World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:7, No:11, 2013

# The Link between Unemployment and Inflation Using Johansen's Co-Integration Approach and Vector Error Correction Modelling

Sagaren Pillay



***Abstract*—**In this paper bi-annual time series data on unemployment rates (from the Labour Force Survey) are expanded to quarterly rates and linked to quarterly unemployment rates (from the Quarterly Labour Force Survey). The resultant linked series and the consumer price index (CPI) series are examined using Johansen's cointegration approach and vector error correction modeling. The study finds that both the series are integrated of order one and are cointegrated. A statistically significant co-integrating relationship is found to exist between the time series of unemployment rates and the CPI. Given this significant relationship, the study models this relationship using Vector Error Correction Models (VECM), one with a restriction on the deterministic term and the other with no restriction.

A formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011 is presented. For the given period, the CPI was found to be an unbiased predictor of the unemployment rate. This relationship can be explored further for the development of appropriate forecasting models incorporating other study variables.

***Keywords*—**Forecasting, lagged, linear, relationship.

## I. Introduction

REFERENCE [8], carried out several formal cointegration tests for the relationships between inflation, unemployment and labour force change rate and obtained an overall confidence in the existence of a true linear and lagged link between the variables. According to [7], the links between inflation and unemployment demonstrate various and even opposite dependencies. In the USA, this dependence is characterized by a positive influence of inflation on unemployment. Effectively, low inflation in the USA leads low unemployment by three years. Studies have also shown that there is very little evidence to support a significant relationship between real wage growth and industry- level employment growth [4].

In explaining the relationship between inflation and unemployment, [9] found that although there are periods where there is a clear trade-off between inflation and unemployment there are periods where both inflation and unemployment change in the same direction. Further, using a panel from ten different OECD countries, from 1950 to 2005, [1] researchers applied panel cointegration methodologies to find statistical evidence for a relationship between these real wages and employment variables. A study has also shown that real wages has a short run negative impact on employment but in the long run this relationship was shown to be positive [12].

In this study, the two variables (unemployment rate and CPI), being non-stationary I (1), were found to be cointegrated in a statistical sense. This means that their residual time series in the vector error correction model (VECM) representation proves to be stationary. The models for both a VECM model with a restriction on the deterministic term and one with no restriction is presented.

In interpreting the two models one must consider that, modeling cointegrated series is difficult because of the need to model systems of equations in which one has to simultaneously specify the deterministic terms and how they enter, determine the lag length, and ensure a congruent representation [6].

## II. Data

The Labour Force Survey (LFS) was introduced in September 2000 and was published on a biannual basis until March 2008 when the Quarterly Labour Force Survey (QLFS) was introduced. This resulted in a discontinuous series that made analysis of unemployment estimates very difficult.

In this study, the LFS biannual unemployment rate time series from September 2000 to September 2007 and the QLFS quarterly unemployment rate time series for the period March 2008 to June 2011 were combined and adjusted for the purpose of analysis. The biannual unemployment rates were converted to quarterly rates using the SAS procedure PROC EXPAND. The procedure uses the SPLINE method by fitting a cubic spline curve to the input values. A cubic spline is a segmented function consisting of third-degree (cubic) polynomial functions joined together so that the whole curve and its first and second derivatives are continuous.

The CPI rates (an economic indicator of inflation) were used for matching quarters with the data for the corresponding unemployment rates.

Sagaren Pillay is with the Statistics South Africa, Pretoria, South Africa (e-mail: sagarenp@statssa.gov.za).

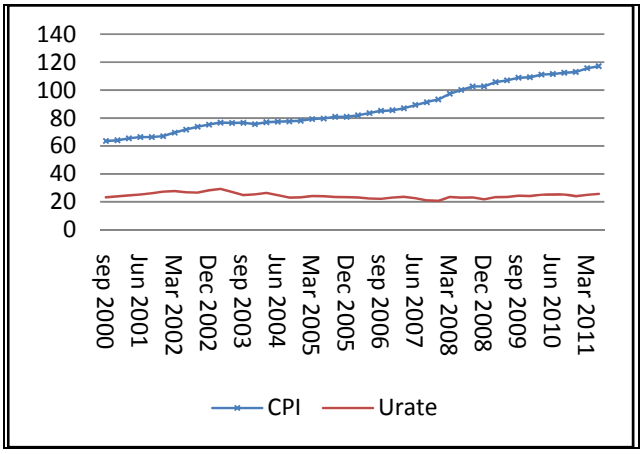


Fig. 1 Graphical representation of unemployment rates and CPI

The graphical representation (Fig. 1) shows the plot of the expanded LFS series combined with the QLFS series and the plot of the CPI for the corresponding months at the end of each quarter.

From the graphs we observe that there are no large changes in the data or outliers that will influence the estimates with a large weight and, hence, potentially bias the estimates, so there is no need for deterministic components, such as intervention dummies in the model specification.

## III. Methodology and Results

### A. Unit Root Test

The first step in the time series analysis was to determine whether the two series are stationary or non-stationary in nature. If the time series are I (1), they have to be characterized by the presence of a unit root and their first difference by the absence of unit roots [6].

The Augmented Dickey Fuller (ADF) unit root test was used to determine whether the series was stationary or non-stationary. The Dickey- Fuller tests for non stationarity of each of the series is shown below (Table I). The null hypothesis is to test a unit root. The ADF test constructs a model with higher order lag terms and tests the significance of the parameter estimates using a non-standard *t-test*. The model used for this routine is $\Delta y_t = \alpha_1 y_{t-1} + \beta_1 \Delta y_{t-1} + \ldots + \beta p\text{-}_1 \Delta y_{t-p+1} + \varepsilon_t$, where the *t-test* checks significance of the $\alpha_1$a term. This procedure is available in most time series analysis software packages.

Consequently, both series have a unit root and their first differences do not have any. Thus, the variables URate and CPI are first order difference stationary and are integrated of order I (1).

TABLE I
Dickey-Fuller Unit Root Test

| Variable | Type | Rho | Pr<Rho | Tau | Pr<Tau |
|---|---|---|---|---|---|
| Urate | Zero Mean | 0.02 | 0.6828 | 0.07 | 0.7002 |
| | Single Mean | -9.54 | 0.1271 | -2.08 | 0.2553 |
| | Trend | -12.61 | 0.2292 | -2.29 | 0.4286 |
| CPI | Zero Mean | 0.61 | 0.826 | 3.99 | 0.9999 |
| | Single Mean | 0.52 | 0.9747 | 0.92 | 0.9948 |
| | Trend | -2.88 | 0.9361 | -1.12 | 0.9131 |

## IV. Cointegration Test

The Johansen and Julius$\lambda_{trace}$ cointegration statistic test for testing the null hypothesis that there are at most r cointegrated vectors is used versus the alternative Hypothesisof more than r cointegratedvectors. Where:

$$\lambda_{trace} = -T \sum_{\lambda=r+1}^{k} log(1-\lambda_i) \quad (1)$$

Theory holds that two time series variables will be cointegrated if they have a long term or equilibrium relationship between them [5]. Given that both series are I (1) imply that their linear combination is I (0). Johansen's test has a number of desirable properties, including the fact that all test variables are treated as endogenous variables [11]. The maximum lag length was set to 7 quarters and an autoregressive order of p=6 were selected based on the partial correlation matrices and partial canonical matrices [10]. The SAS procedure PROCVARMAX was used to test for cointegration and model fitting. The results of the cointegration tests are shown below (Table II).

Tables II A & B show the output from the VARMAX procedure based on the model specified (an intercept term is assumed). In the cointegration rank test using trace (Table II A), we observe that there is no separate drift in the ECM and the process has a constant drift before differencing. These trace statistics are based on the alternate hypothesis (H1) that there is a separate drift and no separate linear trend in the VECM.

The cointegration rank test using trace under restriction (Table II B) shows the trace statistics based on the null hypothesis (H0) that there is no separate drift in the VECM but a constant enters only via the error correction term.

In both cases the series are cointegrated with rank=1 because the trace statistics are smaller than the critical values. In the unrestricted case, Johansen's trace statistic has a value of 16.076 which is greater than the 5% critical value of 15.34, therefore we reject r=0. Further, the test for r=1 versus r>1 does not reject r=1. Thus, Johansen's test indicates a single (r=1) cointegrating vector.

The study proceeds to determine which result, either the model with restriction or the model with no restriction, is appropriate depending on the significance level. Since the cointegration rank is chosen to be 1and the p-value is 0.0549, the hypothesis H0 cannot be rejected at 5% significance level but can be rejected at the significance level of 10% (Table III).

Since U Rate and CPI are cointegrated, according to the Granger representation theorem a cointegrated series can be represented by a vector error correction model (VECM) [2].

For H0, a VECM (6) model with a restriction on the deterministic term will be used and a similar model with no restriction will be used for H1.

A Vector Error Correction Model (VECM) of order p can be written as:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta_1 y_{t-1} + \ldots + \Delta_{p-1} y_{t-p+1} + \varepsilon_t \epsilon Z \qquad (2)$$

where

yt is a k x 1 random vector

the sequence yt is a Var(p) process

yt ~ CI(1)

$\Pi = \alpha\beta^t$ where α is the adjustment coefficient and β the cointegrating vector

$\Gamma_{1\ldots}, \Gamma_k$ are fixed coefficient matrices

$\varepsilon_t$ is a k x 1 white noise process

The results of the fitted VECM (6) model with a restriction on the deterministic term are shown below.

TABLE IIA
COINTEGRATION RANK TEST USING TRACE

| H0: Rank=r | H1: Rank>r | Eigenvalue | Trace | 5% Critical Value | Drift in ECM | Drift in Process |
|---|---|---|---|---|---|---|
| 0 | 0 | 0.2939 | 16.076 | 15.34 | Constant | Linear |
| 1 | 1 | 0.0723 | 2.8521 | 3.84 | | |

TABLE IIB
COINTEGRATION RANK TEST USING TRACE UNDER RESTRICTION

| H0: Rank=r | H1: Rank>r | Eigenvalue | Trace | 5% Critical Value | Drift in ECM | Drift in Process |
|---|---|---|---|---|---|---|
| 0 | 0 | 0.4215 | 27.335 | 19.99 | Constant | Constant |
| 1 | 1 | 0.158 | 6.5366 | 9.13 | | |

TABLEIII
HYPOTHESIS TEST OF THE RESTRICTION

| Rank | Eigenvalue | RestrictedEigenvalue | DF | Chi-Square | Pr > ChiSq |
|---|---|---|---|---|---|
| 0 | 0.2939 | 0.4215 | 2 | 11.26 | 0.0036 |
| 1 | 0.0723 | 0.158 | 1 | 3.68 | 0.0549 |

TABLE IV
ALPHA AND BETA ESTIMATES

| Long-Run Parameter Beta Estimates When RANK=1 | | Adjustment Coefficient Alpha Estimates When RANK=1 |
|---|---|---|
| Variable | 1 | 1 |
| URate ($Y_1$) | 1.000 | 0.00768 |
| CPI ($Y_2$) | 0.02331 | -0.26592 |
| 1 | -31.09951 | --- |

The estimates of the long run parameter β, and the adjustment coefficient, α, are given in the table above. Since the cointegration rank is 1 in the bivariate system, α and $\beta^t$ are (2x1) and (1x3) vectors respectively. The estimated cointegrating vector is $\beta^t$ = [1, 0.02331, -31.09951].The first element of β is 1 since $y_1$is specified as the normalised variable. The impact matrix is: $\pi = \alpha\beta^t$ becomes

$$\begin{bmatrix} 0.00768 \\ -0.26592 \end{bmatrix} [\,1.000 \quad 0.023 \quad -31.09 = \begin{bmatrix} 0{,}07680 & 0{,}00018 & -0{,}23885 \\ -0{,}26592 & -0{,}00620 & 8{,}27006 \end{bmatrix}$$

The long run relationship of the series is

$$\beta^t Y_t = [\,1 \quad 0.023 \; -31.099] \begin{bmatrix} y_{1,} \\ y_2 \\ 1 \end{bmatrix} = Y_{1t} + 0.023 Y_{2t} - 31.099$$

$$Y_{1t} = 31.099 - 0.023\, Y_{2t}$$

Based on the output of the Varmax procedure the model can be written using (2):

$$\Delta y_t =$$
$$\begin{bmatrix} 0{,}0768 & 0{,}00018 & -0{,}23885 \\ -0{,}26592 & -0{,}00620 & 8{,}27006 \end{bmatrix} \begin{bmatrix} y_{1,}t_{-1} \\ y_{2,}t_{-1} \\ 1 \end{bmatrix} +$$
$$\begin{bmatrix} 0{,}12398 & -0{,}02172 \\ -0{,}04139 & 0{,}28424 \end{bmatrix} \Delta y_{t-1} + \begin{bmatrix} -0{,}20272 & 0{,}27722 \\ 0{,}52267 & 0{,}12452 \end{bmatrix} \Delta y_{t-2} +$$
$$\begin{bmatrix} -0{,}41346 & 0{,}16301 \\ -0{,}05825 & -0{,}01603 \end{bmatrix} \Delta y_{t-3} + \begin{bmatrix} 0{,}58462 & -0{,}21349 \\ 0{,}33003 & 0{,}13659 \end{bmatrix} \Delta y_{t-4} +$$
$$\begin{bmatrix} 0{,}01262 & -0{,}17129 \\ 0{,}54388 & -0{,}47806 \end{bmatrix} \Delta y_{t-5} + \epsilon_t$$

## V. MODEL DIAGNOSTICS

Checking the assumptions of the model, (i.e., checking the white-noise requirement of the residuals, and so on), is not only crucial for correct statistical inference, but also for the economic interpretation of the model as a description of the behavior of rational agents [6]. Various tests such as tests for autocorrelation in the squares are able to detect model failures [3].

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ U Rate resulted in a model F test 3.7 and R-square of 0.64. Similarly the regression of Δ CPI resulted in a model F test of 4.3 and R-square of 0.674 (Table V).

The residuals are checked for normality and autoregressive conditional heteroskedasticity or ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are

homoscedastic. The results also show that there are no ARCH effects on the residuals since the "no ARCH" hypothesis cannot be rejected given the F values (Table VI).

There are no AR effects on the residuals - for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table VII).

TABLE V
UNIVARIATE MODEL ANOVA DIAGNOSTICS

| Variable | R-Square | Standard Deviation | F Value | Pr > F |
|---|---|---|---|---|
| URate | 0.6400 | 0.6644 | 3.7 | 0.0028 |
| CPI | 0.6735 | 0.5951 | 4.3 | 0.0010 |

TABLE VI
UNIVARIATE MODEL WHITE NOISE DIAGNOSTICS

| Variable | Durbin Watson | Normality | | ARCH | |
|---|---|---|---|---|---|
| | | Chi-Square | Pr > ChiSq | F Value | Pr > F |
| URate | 1.9628 | 1.71 | 0.4256 | 1.12 | 0.2981 |
| CPI | 1.7549 | 1.27 | 0.5287 | 0.56 | 0.4585 |

TABLE VII
UNIVARIATE MODEL AR DIAGNOSTICS

| Variable | AR1 | | AR2 | | AR3 | | AR4 | |
|---|---|---|---|---|---|---|---|---|
| | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F |
| URate | 0 | 0.9733 | 0.01 | 0.9866 | 0.01 | 0.998 | 0.26 | 0.900 |
| CPI | 0.19 | 0.6673 | 0.07 | 0.9361 | 0.25 | 0.8621 | 0.41 | 0.803 |

TABLE VIII
FITTED MODEL FOR H1: VECM (6) MODEL WITH NO RESTRICTION

| Long-Run Parameter Beta Estimates When RANK=1 | | Adjustment Coefficient Alpha Estimates When RANK=1 |
|---|---|---|
| Variable | 1 | 1 |
| URate ($Y_1$) | 1.000 | -0.11375 |
| CPI ($Y_2$) | -0.00191 | -0.26517 |

The estimates of the long –run parameter β, and the adjustment coefficient, α, are given in the table above. Since the cointegration rank is 1 in the bivariate system, α and β are two dimensional vectors. The estimated cointegrating vector is $\beta^t$ = [1 -0.00191].The first element of β is 1 since $y_1$ is specified as the normalised variable. The impact matrix is: $\pi = \alpha\beta^t$, becomes

$$\begin{bmatrix} -0.11375 \\ -0.26517 \end{bmatrix} [\,1.000 \;\; -0.00191] = \begin{bmatrix} -0.11375 & 0.00022 \\ -0.26517 & 0.00051 \end{bmatrix}$$

The long run relationship of the series is

$$\beta^t Y_t = [\,1 \;\; -0.00191] \begin{bmatrix} y1 \\ y2 \end{bmatrix} = Y_{1t} - 0.00191\, Y_{2t}$$

$$Y_{1t} = 0.00191\, Y_{2t}$$

Based on the output of the Varmax procedure the model can be written using (2):

$$\Delta y_t = \begin{bmatrix} -0{,}11375 & 0{,}00022 \\ -0{,}26517 & 0{,}00051 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0{,}14007 & -0{,}03246 \\ -0{,}07601 & 0{,}29286 \end{bmatrix} \Delta y_{t-1} + \begin{bmatrix} -0{,}12656 & 0{,}27726 \\ 0{,}52433 & 0{,}11732 \end{bmatrix} \Delta y_{t-2} + \begin{bmatrix} -0{,}34873 & 0{,}16542 \\ -0{,}07007 & -0{,}00901 \end{bmatrix} \Delta y_{t-3} + \begin{bmatrix} 0{,}57513 & -0{,}17569 \\ 0{,}31159 & 0{,}13770 \end{bmatrix} \Delta y_{t-4} + \begin{bmatrix} 0{,}10943 & -0{,}16003 \\ 0{,}56371 & -0{,}51045 \end{bmatrix} \Delta y_{t-5} + \begin{bmatrix} 2.61048 \\ 7.66795 \end{bmatrix} + \epsilon_t$$

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ URate resulted in a model F test 4.33 and R-square of 0.675. Similarly the regression of Δ CPI resulted in a model F test of 4.5 and R-square of 0.676 (Table IX).

The residuals are checked for normality and ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals (Table X).

There are no AR effects on the residuals for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table XI).

TABLE IX
UNIVARIATE MODEL ANOVA DIAGNOSTICS

| Variable | R-Square | Standard Deviation | F Value | Pr > F |
|---|---|---|---|---|
| URate | 0.6751 | 0.6292 | 4.33 | 0.001 |
| CPI | 0.676 | 0.5927 | 4.35 | 0.0009 |

TABLE X
UNIVARIATE MODEL WHITE NOISE DIAGNOSTICS

| Variable | Durbin Watson | Normality | | ARCH | |
|---|---|---|---|---|---|
| | | Chi-Square | Pr > ChiSq | F Value | Pr > F |
| URate | 1.9978 | 2.55 | 0.2791 | 3.35 | 0.0757 |
| CPI | 1.7753 | 1.28 | 0.5262 | 0.53 | 0.4713 |

TABLE XI
UNIVARIATE MODEL AR DIAGNOSTICS

| Variable | AR1 | | AR2 | | AR3 | | AR4 | |
|---|---|---|---|---|---|---|---|---|
| | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F |
| URate | 0.06 | 0.8149 | 0.45 | 0.6418 | 0.26 | 0.8565 | 0.35 | 0.8426 |
| CPI | 0.11 | 0.7392 | 0.04 | 0.9634 | 0.25 | 0.8575 | 0.42 | 0.7894 |

### *A. Testing Weak Exogeneity*

Results from the weak exogeneity test indicate that the unemployment rate (URate) is a weak exogeneity of the consumer price index (CPI), whereas the CPI is not a weak exogeneity of URate (Table XII).

TABLE XII
TESTING WEAK EXOGENEITY OF EACH VARIABLE

| Variable | DF | Chi-Square | Pr > ChiSq |
|---|---|---|---|
| URate | 1 | 1.94 | 0.164 |
| CPI | 1 | 10.02 | 0.0016 |

## VI. CONCLUSION

The expected result of the above analysis consists in a formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011. Hence, the two variables, being non-stationary I (1), are cointegrated; i.e. their residual time series in the VECM representation has been proved to be stationary. Johansen's cointegration techniques were applied to investigate the long run relationship between inflation and unemployment. The results indicate the existence of one cointegrating vector amongst the variables. Further, the weak exogeneity results indicate that the unemployment rate is a weak exogeneity of the CPI, whereas the CPI is not a weak exogeneity of the unemployment rate. This relationship found in this study can be explored further for the development of appropriate forecasting models incorporating other study variables.

## REFERENCES

[1] N. Apergis, L. Theodosiou, "The employment-wage relationship: was Keynes right after al",*American review of political economy*, 2008, 6(1): 40-50.
[2] R.F. Engel and C.W.J. Granger, "cointergration and error correction: representation, estimation and testing", *Econometrica*, 1987, 55(2):251-276.
[3] R. Engle, "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics", *Journal of Economic Perpectives*, 2001, 15(4), 157-168.
[4] M. Galizzi, "Wage changes, establishment growth, and the effect of composition bias" *Eastern economic journal*, 2005, 31(2).
[5] D. Gujarati, *Basic Econometrics*, 3rd ed, McGraw Hill Publishers, 2003.
[6] D. F. Hendry, and K. Juselius, Explaining Cointegration Analysis: Part ii. Department of Economics, University of Copenhagen, Denmark, 2001.
[7] I. Kitov, Inflation, unemployment, labor force change in the USA, working papers 28, ECINEQ, society *for the study of economic inequality*, 2006b.
[8] I. Kitov, O. Kitov, S. Dolinskaya, Relationship between inflation, unemployment and labor force change rate in France: cointegration test, MPRA Paper 2736, University library of Munich, Germany, 2007a.
[9] G. Stephanides, "Measuring the NAIRU: Evidence from the European Union, USA and Japan", *International Research journal of Finance and Economics*. 3, 2006.
[10] G.C. Tiao, Modeling multiple time series with applications, *Journal of the American Statistical Association*, 1981, 76:802-812.
[11] C. S. Wassell, jr. P.J. Saunders, Unpublished, Time series on social security and private saving: The issue revised.
[12] S. A. Yusof, "The long-run and dynamic behaviors of wages, productivity and employment in Malaysia", *Journal of Economic studies*, 2007, 35(3), 249-262.