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Global Chaos Synchronization of Identical and Nonidentical Chaotic Systems Using Only Two Nonlinear Controllers

Azizan Bin Saaban, Adyda Binti Ibrahim, Mohammad Shehzad, Israr Ahmad

***Abstract*—**In chaos synchronization, the main goal is to design such controller(s) that synchronizes the states of master and slave system asymptotically globally. This paper studied and investigated the synchronization problem of two identical Chen, and identical Tigan chaotic systems and two non-identical Chen and Tigan chaotic systems using Non-linear active control algorithm. In this study, based on Lyapunov stability theory and using non-linear active control algorithm, it has been shown that the proposed schemes have excellent transient performance using only two nonlinear controllers and have shown analytically as well as graphically that synchronization is asymptotically globally stable.

***Keywords*—**Nonlinear Active Control, Chen and Tigan Chaotic systems, Lyapunov Stability theory, Synchronization.

I. Introduction

In 1960s, Edward Lorenz discovered a simple three dimensional mathematical model for atmospheric convections [1]. In his studies, Lorenz revealed that, a simple 3-D ordinary differential equations with a small changes in initial conditions can result a huge differences in future time. This sensitive dependence on initial conditions is known as butterfly effect. After the pioneering work of Lorenz on chaotic attractor, chaos has become a hot issue among many researchers. For the last three decades, immense researches have been studied on chaos which has divulged features and characteristics of it.

Chaos has another very fascinating phenomenon which is known as chaos synchronization. Synchronization of two chaotic (identical or nonidentical) oscillators is a process where the trajectories of the two chaotic oscillators adjust a common behavior in all future states due to coupling or forcing [2]. This ranges from absolute agreement of trajectories to interlocking of phases. This idea of synchronization was first introduced by Pacora and Carroll [2], since then, synchronization of chaotic dynamical systems has received a great deal of interest among scientists from almost all nonlinear sciences [2]-[8] for more than last two decades.

A wide range of synchronization techniques have been proposed [2], [8]-[14] and are applied theoretically as well as experimentally to achieve the control and synchronize of chaotic systems. Notable among these methods, chaos synchronization using Non-linear active control technique has recently been widely accepted as one of the efficient technique used both for synchronization and anti-synchronization [10]-[15] of chaotic systems. Since Lyapunov exponents are not required for numerical calculations, the nonlinear active algorithm is an effective technique to synchronize two identical as well as non-identical chaotic systems. Non-linear active control algorithm in order to achieve stable synchronization has been applied to many practical systems successfully.

The main objective of this study is to investigate new results for the global chaos synchronization for identical Tigan and unified Chen systems and non-identical Tigan and Chen chaotic systems. This study can be considered as an improvement to the existing results in [16] and [17].

Based on the Lyapunov stability theory and using the approach in [10], a class of non-linear control schemes will be designed to achieve the synchronization between two identical Chen and two identical Tigan chaotic systems and two non-identical Chen and Tigan chaotic systems with less control efforts and enough transient speed. We will establish our results using Lyapunov Stability theory [18] and will achieve asymptotically globally synchronization.

Numerical simulations and graphs will be furnished to show the efficiency and effectiveness of the propose approaches.

The rest of the paper is organized as follows: In Unit II, the problem statement and methodology for nonlinear control is introduced. Unit III discusses the chaos synchronization of identical Chen chaotic system. Unit IV presents the synchronization of identical Tigan chaotic systems. Unit V describes the synchronization of two different Tigan and Chen chaotic systems. In Unit VI numerical simulations are provided and finally in Unit VII, we finished this paper with some conclusion.

II. Designing of Nonlinear Controller

Most of the synchronization procedures belong to the master-slave system arrangement. By master-slave system configuration means that two systems are coupled in the way in which the deportment of the second (slave) system dependent on the first (master) system and the first system is not affected by the action of the second system.

Let us consider a (master-slave) systems configuration for a chaotic system as,

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$$\left.\begin{array}{ll}\dot{x} = A_1 x + f(x) & (\textit{ master system }) \\ \dot{y} = A_2 y + h(y) + \eta & (\textit{ slave system }) \end{array}\right\} \quad (1)$$

where **x, y** ϵ R^n are the corresponding state vectors, $A_1, A_2 \in R^{n\times n}$ are the (***n***×***n***) matrices of system parameters and $f, h : R^n \to R^n$ are the continuous nonlinear sequence functions and η is a nonlinear state feedback controller to be designed later.

If $A_1 = A_2$ and/or $f(\bullet) = h(\bullet)$, then **x** and **y** are the states of two identical chaotic systems.

If $A_1 \neq A_2$ and/or $f(\bullet) \neq h(\bullet)$, then **x** and **y** are the states of two different chaotic systems.

The synchronization problem is to design a controller 'η', which synchronize the states of both the drive and response (master and slave) systems.

The dynamics of synchronization errors can be defined as;

$$\dot{e} = A_2 y - A_1 x + h(y) - f(x) + \eta \quad (2)$$

where, $e_i = y_i - x_i$.

In the absence of a suitable controller ($\eta_i = 0$), the trajectories of two chaotic systems with different initial conditions, $(x_1(0), y_1(0), z_1(0) \neq x_2(0), y_2(0), z_2(0))$ will quickly bifurcate from each other in future time and will become uncorrelated. Thus the aim of synchronization problem is essentially to find such a feedback controller 'η' that stabilizes the error dynamics for all initial conditions [11] in future time.

i.e., $\lim_{t\to\infty}\|e_i(t)\| = \lim_{t\to\infty}\|y_i(t) - x_i(t)\| = 0$, for all $e_i(0)$ ϵ R^n, then the two systems in (1) are said to be synchronized. Let if we select a candidate Lyapunov error function as,

$$V(e) = e^T M e$$

where the matrix ***M*** is a positive definite matrix [10].

We notice that, $V : R^n \to R^n$ is a positive definite function by construction. We further assume that the parameters of the master and slave systems are known and the states of both chaotic systems are measurable.

We may achieve a stable synchronization by selecting a non-linear controller 'η' to make $\dot{V}(e) = -e^T N e$ be a positive definite matrix (i.e., the matrix ***N*** is also a positive definite matrix), then by Lyapunov stability theory [18], the states of the master and slave chaotic systems will be globally asymptotically synchronized.

III. Chaos Synchronization of Two Identical Chen Chaotic Systems

This section focuses on applying non-linear controller scheme to synchronize two identical Chen systems [20]. For this purpose, let us consider a master-slave systems configuration for Chen chaotic system is given as follow;

$$\left.\begin{array}{l}\dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = (c-a)x_1 - x_1 z_1 + c y_1 \\ \dot{z}_1 = x_1 y_1 - b z_1\end{array}\right\} \text{(drive system)} \quad (3)$$

and

$$\left.\begin{array}{l}\dot{x}_2 = a(y_2 - x_2) + \eta_1 \\ \dot{y}_2 = (c-a)x_2 - x_2 z_2 + c y_2 + \eta_2 \\ \dot{z}_2 = x_2 y_2 - b z_2 + \eta_3\end{array}\right\} \text{(response system)} \quad (4)$$

where $x_1, y_1, z_1 \in R^n$ and $x_2, y_2, z_2 \in R^n$ are the corresponding state vectors of drive and response systems respectively, *a*, *b* and *c* are the system parameters, $\eta = [\eta_1, \eta_2, \eta_3]^T$ are the non-linear controllers which have to be designed yet.

The Chen system (3) exhibits a chaotic attractor when the parameter values are taken as, *a* = 35, *b* = 3 and *c* = 28.

The error dynamics is defined as,

$$\left.\begin{array}{l}\dot{e}_1 = a(e_2 - e_1) + \eta_1 \\ \dot{e}_2 = c e_2 + (c-a)e_1 + x_1 z_1 - x_2 z_2 + \eta_2 \\ \dot{e}_3 = -b e_3 + x_2 y_2 - x_1 y_1 + \eta_3\end{array}\right\} \quad (5)$$

where η_1, η_2 and η_3 are the controllers and,

$$\left.\begin{array}{l}e_1 = x_2 - x_1 \\ e_2 = y_2 - y_1 \\ e_3 = z_2 - z_1\end{array}\right\}$$

For the two identical systems (3) and (4) without controller $(\eta_i = 0)$ the trajectories of the two identical systems will diverge exponentially with the course of time and will become unsynchronized on the initial conditions $(x_1(0), y_1(0), z_1(0) \neq x_2(0), y_2(0), z_2(0))$. Thus the aim of the synchronization problem is to design a feedback controller ψ such that $\|e_i\|$ tends to decay to zero as t →∞,

i.e., $\lim_{t\to\infty}\|e(t)\| = 0$, for all $e(0)$ ϵ R^n .

The aim of this study is, to focus on synchronizing two identical chaotic systems by designing a nonlinear controller such that when synchronizing the two chaotic systems, the effect of nonlinearity of chaotic systems should not be neglected and the error dynamics convergence to the origin asymptotical globally with less control effort and enough transient speed. Therefore, using the approach in [10], selecting such a suitable Lyapunov error function candidate that will ensure asymptotically globally stability. For this purpose, let us assume the following theorem.

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Theorem1: The two Systems (3) and (4) will achieve asymptotically and globally synchronization for any initial condition with following control law:

$$\eta_1 = 0,\ \eta_2 = -2ce_2 - x_1 z_1 + x_2 z_2,\ \eta_3 = x_1 y_1 - x_2 y_2$$

Proof: Let us construct a Lyapunov error function candidate as;

$$V(t) = e^T Pe \qquad (6)$$

where $P = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is a positive definite function.

Now the time derivative of the Lyapunov function is,

$$\dot{V}(t) = -7e_1^2 - 28e_2^2 - 6e_3^2 = -e^T \begin{pmatrix} 7 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 6 \end{pmatrix} e \prec 0$$

Therefore,

$$-\dot{V}(t) = e^T Ne \ \text{ and } N = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

where $\boldsymbol{N}$ is also a positive definite matrix.

Hence based on Lyapunov stability theory [18], the error dynamics converges to the origin asymptotically. Thus the master and slave Chen chaotic systems are asymptotically globally synchronized.

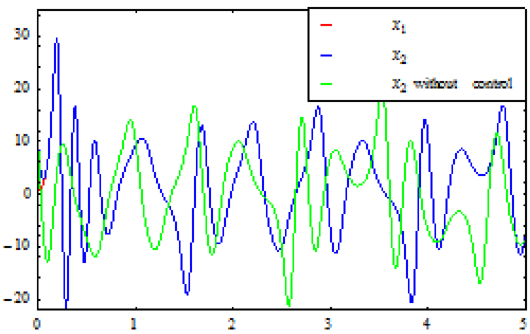

Fig. 1 Time series of x_1 and x_2 (Identical Chen systems)

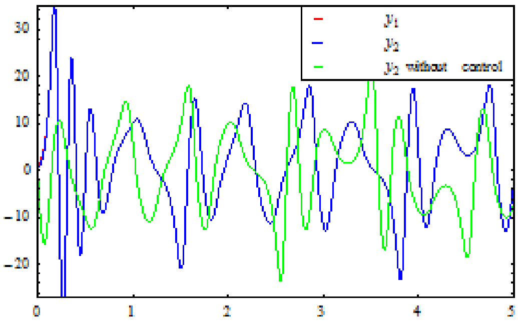

Fig. 2 Time series of y_1 and y_2 (Identical Chen systems)

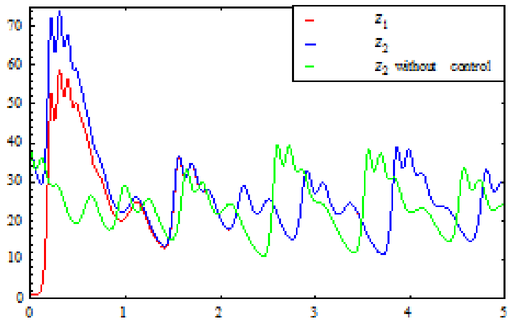

Fig. 3 Time series of z_1 and z_2 (Identical Chen systems)

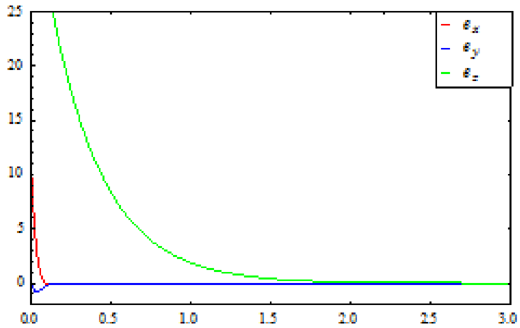

Fig. 4 Time series of errors (Identical Chen systems)

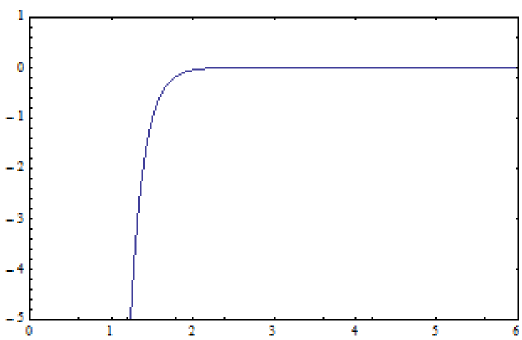

Fig. 5 Derivatives of Lyapunov function (Identical Chen systems)

IV. Chaos Synchronization of Two Identical Tigan Systems

In 2004, G.H. Tigan [21] presented a new 3-D chaotic system in the form of the following differential equations,

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$$\left.\begin{aligned}\dot{x} &= \alpha(y-x)\\ \dot{y} &= (\gamma-\alpha)x-\alpha xz\\ \dot{z} &= xy-\beta z\end{aligned}\right\}$$

where $x_1, y_1, z_1 \in R^n$ the state vectors, α, β and γ are the system parameters. The Tigan system exhibits a chaotic attractor when the parameter values are taken as, α =2 .1, β = 0.6 and γ = 30.

A master-slave systems configuration for Tigan chaotic systems is given as follows;

$$\left.\begin{aligned}\dot{x}_1 &= \alpha(y_1-x_1)\\ \dot{y}_1 &= (\gamma-\alpha)x_1-\alpha x_1 z_1\\ \dot{z}_1 &= x_1 y_1-\beta z_1\end{aligned}\right\} \quad \text{(master system)} \qquad (7)$$

and

$$\left.\begin{aligned}\dot{x}_2 &= \alpha(y_2-x_2)+\eta_1\\ \dot{y}_2 &= (\gamma-\alpha)x_2-\alpha x_2 z_2+\eta_2\\ \dot{z}_2 &= x_2 y_2-\beta z_2+\eta_3\end{aligned}\right\} \quad \text{(slave system)} \qquad (8)$$

where $x_1, y_1, z_1 \in R^n$ and $x_2, y_2, z_2 \in R^n$ are the corresponding state vectors of drive and response systems respectively, α, β and γ are the system parameters, $\eta(t)=[\eta_1,\eta_2,\eta_3]^T$ are the non-linear controllers that is to be designed yet.

For chaotic synchronization of the above drive-response systems, the error dynamics is described as,

$$\left.\begin{aligned}\dot{e}_1 &= \alpha(e_2-e_1)+\eta_1\\ \dot{e}_2 &= (\gamma-\alpha)e_1-\alpha x_2 z_2+\alpha x_1 z_1+\eta_2\\ \dot{e}_3 &= -\beta e_3+x_2 y_2-x_1 y_1+\eta_3\end{aligned}\right\} \qquad (9)$$

Let us propose the following theorem.

Theorem 2: The two Systems (7) and (8) will approach global and asymptotical synchronization for any initial condition with following control law;

$$\eta_1=0,\ \eta_2=-\gamma e_1-e_2+\alpha(x_2z_2-x_1z_1)\ ,\ \eta_3=-\beta e_3-x_2y_2+x_1y_1$$

Proof: Substituting the proposed controllers in (9), we have,

$$\left.\begin{aligned}\dot{e}_1 &= \alpha(e_2-e_1)\\ \dot{e}_2 &= -\alpha e_1-e_2\\ \dot{e}_3 &= -2\beta e_3\end{aligned}\right\}$$

Let us construct a Lyapunov function candidate as;

$$V(t)=e^T P e$$

where, $P=\begin{pmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 0 & 0.5\end{pmatrix}$ is a positive definite function.

Now the time derivative of the Lyapunov function is,

$$\dot{V}(t)=-2.1{e_1}^2-{e_2}^2-1.2{e_3}^2=-e^T\begin{pmatrix}2.1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1.2\end{pmatrix}e \prec 0$$

i.e., $-\dot{V}(t)=e^T N e$ and $N=\begin{pmatrix}2.1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1.2\end{pmatrix}$ which is also a positive definite matrix.

We can see that V(e) and $-\dot{V}(e)$ are positive definite functions. Hence the error states , $\lim_{t\to\infty}\|e_i(t)\|=0$

Thus the drive and response Tigan systems are globally asymptotically synchronized.

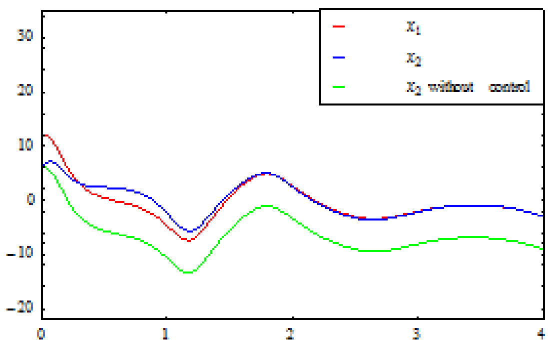

Fig. 6 Time series of x_1 and x_2 (Identical Tigan system)

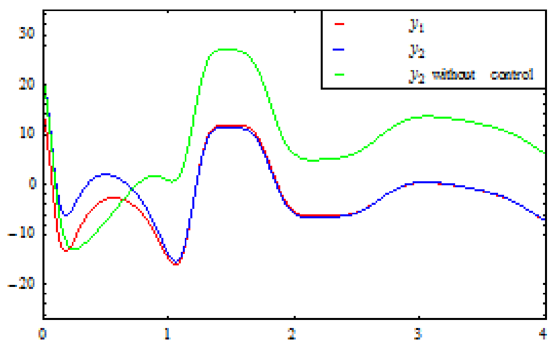

Fig. 7 Time series of y_1 and y_2 (Identical Tigan system)

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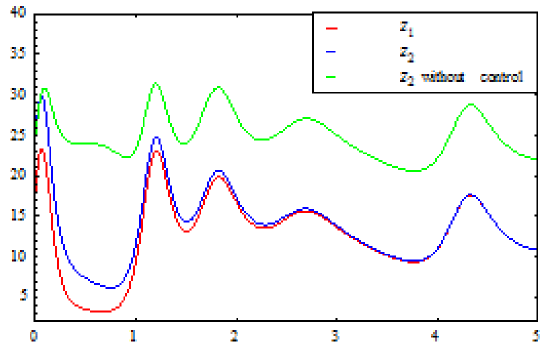

Fig. 8 Time series of z_1 and z_2 (Identical Tigan system)

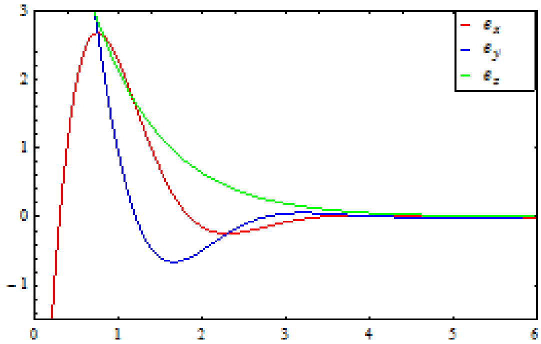

Fig. 9 Time series of errors (Identical Tigan system)

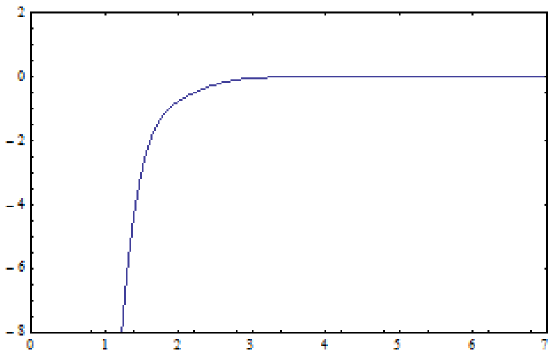

Fig. 10 Time derivative of Lyapunov function (Identical Tigan systems)

V. Synchronization of Two Different Tigan and Chen Chaotic Systems

It is assume that the Tigan chaotic system drive the Chen chaotic system. Therefore, the master and slave systems arrangement is described as;

$$\left.\begin{aligned} \dot{x}_1 &= \alpha(y_1 - x_1) \\ \dot{y}_1 &= (\gamma-\alpha)x_1 - \alpha x_1 z_1 \\ \dot{z}_1 &= x_1 y_1 - \beta z_1 \end{aligned}\right\} \quad \text{(master system)} \qquad (10)$$

and

$$\left.\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + \eta_1 \\ \dot{y}_2 &= (c-a)x_2 - x_2 z_2 + c y_2 + \eta_2 \\ \dot{z}_2 &= x_2 y_2 - b z_2 + \eta_3 \end{aligned}\right\} \quad \text{(slave system)} \qquad (11)$$

For chaotic synchronization of the above master-slave systems, the error dynamics is described as,

$$\left.\begin{aligned} \dot{e}_1 &= -a e_1 + (\alpha - a)x_1 - \alpha y_1 + a y_2 + \eta_1 \\ \dot{e}_2 &= c e_2 + c y_1 + x_2(c - a - z_2) - (\gamma - \alpha - \alpha z_1)x_1 + \eta_2 \\ \dot{e}_3 &= -b e_3 - b z_1 + \beta z_1 + x_2 y_2 - x_1 y_1 + \eta_3 \end{aligned}\right\} \qquad (12)$$

The aim of the synchronization problem is to design a feedback controller '$\eta(t)$' such that $\|e_i\|$ tends to zero . By defining the controller, $\eta = [\eta_1, \eta_2, \eta_3]^T$ as;

$$\left.\begin{aligned} \eta_1 &= (a-\alpha)x_1 - a y_2 + \alpha y_1 \\ \eta_2 &= -2c e_2 - c y_1 - x_2(c - a - z_2) + (\gamma - \alpha - \alpha z_1)x_1 \\ \eta_3 &= b z_1 - \beta z_1 - x_2 y_2 + x_1 y_1 \end{aligned}\right\} \qquad (13)$$

Therefore, the error system (12) becomes,

$$\left.\begin{aligned} \dot{e}_1 &= -a e_1 \\ \dot{e}_2 &= -c e_2 \\ \dot{e}_3 &= -b e_3 \end{aligned}\right\} \qquad (14)$$

From (14), one can see that the error system (14) is a linear system of the form, $\dot{e} = Ae$.

Thus by linear control theory, the system matrix $\boldsymbol{A}$ has Hurwitz [19], and so the all the eigenvalues of the system matrix $\boldsymbol{A}$ are negative,

i.e., $$A = \begin{pmatrix} -35 & 0 & 0 \\ 0 & -28 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$ is Hurwitz.

Hence the system (14) is asymptotically stable, which implies that the two chaotic Tigan and Chen systems are synchronized asymptotically globally.

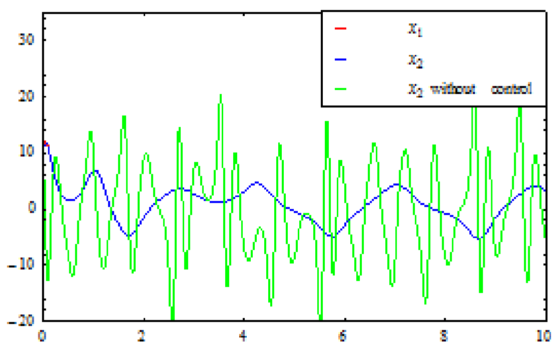

Fig. 11 Time series of x_1 and x_2 (Nonidentical Tigan & Chen chaotic systems)

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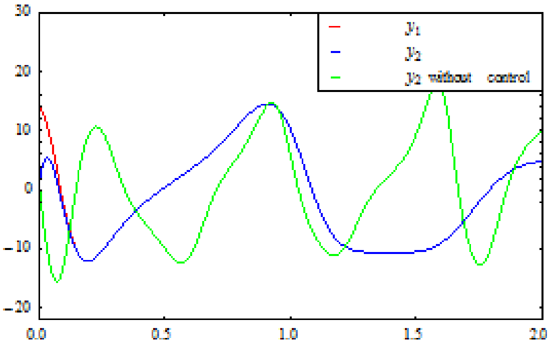

Fig. 12 Time series of y_1 and y_2 (Nonidentical Tigan & Chen chaotic systems)

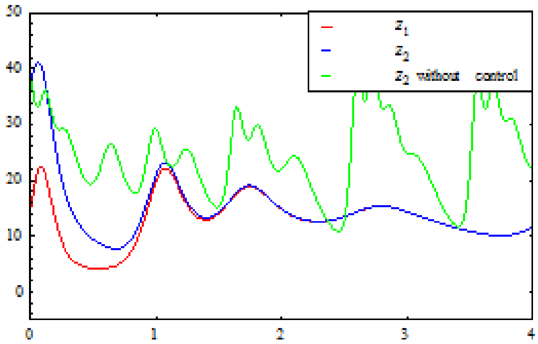

Fig. 13 Time series of z_1 and z_2 (Nonidentical Tigan & Chen chaotic systems)

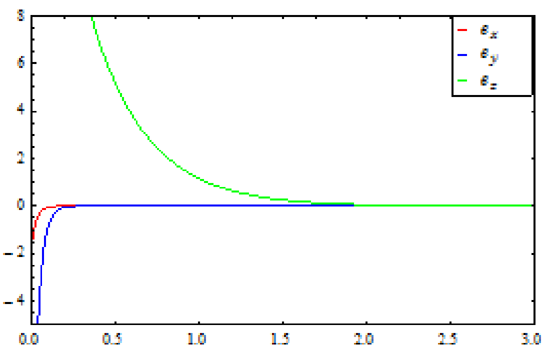

Fig. 14 Time series of errors (Nonidentical Tigan & Chen chaotic systems)

VI. Numerical Simulations

In this unit, numerical simulations are presented to verify the effectiveness of the proposed method. The parameters for Chen chaotic systems are selected as, a = 35, b = 3 and c = 28 with initial conditions are taken as,

$$(x_1(0), y_1(0), z_1(0)) = (0.5, 1, 1)$$

and

$$(x_2(0), y_2(0), z_2(0)) = (10.5, 1, 38)$$

The parameters for Tigan chaotic systems are selected as, α=2.1, β=0.6 and γ=30, with initial conditions are $(x_1(0), y_1(0), z_1(0)) = (12,14,18)$ and $(x_2(0), y_2(0), z_2(0)) = (6,21,25)$.

VII. Conclusion

In this study, it was found that only two controllers and less efforts were utilized to achieve the asymptotically globally synchronization between two identical and nonidentical chaotic systems. This study has shown excellent transient performance for identical Chen and identical Tigan chaotic systems using only two nonlinear controllers. This study focused on selecting such a suitable Lyapunov error function candidate that ensured asymptotically globally stability.

- Fig. 4 shows the synchronization error for Tigan chaotic systems when the controls are switched on at t= 0s. It has been shown that the synchronization error has already achieved at t=1.6s, while the synchronization error was achieved at t= 5s for [16], and thus the time delay is almost 3.4s and only two controllers and less effort were applied to synchronize two identical Tigan chaotic systems.
- Fig. 9 shows the synchronization error for Tigan chaotic systems when the controls are switched on at t= 0s. It has been shown that the synchronization error has already achieved at t=4s, while the synchronization error was achieved at t= 4.8s for [17], and thus the time delay is almost 0.8s. On the other side, only two controllers were applied to synchronize two identical Tigan chaotic systems.
- Fig. 14 shows the synchronization of two nonidentical Tigan and Chen chaotic systems. For the two different chaotic systems (Tigan and Chen systems), that contain parameters mismatch and different structures, the controller was used for synchronizing the states of drive and response systems asymptotically globally, which shows that the investigated controller is more robust to accidental mismatch in the transmitter and receiver.

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