Revolving Ferrofluid Flow in Porous Medium with Rotating Disk
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Abstract—An attempt has been made to study the effect of rotation on incompressible, electrically non-conducting ferrofluid in porous medium on Axi-symmetric steady flow over a rotating disk excluding thermal effects. Here, we solved the boundary layer equations with the variation of velocity components radial, tangential and vertical in space, ferrofluids in the field of fluid mechanics, having all three medical applications also. Surgery. Therefore, ferrofluids play an important role in bio-positioning tamponade for retinal detachment repair in eye used in the contrast medium in X-ray examinations and for geometrical shape of the speaker system. Magnetic fluids are and increases the acoustical power without any change in the controlling of heat in loudspeakers which makes its life longer since last five decades. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [1], [2]. Ferrofluids are widely coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them fascinated the researchers and engineers vigorously since last five decades. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [1], [2]. Ferrofluids are widely used in sealing of the hard disc drives, rotating x-ray tubes under engineering applications. Sealing of the rotating shafts is the most known application of the magnetic fluid. The major applications of ferrofluid in electric field is that controlling of heat in loudspeakers which makes its life longer and increases the acoustical power without any change in the geometrical shape of the speaker system. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in biomedical applications also.

There are rotationally symmetric flows of incompressible ferrofluids in the field of fluid mechanics, having all three velocity components radial, tangential and vertical in space, different from zero. In such types of flow, the variables are independent of the angular coordinates. We consider this type of flow for an incompressible ferrofluid when the plate is subjected to the magnetic field \( H_0 \), using Neuringer-Rosensweig model \([3]\). This model has been used by Verma et al. \([4]–[6]\) for solving paramagnetic couette flow, helical flow with heat conduction by taking into account the interactions of external magnetic field with ferrofluid. A detail account of magneto-viscous effects in ferrofluids has been given in a monograph by Odenbach \([7]\). Rosensweig \([8]\) has given an authoritative introduction to the research on magnetic liquids in his monograph and study of the effect of magnetization yields the interesting information.

The pioneering study of ordinary viscous fluid flow due to the infinite rotating disk was first carried out by Karman \([9]\). He introduced the famous similarity transformation which reduces the governing partial differential equations into the ordinary differential equations. Cochran \([10]\) obtained asymptotic solutions for the steady hydrodynamic problem formulated by Von Karman. Benton \([11]\) improved Cochran’s solutions and solved the unsteady case. Attia \([12]\) studied the unsteady state in the presence of an applied uniform magnetic field. Ram et al. \([13]\) investigated the ferrofluid flow in a porous medium due to an infinite rotating disk.

Attia \([14]\) investigated steady laminar flow of an incompressible viscous non-Newtonian fluid due to the uniform rotation of porous disk of infinite extent in porous medium with heat transfer. Sunil and Mahajan \([15]\) studied the nonlinear stability analysis of magnetized ferrofluid heated and soluted from below with MFD viscosity via generalized energy method. Ram et al. \([16]\) solved the non-linear differential equations under Neuringer-Rosensweig model for ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on velocity components and pressure profile. Further, the effect of porosity on velocity components and pressure profile has been studied by Ram et al. \([17]\). The magnetic effects on heat fluid and entropy generation interactions in a porous medium due to an infinite rotating disk was first carried out by Karman \([18]\). Ram and Kumar \([19]\) investigated the effects of field dependent viscosity on ferrofluid flow saturating the porous medium over a rotating disk. And, ferrofluid flow with heat transfer over a stretchable rotating disk with magnetic field dependent viscosity has been investigated by Ram and Kumar \([20]\).

In the present paper, we take cylindrical coordinates \( r, \theta, z \) where the \( z \)-axis is normal to the plane and this axis is considered as the axis of rotation. We have presented the
boundary layer equations along with boundary conditions. These equations along with Maxwell’s relations are solved theoretically as well as numerically. Also, it is found that there is a large variation in the boundary layer thickness as compared to the ordinary viscous fluid flow case. We have also given the expression for the total volume flowing outwards the axis taken over a cylinder of radius R around the z-axis by using the description given on page 229 in Schlichting [21]. The effect of vertically applied magnetic field in a circular layer of ferrofluid within the rotating disk is studied within the framework of the Neuringer-Rosensweig approach. This problem, to the best of our knowledge, has not been investigated yet.

II. FORMULATION AND SOLUTION OF THE PROBLEM

The ferrofluid flow under consideration is represented by the following basic governing equations.

Equation of continuity

\[ \nabla \cdot \vec{V} = 0 \]  

Equation of motion

\[ \rho \left( \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} \right) = -\nabla p + \mu_0 (M \cdot \nabla \vec{H}) + \mu \nabla^2 \vec{V} \]

\[ + 2\rho (\vec{H} \times \vec{V}) + \frac{\rho \vec{V}}{2} \]  

The effect of rotation includes two terms: (a) Centrifugal force \(-\frac{1}{2} \nabla \times \vec{V} \times \vec{r}^2\) and (b) Coriolis acceleration \(2(\vec{H} \times \vec{V})\). In (2), \(p' - \frac{\rho \vec{H}}{2} \nabla \times \vec{r}^2 = \bar{p}\) is the reduced pressure, where \(p'\) stands for fluid pressure.

Maxwell’s relations

\[ \nabla \times \vec{H} = 0; \quad \nabla \cdot (\vec{H} + \vec{M}) = 0 \]

Assumptions

\[ \vec{M} = \chi \vec{H}, \quad \vec{M} \times \vec{H} = 0 \]

The revolving ferrofluid flow is subjected to following boundary conditions

\[ V_r = 0, \quad V_\theta = 1, \quad V_z = 0 \quad \text{at} \quad z = 0 \]

\[ V_r, \quad V_\theta \to 0 \quad \text{and} \quad V_z \to -C \quad \text{as} \quad z \to \infty \]

On considering the assumptions that the flow is steady (i.e. \( \frac{\partial}{\partial t} \) is zero) and axisymmetric (i.e. \( \frac{\partial}{\partial \theta} \) is zero), negligible variation in magnetic field in axial direction, the boundary layer approximation

\[ -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial r} + \kappa \mu \frac{\partial}{\partial r} \left( \frac{\bar{p}}{\rho} \right) = -r \omega^2 \]

in radial direction and using the Karman’s similarity transformations

\[ V_r = r \omega E(\alpha), \quad V_\theta = r \omega F(\alpha), \quad V_z = \sqrt{2} \omega G(\alpha), \]

\[ p = \rho_0 \omega P(\alpha) \quad \text{where} \quad \alpha = \frac{\omega}{\sqrt{\nu} z} \]

We get a system of non linear coupled ordinary differential equations in the dimensionless variables \(E, F, G\) and \(P\) as:

\[ E^* - GE^* - E^2 + F^2 + 2 F - \beta E - 1 = 0 \]  

\[ F^* - GF^* - 2 EF - 2 E - \beta F = 0 \]  

\[ P^* - G^* + GG^* + \beta G = 0 \]  

\[ G^2 + 2 E = 0 \]

The boundary conditions for the flow are

\[ E(0) = 0, \quad F(0) = 1, \quad G(0) = 0, \quad P(0) = P_0 \]

\[ E, \quad F \to 0 \quad \text{and} \quad G \to -E \quad \text{as} \quad \alpha \to \infty \]

Cochran indicated that formal asymptotic expansions (for large \( \alpha \)) of the system of (7)-(10) are the power series in \( \exp (-\alpha \alpha) \), i.e.

\[ E(\alpha) = \sum_{i=0}^{\infty} A_i e^{-i \alpha} \]  

\[ F(\alpha) = \sum_{i=0}^{\infty} B_i e^{-i \alpha} \]  

\[ G(\alpha) = G(0) + \sum_{i=0}^{\infty} C_i e^{-i \alpha} \]  

\[ (P - P_0)(\alpha) \approx \sum_{i=0}^{\infty} D_i e^{-i \alpha} \]

Let \( E'(0) = a \) and \( F'(0) = b \). Using this supposition and (12)-(15), we get the following boundary conditions for the approximate solution:

\[ E^*(0) = -2; \quad E^*(0) = a \beta - 4 b; \quad E^*(0) = -2(b^2 + 3 \beta) \]  

\[ F^*(0) = \beta; \quad F^*(0) = 4 a + b; \quad F^*(0) = 2 a b + \beta^2 - 8 \]
\[ G'(0) = 0; \quad G''(0) = -2a; \quad G'''(0) = 4; \]
\[ G^*(0) = 2(4b - a \beta); \]
\[ P'(0) = -2a; \quad P''(0) = 4; \quad P'''(0) = 8b; \]
\[ P^*(0) = 4(b^2 + 2 \beta - 3 \alpha^2) \]  

First five coefficients in (11) are calculated with the help of (10) and the boundary conditions for \( E(\alpha) \) in (15), which are as follows:

\[ A_1 = \left( -\frac{b^2 + 3 \beta}{12 \alpha^4} + \frac{7(4b - a \beta)}{12 \alpha^3} + \frac{71}{12 \alpha^2} + \frac{77 \alpha}{12} \right) \]
\[ A_2 = \left( -\frac{b^2 + 3 \beta}{3 \alpha^4} + \frac{13(4b - a \beta)}{6 \alpha^3} + \frac{59}{3 \alpha^2} + \frac{-107 \alpha}{6} \right) \]
\[ A_3 = \left( -\frac{b^2 + 3 \beta}{2 \alpha^4} - \frac{3(4b - a \beta)}{2 \alpha^3} + \frac{49}{2 \alpha^2} + \frac{39 \alpha}{2} \right) \]
\[ A_4 = \left( -\frac{b^2 + 3 \beta}{3 \alpha^4} + \frac{11(4b - a \beta)}{6 \alpha^3} + \frac{41}{3 \alpha^2} + \frac{61 \alpha}{6} \right) \]
\[ A_5 = \left( -\frac{b^2 + 3 \beta}{12 \alpha^4} + \frac{5(4b - a \beta)}{12 \alpha^3} - \frac{35}{12 \alpha^2} + \frac{25 \alpha}{12} \right) \]

Similarly we can find other coefficients involving in (17) - (19). Using the values \( a = 0.54, b = -0.62 \) and \( c = 0.886 \) from Cochran [10], we calculate the values of the coefficients \( A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4, B_5, C_1, C_2, C_3, C_4, C_5, D_1, D_2, D_3, D_4 \) and \( D_5 \) numerically. We draw the graphs of velocity components and asymptotic pressure with the dimensionless parameter \( \alpha \).

The boundary layer displacement thickness is calculated as:

\[ d = \frac{1}{\omega_0} \int_{z=0}^{\infty} v_\alpha \, dz = \int_{a=0}^{\infty} F(\alpha) \, d\alpha \]  

Total volume flowing outward the z-axis,

\[ Q = 2\pi R \int_{z=0}^{\infty} v_\alpha \, dz = 2\pi R^2 \int_{a=0}^{\infty} \omega E(\alpha) \sqrt{v_\alpha} \, d\alpha \]
\[ = -2\pi R^2 \sqrt{\omega v} G(\alpha) = 2.786094 R^2 \sqrt{\omega v} \]
\[ = 2.786094 R^2 \sqrt{\frac{\alpha}{z}} \]

The fluid is taken to rotate at a large distance from the wall, the angle becomes
Fig. 1 Effect of rotation on radial velocity for variation in porosity parameter $\beta$

Fig. 2 Effect of rotation on tangential velocity for variation in porosity parameter $\beta$

Fig. 3 Effect of rotation on axial velocity for variation in porosity parameter $\beta$

Fig. 4 Effect of rotation on pressure for variation in porosity parameter $\beta$

Comparing Figs. 1 and 4, we conclude that when the radial velocity increases, the pressure decreases and when the radial velocity decreases, the pressure increases. These figures have converse behavior to each other. The change in the curve for the radial velocity is faster due to external magnetic field, and magnetic field reduces the time required for velocity profile to reach their convergence level. In the present work, we have
calculated the displacement thicknesses numerically for various values of porosity parameter. Here, the displacement thicknesses are $d_1 = 1.4020582$, $d_2 = 2.309985$, $d_3 = 3.248487$ and $d_4 = 4.217463$ for $\beta = 0, 1, 2$ and 3, respectively. Whereas in the similar study conducted by Benton [11] in absence of revolutionary effect and porous medium both, the displacement thickness was 1.2714. This much difference comes out due to the effect of porous medium. In nut shell, we can conclude that the boundary layer displacement thickness is increasing with increase in porosity parameter $\beta$.

Also, we have calculated the angle between the wall and ferrofluid, which is 41°.

IV. CONCLUDING REMARKS

In nut shell, the porosity parameter and revolution of ferrofluid have appreciable effects on the ferrofluid flow. The values of radial, tangential velocity components and pressure get increased on increasing the porosity parameter whereas the axial velocity remains almost unaffected. Also revolution of ferrofluid in porous medium leads to a slow convergence rate of the various flow characteristics. Also, the displacement boundary layer gets thicker due to porous medium

NOMENCLATURE

$\vec{H}$ Magnetic field intensity
$M$ Magnetization
$p'$ Fluid pressure
$p$ Reduced fluid pressure
$\vec{V}$ Velocity of ferrofluid
$\nu$ Kinematic viscosity
$\mu_1$ Reference viscosity of fluid
$\mu_0$ Magnetic permeability of free space
$\rho$ Fluid density
$\chi$ Magnetic susceptibility
$\nabla$ Gradient operator
$K$ Darcy Permeability
$\alpha$ Dimensionless parameter
$\beta$ Viscosity variation parameter
$r$ Radial direction
$\theta$ Tangential direction
$z$ Axial direction
$\omega$ Angular velocity of disk
$\omega$ Velocity of fluid revolution
$V_r$ Radial velocity
$V_\theta$ Tangential velocity
$V_z$ Axial velocity
$\phi_0$ Angle of fluid rotation
$Q$ Total volume of fluid flowing outward the z-axis

REFERENCES


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