Investigation of Thermal and Mechanical Loading on 
Functional Graded Material Plates

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Abstract—This paper interested in the mechanical deformation behavior of shear deformable functionally graded ceramic-metal (FGM) plates. Theoretical formulations are based on power law theory when build up functional graded material. The mechanical properties of the plate are graded in the thickness direction according to a power-law. Displacement and stress is obtained using finite element method (FEM). The load is supposed to be a uniform distribution over the plane surface (XY plane) and varied in the thickness direction only. An FGM’s gradation in material properties allows the designer to tailor material response to meet design criteria. An FGM made of ceramic and metal can provide the thermal protection and load carrying capability in one material thus eliminating the problem of thermo-mechanical deformation behavior. This thesis will explore analysis of FGM flat plates and shell panels, and their applications to structural problems. FGMs are first characterized as flat plates under pressure in order to understand the effect variation of material properties has on structural response. In addition, results are compared to published results in order to show the accuracy of modeling FGMs using ABAQUS software.

Keywords—Functionally graded material, finite element method, thermal and structural loading.

I. INTRODUCTION

This study, focus on functionally graded materials with mechanical loadings. The term “functionally graded materials (FGMs)” is now widely used by the materials community for a class of materials exhibiting spatially inhomogeneous microstructures and properties. Graded materials in themselves are not something new, but what is exciting about them is the realization that gradients can be designed at a micro structural level to tailor specific materials for their functional performance in particular applications.

Functionally Graded Materials (FGM) have continuous variation of material properties from one surface to another unlike a composite which has stepped (or discontinuous) material properties. The gradation of properties in an FGM reduces the thermal stresses, residual stresses, and stress concentrations found in traditional composites. An advanced composite materials known as functionally graded material (FGM) have received an appreciable consideration in structural engineering design, especially when the materials are subjected to extremely high thermal loading. The material property of the FGM can be tailored to accomplish the specific demands in different engineering utilization to achieve the advantage of the properties of individual material. This is possible due to the material composition of the FGM changes sequentially in a preferred direction. The applicability of this material is that it eliminates the interface problem due to proficient and continuous change of material properties from one surface to the other [1], [2].

The mechanical deformation of FGM structures have attracted the attention of many researchers in the past few years in different engineering applications which include design of nuclear power plants, heat engine components and aerospace structures etc. A number of plate theories are available to analyze the deformations of composite plates. The foremost constraint of using the classical Kirchhoff plate theory (CLPT) is that it ignores transverse shear effects and consequently provides reasonable results for relatively thin plates [3]. To abstain from the said complication, earlier attempts were made by Reissner [4] and Mindlin [5]. However, a shear-correction factor is needed to eliminate the problem of a constant transverse shear stress distribution. Several authors have used the first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) kinematics to analyze the deformation characteristics of FGM plates. For example, Reddy [6] presented the mathematical formulation in conjunction with finite element model, based on third order shear deformation theory for static and dynamic analysis of the FGM plates.

"Functionally Graded Materials (FGM) are a class of composites that have continuous variation of material properties from one surface to another and thus alleviate the stress concentration found in laminated composites. The gradation in properties of the material reduces thermal stresses, residual stresses, and stress concentrations" [7].

A functionally graded structure is defined as, "those in which the volume fractions of two or more materials are varied continuously as a function of position along certain dimension(s) of the structure to achieve a required function" [6]. Because the properties change throughout the dimension (typically the thickness) of a material, FGMs can provide designers with tailored material response and exceptional performance in thermal and mechanical environments. For example, the Space Shuttle utilizes ceramic tiles as thermal protection from heat generated during re-entry into the Earth’s atmosphere. However, these tiles are prone to cracking at the tile / superstructure interface due to differences in thermal expansion coefficients. Typically, FGMs are made from a mixture of metals and ceramics and are further characterized by a smooth and continuous change of the mechanical properties..."
properties from one surface to another. The ceramic constituents of FGMs are able to withstand high temperature environments due to their better thermal resistance characteristics, while the metal constituents provide stronger mechanical performance and reduce the possibility of catastrophic fracture. FGMs are now developed for an increased use as face sheets of sandwich structures, and in such a case the face sheet may be modeled as an FGM plate resting on an elastic foundation.

An FGM made of ceramic and metal can provide the thermal protection and load carrying capability in one material thus eliminating the problem of cracked tiles found on the Space Shuttle.

An FGM composed of ceramic on the outside surface and metal on the inside surface eliminates the abrupt transition between coefficients of thermal expansion, offers thermal/corrosion protection, and provides load carrying capability. This is possible because the material composition of an FGM changes gradually through-the-thickness; therefore, stress concentrations from abrupt changes in material properties (i.e., coefficients of thermal expansion) are eliminated. A real-world application example was given by [8].

Conventional aluminum was selected for the main structure and a layer of heat resistant material for protecting it. The properties of aluminum demand that the maximum temperature of the vehicle's structure be kept below 175°C in operation. But aero thermal heating during the re-entry process creates high surface temperature which is well above the melting point of aluminum (660°C). Thus, an effective insulator was needed. A silica-based insulation material was decided for the heat-resistant tiles and other coverings to protect the Shuttle’s airframe. Fig. 1 shows seven different materials which cover the external surface of the Space Shuttle according to the temperature variation during the re-entry.

The materials were chosen by their weight efficiency and stability at high temperature. The areas of the highest surface temperature in the Shuttle, the forward nose cap and the leading edge of the wings, are made with Reinforced Carbon-Carbon (RCC). There are two main types of tiles, referred to as Low-temperature Reusable Surface Insulation (LRSI) and High-temperature Reusable Surface Insulation (HRSI). Relatively low temperature of surface where the maximum surface temperature runs between 370 and 650°C is covered by LRSI. HRSI covers the areas where the maximum surface temperature runs between 650 and 1,260°C. Metallic thermal production system (TPS) is considered as a much-needed alternative to the ceramic-based brittle tile and thermal-blanket surface insulation currently used on the Space Shuttle. Metallic thermal production system offers the significant advantages [9]. This study will explore FGM applications in severe thermal environments, such as aerospace and space vehicles. FGM performance is first characterized under thermal environments and mechanical loading in order to understand the unique characteristics of FGMs and to compare FGM structural response to traditional metal structure. The results for simply supported square and rectangular plates based on the finite element approach are compared with those obtained analytically for classical laminated plate theory [10] and the higher-order shear deformation theories [7], [11]. The results are completely similar to those analytical solutions.

II. METHODOLOGY

A. Research Focus and Developments

In this study, to establish the methodology used to model FGMs with finite elements. Coordinate systems, boundary conditions, theoretical formulation and details of the finite element analysis are discussed or defined. Units are as follows: length [m], pressure [N/m²], temperature [°C], expansion [1/°C], density [kg/m³], conductivity [W/m°C], and heat transfer film coefficient [W/m²°C].

Displacements in the x, y, and z directions are noted by u, v, and w, respectively. Rotation about the x and y axes are noted by dw/dx and dw/dy, respectively.

Because the material properties of the FGM change throughout the thickness, the numerical model must be broken up into various “slices” in order to capture the change in properties. These "slices" capture a finite portion of the thickness and are treated like isotropic materials. Material properties are calculated at the mid-plane of each of these "slices" using the power law equation previously outlined. The "slices" and their associated properties are then layered together to establish the through-the-thickness variation of material properties. Although the layered "slices" do not reflect the gradual change in material properties, a sufficient number of "slices" can reasonably approximate the material gradation. Fig. 2 shows how the thickness has been discretized into nine slices, that ply one is at the bottom of the plate, and that the origin of the z axis is at the mid-plane of the plate with +z in the direction of the top surface.

Fig. 1 Thermal protection system in Space Shuttle, (a) Temperature variation during re-entry (b) Location of different materials [8]
An FGM is defined to be a material which has a continuous gradation through the thickness (h). One side of the material is typically ceramic and the other side is typically metal. A mixture of the two materials composes the through-the-thickness characteristics. This material variation is dictated by a parameter, “n.” At n = 0 the plate is a fully ceramic plate while at n = ∞ the plate is fully metal. Material properties are dependent on the n value and the position in the plate and vary according to a power law. "The typical material property P is varied through the plate thickness according to the expressions (a power law).

\[ P = P_t V_t(z) + P_b V_b(z) \]  \hspace{1cm} (1)

where \( P_t \) and \( P_b \) denote the particular property being considered at the top and bottom faces of the plate, respectively, and \( n \) is the parameter that dictates the material variation profile through-the-thickness [7]. The variation of the volume fraction, \( V_f \) of ceramic through-the-thickness of the plate is an indicator of the volumetric fraction of ceramic at a given location. This volume fraction is based on the mixture of metal and ceramic and is an indicator of the material composition (volumetric wise) at any given location in the thickness. If the volume fraction of ceramic is defined as \( V_f \) then the volume fraction of metal is the remainder of the material, or \( 1 - V_f \). E, G, \( \nu \), \( \rho \), \( \alpha \), and \( k \) vary according to the power law and their calculated values are entered into ABAQUS accordingly.

\[ V_t(z) + V_b(z) = 1 \]  \hspace{1cm} (2)

The effective properties of functionally graded material are obtained according to a simple power–law. The volume fraction of the constituent of the top surface of the plate follows a simple power–law as,

\[ V_f = \left( \frac{2z + h}{2h} \right)^n \]  \hspace{1cm} (3)

where \( n \) is the non-negative volume fraction index which prescribes the material variation profile through the thickness of the plate and may be adjusted to obtain the optimum distribution of the constituent material. It is ascertained that the effective Young’s modulus \( E \) and thermal expansion coefficient \( \alpha \) are the temperature dependent. However, the mass density \( \rho \) and the thermal conductivity \( k \) are independent of the temperature.

Poisson’s ratio \( \nu \) is assumed to be constant as it weakly depends on the temperature changes. From (1) and (3), the effective material properties with two constituents for graded plates can be expressed as,

Material properties are given Table I. Ceramics and metal combinations are Zircon (ZrO2) - Aluminum, Aluminum Oxide (Al2O3) - Steel (SUS304), Silicon Nitride (Si3N4) - Nickel (Ti-6Al-4V) in general. In this study, zircon and aluminum are used when functional graded material is created. Material property varied though the plate thickness.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>E (MPa)</th>
<th>( \nu )</th>
<th>( \rho ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramics</td>
<td>Zircon (ZrO2)</td>
<td>151</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>Aluminum Oxide (Al2O3)</td>
<td>320.24</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Silicon Nitride (Si3N4)</td>
<td>322.27</td>
<td>0.24</td>
</tr>
<tr>
<td>Metal</td>
<td>Aluminum Steel (SUS304)</td>
<td>70</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Nickel (Ti-6Al-4V)</td>
<td>105.70</td>
<td>0.29</td>
</tr>
</tbody>
</table>

III. PROBLEM FORMULATION

A. Theoretical Formulation of FGM
The “n” value is of significance because it is an exponent of the volume fraction equation. "n" essentially dictates the amount and distribution of ceramic in the plate. With higher values of "n" the plate tends toward metal (the lower surface) while lower values of "n" tend toward ceramic (the upper surface). Designers can vary the "n" value to tailor the FGM to specific applications. This thesis will characterize "n" for each of the models studied in order to provide designers with general value of "n" that will best suit their needs. Fig. 4 details the change in volume fraction through-the-thickness for the values of "n" studied (n=0.0 (fully ceramic), n=0.2, n=0.5, n=1.0, n=2.0, n=∞ (fully metal)).

It is noted that at n=0.2 there is very little metal in the material and a sharp transition from majority metal to majority ceramic is located at z/h = -0.4. As the "n" value increases the material composition tends more toward metal. It is apparent that structural designers requiring significant thermal protection should consider low values of "n" which will yield a ceramic rich plate. Designers that desire corrosion protection with high load carrying capability should consider higher values on “n” which yield a metal rich plate.

IV. SOLUTIONS METHODOLOGY

A. Finite Element Model

The goal of analyzing flat plates under thermal loading is to characterize the effect "n” has on the structural response to thermal loading. Following work published by J.N. Reddy [7], a flat Aluminum-Zircon with sides a=0.2 m and thickness h = 0.01 m is exposed to various surface temperatures. The top surface is exposed to isothermal temperatures in a range 0°C to 600°C and the bottom temperature is exposed to a constant temperature of 20°C. Note: each top surface temperature is noted that at n=0.2 there is very little metal in the material and a sharp transition from majority metal to majority ceramic is located at z/h = -0.4. As the "n” value increases the material composition tends more toward metal. It is apparent that structural designers requiring significant thermal protection should consider low values of "n" which will yield a ceramic rich plate. Designers that desire corrosion protection with high load carrying capability should consider higher values on “n” which yield a metal rich plate.

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post-processor. However, due to the unique techniques required to models FGMS, Matlab is used as a pre-processing tool to create input files more efficiently. Matlab is used to automatically generate ABAQUS input files. This powerful tool allows one to focus on the study of FGMS rather than the mundane task of entering properties by hand.

The second step was to analyze the structural response to the applied temperature profile created in the heat transfer analysis. The goal of analyzing flat plates under distributed pressure loading ($q_0a^2/Ep_th^4$) is to characterize the effect “$n$” has on the structural response to mechanical loading. Again following work published by J.N. Reddy [3], the same flat plate previously analyzed was exposed to a distributed pressure load applied to the elements and in the $-z$ direction. Nodal temperatures are applied to a structural problem with simply support boundary conditions. The difference in applied temperature and stress free reference temperature causes the material to expand, thus creating stress and out of plane displacements. This analysis was performed for both linear and non-linear geometries and for all simply supported boundary conditions.

B. Non-Linearity Background and Analysis

Knowing that plates and shells subject to thermal loading will likely experience deformation that is over 50% of the thickness, it is of interest to compare linear and non-linear solutions when characterizing FGMS. Major sources of non-linearity are geometric stiffening and material non-linearity (yielding). Geometric stiffening is caused by structural deformation which affects the structural stiffness. “An example of geometric nonlinearity is a thin flat disk, clamped around its circular boundary, and loaded by lateral pressure. If lateral deflection at the center is more than about half the thickness, the disk develops membrane stretching forces that carry a considerable portion of the load” [12].

In the examples, the stiffness matrix and reaction matrix are dependent on the displacement. Both matrices must be updated after every iteration in order to account for these stress stiffening effects. In other words, as the structure undergoes large displacements, the structural stiffness is affected by the displacement, as shown in the equation below:

\[
\left[ K + K(\delta, \delta^2) \right] \{\delta\} = \{F\} \quad (5)
\]

The non-linear stiffness matrix (Greens’ strain) is proportional to the displacement and the square of the displacement, as shown below for strain in the $x$ direction; [12]

\[
\varepsilon_{11} = \frac{\partial u_t}{\partial x_1} + \frac{1}{2} \left( \frac{\partial u_t}{\partial x_2} \right)^2 + \frac{1}{2} \left( \frac{\partial u_t}{\partial x_3} \right)^2 \quad (6)
\]

In order to evaluate the non-linear portion of the stiffness matrix, displacements are required. However, these displacements are affected by the non-linear portion of the stiffness matrix therefore the non-linear analysis becomes an interactive process. In ABAQUS, non-linear geometry is turned on using "NLGEOM = YES" in the "Step" card.

This tells ABAQUS to check the stiffness matrix to ensure that the internal and external forces balance as the load is incremented toward the desired load. If the internal / external forces and displacement do not balance with the stiffness matrix, as outlined above, then ABAQUS updates the stiffness matrix using the incremental displacement and tries again. This increment control is handled automatically by ABAQUS; however the user can provide a means for direct control. ABAQUS/Standard generally uses Newton's method as a numerical technique for solving the nonlinear equilibrium equations because convergence is much faster than with other solvers (usually modified Newton or quasi-Newton methods) for the types of nonlinear problems most often studied with ABAQUS [13]. ABAQUS/Standard uses a scheme based predominantly on the maximum force residuals following each iteration. By comparing consecutive values of these quantities, ABAQUS/Standard determines whether convergence is likely in a reasonable number of iterations.

If convergence is deemed unlikely, ABAQUS/Standard adjusts the load increment; if convergence is deemed likely, ABAQUS/Standard continues with the iteration process. In this way excessive iteration is eliminated in cases where convergence is unlikely, and an increment that appears to be converging is not aborted because it needed a few more iterations. One other ingredient in this algorithm is that a minimum increment size is specified (0.01), which prevents excessive computation in cases where buckling, limit load, or some modeling error causes the solution to stall. This control is handled internally, with user override if needed. Several other controls are built into the algorithm; for example, it will cut back the increment size if an element inverts due to excessively large geometry changes. These detailed controls are based on empirical testing [13].

V. RESULTS AND DISCUSSIONS

This part will present results from finite element models for flat plates under thermal loading and pressure loading.

Linear and non-linear plots of non-dimensional deflections (w/h) vs. load parameter ($q_0a^2/Ep_th^4$) are shown in Figs. 6 and 7.
Ceramic provides the lowest deflection because it is the stiffest material while aluminum provides the highest deflection because it is the softest material. FGM plate deflections fall in the middle with n=0.2 providing the minimum deflection. Lowering “n” tends to lower the center deflection. Of particular importance is the drastic reduction of deflections by functionally graded materials. For all “n” values, FGMs reduced deflections by approximately 50% over a metal plate. The trend of lowering “n” to lower deflection follows the same trend found in thermally loaded plates. In addition, n=0.2 provides the lowest deflection on both thermally loaded plates and pressure loaded plates.

<table>
<thead>
<tr>
<th>Displacement (z)</th>
<th>n=2</th>
<th>n=1</th>
<th>n=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=b</td>
<td>5.88E-06</td>
<td>5.38E-06</td>
<td>4.79E-06</td>
</tr>
<tr>
<td>a=b/2</td>
<td>1.51E-05</td>
<td>1.38E-05</td>
<td>1.23E-05</td>
</tr>
<tr>
<td>a=b/4</td>
<td>2.08E-05</td>
<td>1.91E-05</td>
<td>1.70E-05</td>
</tr>
</tbody>
</table>

When value of "n" is increased, both functional graded material's density and material hardness increase for these reason displacements of the center point decreases.

Simple support boundary conditional and under the uniform load (q=10^5 N) FGM plate displacement is investigated. Fig. 8 is show that displacement is according to distance of plate edge on x axis and also variant 'n' value. Centre of the plate displacement maximum is seen Fig. 8.

FGMs provide designers the ability to customize material response to meet their design objectives. In addition, FGMs provide thermal protection and load carrying capability, all while alleviating the stress concentrations found in thermal protection systems. In this study, rectangular FGM plate which is loaded both temperature and pressure loading. Rectangular Zr-Al FGM flat plates with a temperature gradient of 580°C it was found that a plate with n=0.2 provides the lowest deflection and n=0.5 provides the lowest mid-plane stress (only 2.4% lower than mid-plane stress for n=0.2). These displacements and stresses are 68% and 54%, respectively, lower than a metal plate. It can be concluded that a non-linear analysis with a range of “n” from 0.2 to 0.5 provides the optimum solution to thermally loaded plates.
studied in this thesis (n=0.2 corresponds to a plate that is 83.3\% by volume Z_{i}).

For rectangular Zi-Al FGM flat plates under a non-dimensional pressure loading of 30 it was found that a plate with n=0.2 provides the lowest deflection and lowest mid plane stress; the displacements and stresses are 50\% and 4\%, respectively, lower than a metal plate. It can be concluded that a non-linear analysis with n=0.2 provides the optimum solution to pressure loaded plates studied in this thesis.

It is important to note that n=0.2 performed best in nearly every FGM plate and panel analyzed in this thesis, both under pressure and thermal loading.

REFERENCES