image according to it characteristics. This makes the proposed algorithm to have higher PSNR than the standard Gaussian smoothing filter. The proposed algorithm could probably be employed in a number of applications in different areas of image processing, such as image enhancements, medical diagnostics, remote sensing, etc.

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