

Linear Programming Application in Unit Commitment of Wind Farms with Considering Uncertainties

M. Esmaeeli Shahrakht, A. Kazemi

Abstract—Due to uncertainty of wind velocity, wind power generators don't have deterministic output power. Utilizing wind power generation and thermal power plants together create new concerns for operation engineers of power systems. In this paper, a model is presented to implement the uncertainty of load and generated wind power which can be utilized in power system operation planning. Stochastic behavior of parameters is simulated by generating scenarios that can be solved by deterministic method. A mixed-integer linear programming method is used for solving deterministic generation scheduling problem. The proposed approach is applied to a 12-unit test system including 10 thermal units and 2 wind farms. The results show affectivity of piecewise linear model in unit commitment problems. Also using linear programming causes a considerable reduction in calculation times and guarantees convergence to the global optimum. Neglecting the uncertainty of wind velocity causes higher cost assessment of generation scheduling.

Keywords—Load uncertainty, linear programming, scenario generation, unit commitment, wind farm.

I. INTRODUCTION

DUE to increasing cost of fuels, there is a profound trend in using wind power. As the amount of wind power generation increases, concerns due to its effects on power system operation and expenses increases. As the wind is an intermittent power supply, developing forecasting tools is so useful. Though the forecasting tools become more accurate day by day, in the absence of a perfect estimation, decision about unit commitment should be done with uncertainties. Researchers have considered uncertainty of imprecise parameters using one of these two methods: fuzzy systems approach and probabilistic (stochastic) approach. Stochastic programming considers stochastic variables and uncertainty in conventional linear and nonlinear programming. Discussed randomness and uncertainty are generally represented using a probability density function (PDF).

Unit commitment problem is an important optimization process to determine on/off scheduling of units in power system operation. The purpose is to minimize the total cost of system operation in order to satisfy system load demand and other constraints. Unit commitment problem is generally a nonlinear, large scale, and mixed integer combinatorial problem. This problem has been a dynamic research topic in the recent decades because of potential savings in operation

costs. As a result numerous methods including heuristics methods [1], [2], dynamic programming [3], genetic algorithm [4], Neural Networks, Fuzzy Systems [5], mixed integer linear programming [6], and Lagrangian relaxation [7]-[10] have been suggested. Among these methods Lagrangian relaxation is the most widely used approach because of its capability in solving large scaled problems. The most important defect of this approach is that it is required heuristics methods to achieve feasible solutions which might be suboptimal, due to nonconvexities of unit commitment problem. In contrast, mixed integer linear programming guarantees convergence to an optimal solution in a few steps. Moreover, it provides precise and flexible modeling framework [11]. One of the first efforts for solving unit commitment problem using stochastic optimization approach is presented in [12]. In this work the possible future result for the demand are shown with a scenario tree. In this method, it is assumed that only a limited number of scenarios are possible because using all the possible scenarios is computationally impossible. Authors have tested their method using 100 scenarios and a 48-hour time horizon. A stochastic model for long-term security constrained unit commitment is used in [13], which outage of generating units, transmission lines and load prediction errors are considered.

In this paper, stochastic unit commitment of thermal power plants and wind farms is implemented with constraints for wind power generation. Wind speed uncertainties during each period of operation time is implemented through several scenario based on the wind speed PDF. To employ the mixed integer linear programming method and achieving global optimum, fuel cost function of thermal units is approximated by a set of piecewise block such that it is not distinguishable from the nonlinear model.

The main contributions of this work are as follows:

1. Presenting a stochastic scheduling method for solving stochastic wind-thermal unit commitment problem.
2. Presenting a linear model of unit commitment problem to solve the problem using linear programming tools.
3. Appropriate envelopment of forecasting error using application of Monte Carlo simulation for scenario generation.

The rest of this paper is organized as follows. In Section II, the proposed approach to model stochastic unit commitment is introduced. In Section III, problem formulation and problem constrained are discussed. As a sample problem, 12-unit test system (including 10 thermal generating units and 2 wind farms) is employed to be solved using optimization method in Section IV. Conclusions are given in Section V.

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II. IMPLEMENTATION OF THE PROPOSED MODEL

Load uncertainty and inaccuracy of forecasting for wind speed at all time intervals, faces wind-thermal unit commitment problem to the uncertainty. To model this uncertainty a set of scenarios are generated using Monte Carlo simulation method based on load and wind speed probability density functions. Then cost of operation for these scenarios is evaluated using linear model of mid-term unit commitment. Expected values of parameters are determined according to their values in each scenario and probability of the scenario.

A. Load Uncertainty Modeling

Due to imprecise load forecasting, there is an uncertainty in expected future load. Typically in literature normal distribution is used for modeling load forecasting error. In this paper, $P_d(t)$ is the forecasted weekly peak load in time interval t and its error assumes that it has a normal distribution of $N(\mu, \sigma)$ where $\mu(t)$ and $\sigma(t)$ are its mean and standard deviation respectively. These quantities in each time interval of t can be determined according to statistic studies.

B. Wind Power Uncertainty Modeling

Wind speed error can be represented by a suitable PDF. The PDF related to the difference between predicted and measured wind velocity, can be detected with a Gaussian distribution. In [14] the authors show that PDF of wind speed prediction error, in the speeds range suitable for wind energy applications, is a Gaussian distribution.

We assume that the uncertainty of wind speed forecasting in all wind farms is a normal distribution with two known parameters, $\mu_{w_i}(t)$ and $\sigma_{w_i}(t)$, respectively for mean and standard deviation of i th wind farm. The generated wind power varies with wind speed at the site of wind farms. The output power of wind turbine could be determined from its power curve which is a plot of output power against wind speed [15]. Fig. 1 shows conventional curve of wind turbine.

It is designed to start power generation at cut-in wind speed (V_{ci}) and be shut down at the cut-out wind speed (V_{co}) for safety. When the wind speed is higher than rated wind speed (V_r) and lower than cut-out wind speed (V_{co}), the rated power is generated. As shown in Fig. 1 there is a nonlinear relationship between generated output power and wind speed when wind speed is located between V_{ci} and V_r . Therefore the generated wind power by such turbine will be found by:

$$P_i = \begin{cases} 0 & 0 \leq SW_i \leq V_{ci} \\ P_r \times (A + B * SW_i + C * SW_i^2) & V_{ci} \leq SW_i \leq V_r \\ P_r & V_r \leq SW_i \leq V_{co} \\ 0 & SW_i \geq V_{co} \end{cases} \quad (1)$$

The calculation of constants A, B and C is discussed in [16]. In this paper typical model of wind turbine with cut-in, rated and cut-out wind speed respectively equal to 3, 11.5 and

25 m/s and rated output power equal to 2 MW is used. It is assumed that there are 40 turbines in each wind farm.

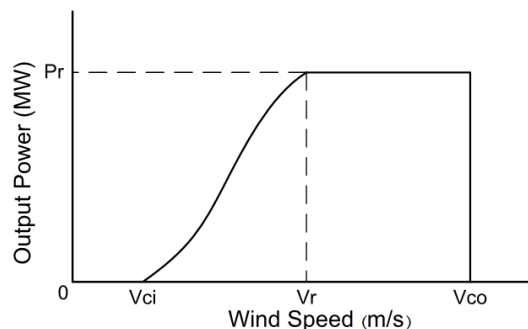


Fig. 1 Typical power curve of a wind turbine

C. Scenario Generation

Inverse transform method is used for scenario generation. It is assumed that PDFs of loads and wind speeds in each time interval is available. To determine a random value for these parameters, cumulative distribution functions (CDFs) is plotted based on their PDFs. Then, a random decimal number between zero and one is selected and corresponding value of the parameter is determined from the cumulative curve.

There are 3 stochastic parameter (load and wind speed of 2 wind farms) and 12 time intervals. So each scenario has 36 components (3*12). Each component is a random variable generated based on above approach.

III. UNCERTAINTY MODELING IN UNIT COMMITMENT PROBLEM FORMULATION

In this section a model for mid-term unit commitment scheduling is presented in which the uncertainty for main elements is modeled. The time horizon of scheduling for this problem is one season with a weekly interval. Due to time scheduling of this problem in its formulation, ramping and minimum up/down constraints are neglected.

A. Problem Formulation

The objective function of mid-term unit commitment problem is minimizing the cost of demand supply, while the constraints of required reserve, wind power generation and units capacity are satisfied. The objective function of scenario s is defined as total cost including production cost of thermal units and the operation and maintenance cost of thermal units and wind farms. It can be formulated as:

$$\begin{aligned} & \text{Min } J(x_{i,t}^s, u_{i,t}^s, \xi^s) \\ & = \sum_{t=1}^T \sum_{g=1}^{N_G} \{FC(P_{GD}^S(g, t)).n(t)\} \\ & + \sum_{t=1}^T \sum_{g=1}^{N_G} \{(P_{GD}^S(g, t) + P_{GR}^S(g, t)).OMVCT(g).n(t)\} \\ & + \sum_{t=1}^T \sum_{g=1}^{N_G} \{P_G^{\max}(g).OMFCT(g).\frac{n(t)}{8760}\} \\ & + \sum_{t=1}^T \sum_{w=1}^{N_W} \{P_W^S(w, t).OMVCW(w).n(t)\} \\ & + \sum_{t=1}^T \sum_{w=1}^{N_W} \{P_W^{\max}(w).OMFCW(w).\frac{n(t)}{8760}\} \end{aligned} \quad (2)$$

System constraints include supplying the system demand, required reserve and generating constraints of thermal units. These constraints can be formulated as:

$$\sum_{g=1}^{N_G} P_{GD}^S(g, t) + \sum_{w=1}^{N_W} P_W^S(w, t) = P_d^s(t) \quad t=1,2,\dots,T \quad (3)$$

$$\sum_{g=1}^{N_G} P_{GR}^S(g, t) \geq \text{RESW} \times \sum_{w=1}^{N_W} P_W^S(w, t) + P_R^S(t) \quad (4)$$

$$P_{GD}^S(g, t) + P_{GR}^S(g, t) \leq P_G^{\max} U^s(g, t) \quad (5)$$

$$P_G^{\min} \cdot U^s(g, t) \leq P_{GD}^S(g, t) \quad (6)$$

For compensating possible fluctuations of generated wind power, in (4) a fixed percentage of the total wind power generation (e.g. 10%) in requirement of system reserve is considered. The second term of system reserve is a percentage of total system loads (e.g. 5%) which is applied for forecasting error compensating.

B. Piecewise Linear Fuel Cost Function

In unit commitment problems often quadratic function are used as fuel cost function of thermal units. It is represented by the following equation:

$$FC(P_G(g, t)) = a_g + b_g \cdot P_G(g, t) + c_g \cdot (P_G(g, t))^2 \quad (7)$$

As shown in Fig. 2, cost function represented in (7) can be approximated accurately by a set of piecewise blocks. For practical applications, piecewise linear function is not distinguishable from nonlinear model if enough pieces are used [11].

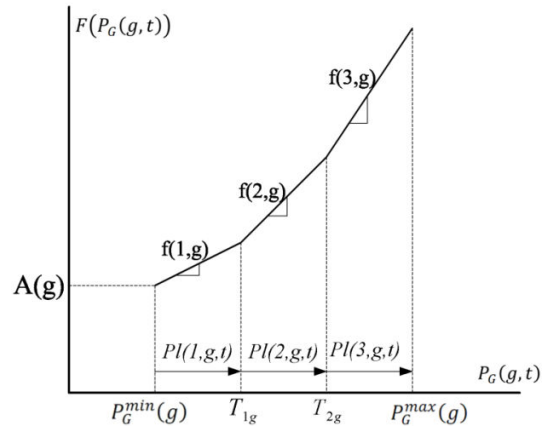


Fig. 2 Piecewise linear fuel cost function

Analytical representation of this approximation is as follow:

$$FC(P_G(g, t)) = A_g U(g, t) + \sum_{l=1}^{NL} f(l, g) \cdot Pl(l, g, t) \quad (8)$$

$$A(g) = a_g + b_g \cdot P_G^{\min}(g) + c_g \cdot (P_G^{\min}(g))^2 \quad (9)$$

$$P_G(g, t) \approx \sum_{l=1}^{NL} Pl(l, g, t) + P_G^{\min}(g) U(g, t) \quad (10)$$

$$Pl(1, g, t) \leq T_1 - P_G^{\min}(g) \quad (11)$$

$$Pl(l, g, t) \leq T_g(l) - T_g(l-1) \quad , l = 2, \dots, NL - 1 \quad (12)$$

$$Pl(NL, g, t) \leq P_G^{\max}(g) - T_g(NL - 1) \quad (13)$$

$$Pl(l, g, t) \geq 0 \quad (14)$$

C. Determining of Output Variables

Stochastic scheduling idea of unit commitment is based on developing or simulating possible choices in stochastic conditions to solve unit commitment problem for those choices. Then a combination of the results is selected to represent the stochastic solution. Mathematical formulas of the problem, under a set of scenarios, require solving deterministic unit commitment problem with the following objective function subjected to system constraints and limitations of thermal units and wind farms.

$$\text{Min } Pr^s \times J(x_{i,t}^s, u_{i,t}^s, \xi_t^s) \quad (15)$$

Thus with respect to the number of scenarios, there are several optimal solution such as $u_{i,t}^s = u_{i,1}^s, u_{i,2}^s, \dots, u_{i,T}^s$ and expected output for each variable will be weighted average of those solutions in each time interval.

Simulation process of the proposed approach consists of the following steps:

1. Generating scenarios which include two parts: system demands and wind speed of the wind farms in different time periods.
2. Using the proposed approach to solve deterministic unit commitment problem for each of above mentioned

scenarios.

- Determining desired values of output variables based on their values in different scenarios and the occurrence probability of each scenario.

IV. SIMULATIONS AND NUMERICAL RESULTS

The Proposed optimization algorithm is applied to a 12-unit test system including 10 thermal units and 2 wind farms. The fuel cost function of thermal units is fitted using a piecewise linear function with four segments. The time horizon of scheduling for this problem is one season with a weekly interval. The input data for thermal units are given in [17] and information of two wind farms are added to them. The maximum output power of each wind farm is 80MW. The mean values and standard deviations of weekly peak load and estimated wind speed for each farm in all time intervals are given in [15].

In this paper, first we compare the results of piecewise linear and nonlinear models in a deterministic unit commitment problem. It will also be shown the effectiveness of the proposed piecewise linear model for unit commitment problem which considerably reduces the calculation time and guarantees the problem convergence. Then the linear model of stochastic unit commitment will be solved and the results will be compared with the results of problem deterministic solution. The effect of installed wind power is another subject that is analyzed in this paper.

A. Deterministic Solution of Unit Commitment

In this section a deterministic unit commitment will be considered based on mean values of parameters in each period neglecting their uncertainties. The deterministic mid-term unit commitment problem for the test system based on nonlinear model is solved. The results of committing states of units are shown in Table I. Then the problem will be solved based on piecewise linear model and the results of committing states of units are shown in Table II. As seen in these tables, the results of linear programming obtained using piecewise linear approximation of fuel cost function are pretty close to those of nonlinear programming obtained using quadratic fuel cost function. Because suitable precision is considered in the linear approximation and the number of pieces are selected accurately.

The cost of operation and calculation time for solving the problem in both cases is presented in Table III. The results confirm the effectiveness and usefulness of piecewise linear model. The noticeable reduction of calculation time in linear model is the very important advantage of this model while the results are similar in both models.

Also, the effect of the amount of installed wind power on the scheduling cost of UC is studied. For this purpose, once installed wind power is increased to 50% (i.e. installed wind power is 2*120 MW) and once it is decreased to 50% (i.e. installed wind power is 2*40 MW). The cost of UC in the absence of wind farms is also calculated. The results of these studies are given Fig. 3. Although increasing in installed wind power causes reduction in cost of UC, but it does not reduce

according to the constant rate.

TABLE I
 COMMITTING STATES OF UNITS IN NONLINEAR MODEL OF DETERMINISTIC UC
 (1=ON, 0=OFF)

Units	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
Th1	1	1	1	1	1	1	1	1	1	1	1	1
Th2	1	1	1	1	1	1	1	1	1	1	1	1
Th3	1	1	0	1	1	1	1	1	1	1	1	1
Th4	1	1	1	1	1	1	1	1	1	1	1	1
Th5	1	1	1	1	1	1	1	1	1	1	1	1
Th6	1	1	0	0	1	1	1	0	0	1	1	1
Th7	0	0	0	0	0	0	0	0	0	0	1	1
Th8	0	0	0	0	0	0	0	0	0	0	0	1
Th9	0	0	0	0	0	0	0	0	0	0	0	0
Th10	0	0	0	0	0	0	0	0	0	0	0	0

TABLE II
 COMMITTING STATES OF UNITS IN PIECEWISE LINEAR MODEL OF
 DETERMINISTIC UC (1=ON, 0=OFF)

Units	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
Th1	1	1	1	1	1	1	1	1	1	1	1	1
Th2	1	1	1	1	1	1	1	1	1	1	1	1
Th3	1	1	0	1	1	1	1	1	0	1	1	1
Th4	1	1	1	1	1	1	1	1	1	1	1	1
Th5	1	1	1	1	1	1	1	0	0	1	1	1
Th6	1	1	0	0	1	1	1	1	1	1	1	1
Th7	0	0	0	0	0	0	0	0	0	0	1	1
Th8	0	0	0	0	0	0	0	0	0	0	0	1
Th9	0	0	0	0	0	0	0	0	0	0	0	0
Th10	0	0	0	0	0	0	0	0	0	0	0	0

TABLE III
 DETERMINISTIC UC RESULTS

Modeling method	Cost	Calculation time
Nonlinear model	M\$ 56.67	6.5 s
Piecewise linear model	M\$ 56.66	0.5 s

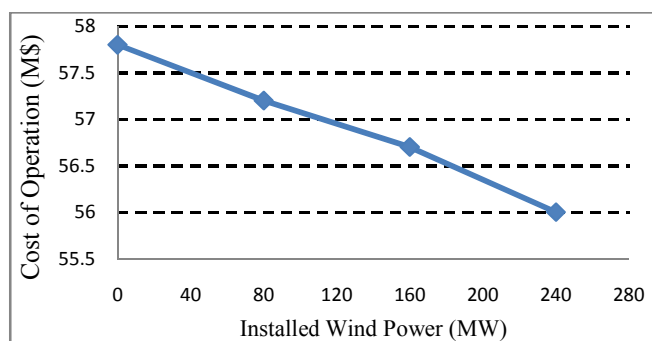


Fig. 3 The effect of installed wind power on operation cost

B. Stochastic Solution of Unit Commitment

The main idea of stochastic optimization is extension and simulation of possible choices. Scenario generation is a conventional way to model these probable conditions. Load and wind speed are two stochastic parameters in unit commitment problem. To simulate these uncertainties scenario generation based on inverse transform is used. Then

deterministic UC problem is solved for each scenario.

A combination of results, usually the weighted average of results, is selected to present stochastic solution. In this study, 180 scenarios are generated to simulate the uncertainty of the problem. All scenarios have the same probabilities.

The piecewise linear model of fuel cost function is used in this work and linear programming is applied to solve UC problem. For precise analysis of stochastic unit commitment problem, 6 scenario groups each containing 30 scenarios have been considered. The indexes related to total cost of stochastic unit commitment are presented in Table IV. These indexes are including mean value, standard deviation and variation coefficient of total cost.

TABLE IV

STOCHASTIC UC RESULTS WITH DIFFERENT SCENARIOS IN LINEAR MODEL

Number of scenarios	Total cost (M\$)		Variation coefficient (%)
	Mean	Standard Deviation	
1*30	57.414	1.762	0.560
2*30	56.962	1.699	0.385
3*30	56.952	1.562	0.289
4*30	56.827	1.612	0.259
5*30	56.731	1.619	0.233
6*30	56.688	1.638	0.215

As seen, the total cost by increasing the number of scenarios has been decreased because the model has become more precise and it more appropriately covers the uncertainties of the problem. In each stage by increasing the number of scenarios, the mean of total operation cost and variation coefficient are decreased.

The load supplying contribution of units in stochastic UC is shown in Fig. 4. As seen, the most of units are committed at last week when the system demand is at the highest level. Also, all of generated wind powers have been used in each period.

To compare stochastic and deterministic approaches the load supplying contribution of units in deterministic UC is presented in Fig. 5.

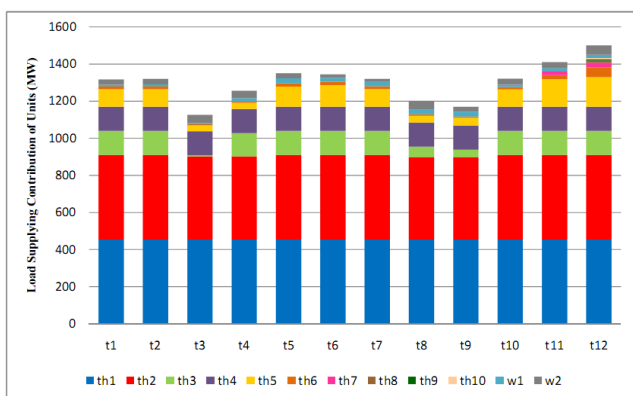


Fig. 4 Mid-term stochastic UC results for load contribution

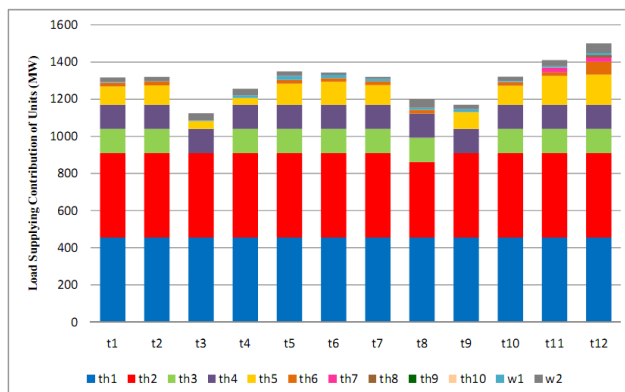


Fig. 5 Mid-term deterministic UC results for load contribution

The calculation time for solving linear model of stochastic unit commitment is 19 minutes. In order to scenarios reduction and hence calculation time reduction, a composed algorithm called backward reduction and forward selection has been used. Three different cases for scenarios reduction have been investigated by this method. The results of these cases are presented in Table V. As seen in Table V, more reduction in the number of scenarios leads to more inaccuracy. But the calculation time is decreased in accordance with percentage of reduction. Therefore, a trade-off between the accuracy and the computation time must be considered.

TABLE V

RESULTS OF STOCHASTIC UC IN DIFFERENT PERCENTAGE OF REDUCTION

Cases	Number of Scenarios	Operation Cost	Inaccuracy (%)
Base Case	180	56.688	-
85% reduction in base case	27	56.676	0.02
90% reduction in base case	18	56.663	0.04
95% reduction in base case	9	56.653	0.06

V. CONCLUSION

In this paper a linear optimization method considering uncertainty in power systems was proposed to solve stochastic UC problem. Utilizing linear optimization method leads to achieving a global optimum without divergence problems seen in nonlinear problems. UC uncertainty is caused by load prediction error and stochastic nature of wind speed in wind farms.

In this paper first, mid-term stochastic UC problem as a deterministic optimization problem is solved using linear and nonlinear programming. The results show affectivity of piecewise linear model in unit commitment problems. Also using linear programming causes a considerable reduction in calculation times. Then the uncertainties are simulated by utilizing a set of scenarios. Results show reduction in the number of scenarios leads to inaccuracy. But the calculation time is decreased in accordance with percentage of scenario reduction. In addition, using linear methods causes the average of the scenarios solution, unlike nonlinear methods, does not lead to unfeasible or suboptimal solutions and the global optimum could be accessed.

NOMENCLATURE

Constants

$A(g)$	Coefficient of the piecewise linear production cost function of unit g .
a_g, b_g, c_g	Coefficients of the quadratic production cost function of unit g .
$f(l, g)$	Slope of block l of the piecewise linear production cost function of unit g .
NL	Number of segments of the piecewise linear production cost function.
OMFCT(g)	Operation and maintenance fixed cost of thermal unit g .
OMFCW(w)	Operation and maintenance fixed cost of wind farm w .
OMVCT(g)	Operation and maintenance variable cost of thermal unit g .
OMVCW(w)	Operation and maintenance variable cost of wind farm w .
P_G^{\max}	Capacity of unit g .
P_G^{\min}	Minimum power output of unit g .
Pr^s	Probability of scenario s .
RESW	A fraction of total wind power employed to compensate wind power prediction errors.
$T_g(l)$	Upper limit of block of the piecewise linear production cost function of unit g .
$n(t)$	Number of hours at time t (e.g. 168 h).
<i>Variables</i>	
$P_{GD}^S(g, t)$	Load contribution of thermal unit g at time t in scenario s .
$P_R^S(t)$	A fraction of total system load for system reserve requirement at time t in scenario s .
$P_{GR}^S(g, t)$	Reserve contribution of thermal unit g at time t in scenario s .
$P_W^S(w, t)$	Generation of wind farm w at time t in scenario s .
$U^S(g, t)$	Commitment state of unit g at time t in scenario s (on=1, off=0).
ξ^s	Vector of scenario s .
<i>Sets</i>	
g	Set of indexes of the generating units.
t	Set of indexes of the time periods.
s	Set of indexes for scenarios.
w	Set of indexes of wind farms.

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