

Simulating Flow Transients in Conveying Pipeline Systems by Rigid Column and Full Elastic Methods: Pump Combined with Air Chamber

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Abstract—In water pipeline systems, the flow control is an integrated part of the operation, for instance, opening and closing the valves, starting and stopping the pumps, when these operations very quickly performed, they shall cause the hydraulic transient phenomena, which may cause pump and, valve failures and catastrophic pipe ruptures. Fluid transient analysis is one of the more challenging and complicated flow problems in the design and the operation of water pipeline systems. Transient control has become an essential requirement for ensuring safe operation of water pipeline systems. An accurate analysis and suitable protection devices should be used to protect water pipeline systems. The fourth-order Runge-Kutta method has been used to solve the dynamic and continuity equations in the rigid column method, while the characteristics method used to solve these equations in the full elastic methods. This paper presents the problem of modeling and simulating of transient phenomena in conveying pipeline systems based on the rigid column and full elastic methods. Also, it provides the influence of using the protection devices to protect the pipeline systems from damaging due to the gain pressure which occur in the transient state. The results obtained provide that the model is an efficient tool for flow transient analysis and provide approximately identical results by using these two methods. Moreover, using the closed surge tank reduces the unfavorable effects of transients.

Keywords—Flow transient, Pipeline, Air chamber, Numerical model, Protection devices, Elastic method, Rigid column method.

I. INTRODUCTION

IRRIGATION system design includes pumps, valves and pipes should be properly selected, sized and placed of many other components. In the design process like any hydraulic piping system, pipes should be selected with a pressure rating equal to or greater than the combination of operating plus transients pressures.

Transients can introduce large pressure forces and rapid fluid accelerations into a piping system. Due to the devastating effects that a hydraulic transient can cause, its analysis is very important in determining the values of transient pressures that can result from flow control operations and to establish the design criteria for system equipment and devices, so as to provide an acceptable level of protection against system failure due to pipe collapse or bursting.

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To reduce or even to eliminate the dangerous effects of the water-hammer; the surge devices have been added to the pipe systems. Most of these protection equipments aims to protect against unfavorable large pressure fluctuations tend to maintain the pressure at a nearly constant value at some fixed places, or tend to keep the pressure from exceeding a predetermined value [1]-[3].

Several criteria can be adopted to determine which surge devices are to be used such as the effectiveness, dependability, evaluation of cost character and frequency of maintenance requirement over an exceeded period [4].

Closed surge tank "Air chamber" is a relatively small pressurized reservoir that contains water and air. Generally, it is connected to the main pipe just at the discharge point of the pumps. Its primary purpose is to prevent the negative pressures and the column separation downstream the pumps [5], [6]. After the energy failure, the liquid drains from the air chamber to the pipe and the air volume inside the chamber expands causing a pressure drop.

When the head is very high, then it is not practical to use a surge tank with large height, thus, the air chamber (closed surge tank) can be used [7], [8], it may be one chamber or two chamber, in the case of one chamber, the amplitude of pressure oscillation during water-hammer is smaller than the case of the double air chamber [9], [10]. Some transient solutions are safest and maintenance-free, like the flow control and pressure reducing device such as relief valves and check valves [4], [11].

The use of digital computers for analyze hydraulic transients has been used tenths of years ago [1]-[3], [6], [7], [12]-[16], and increased considerably in recent years, also sophisticated numerical methods has been introduced for such analyses [1], [17], [18]. The computer simulation allows to use computer in controlling the pipe system devices [19], [20]. In this article a computer program has been developed in order to simulate and design hydraulic transients in pipeline including air chamber with two analyzing methods.

II. MATERIALS AND METHODS

Note: Most of the formulations shown in this document are taken from three references [1], [21], [8]:

1. M. H. Chaudhry, Applied hydraulic transient Second edition Ed., Van Nostrand Reinhold Company Inc, New York (1987).

2. A. Oulhaj, Ecoulements transitoires dans les conduites en charge, Institute of Agronomy and Veterinary Hassan II, Rabat, Morocco. (1998).
3. J. P. Tullis, Hydraulics of Pipelines – Pumps, Valves, Cavitation, Transients, John Wiley & Sons, New York (1989).

A. Closed Surge Tank at Pump Downstream “Instantaneous Pump Stopping”

The configuration considered in this article is shown in Figs. 1 and 2 consist of a long pipeline in which water is flowing due to effect of pump. Two Methods (rigid column and elastic) have been used to simulate and to analyze the flow transient in pipeline systems.

1. Elastic Method

When changes in velocity, and consequently pressure, occur rapidly, both the compressibility of the liquid and the elasticity of the pipe must be included in the analysis. This procedure is often called "elastic" or "water-hammer" analysis and involves acoustic pressure waves traveling through the pipe and the solution of partial differential equations. Even though the term transient refers to all unsteady flows, it is generally used to identify the "elastic" case specifically [8].

The simplified equations that govern unsteady flow in pipeline system are motion and continuity equations which solved together ((1) and (2)).

$$\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{fV|V|}{2gD} = 0 \quad (1)$$

$$\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0 \quad (2)$$

where: H is the piezometric head, V is the flow velocity, x is the distance along the pipeline, t is the time, g is the acceleration of gravity, f is the pipeline friction factor (assumed constant), D is the pipeline diameter, and a is the celerity of a pressure wave in the pipeline.

By multiplying (2) by unknown constant λ , adding it to (1), and by rearranging and taking the total derivative to obtain the compatibility (3) and (5):

$$\frac{g}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \quad C^+ \text{ equation} \quad (3)$$

$$\text{For } \frac{dx}{dt} = +a \quad (4)$$

$$\frac{g}{a} \frac{dH}{dt} - \frac{dV}{dt} - \frac{fV|V|}{2D} = 0 \quad C^- \text{ equation} \quad (5)$$

$$\text{For } \frac{dx}{dt} = -a \quad (6)$$

Solution of (3) and (5) is done by using finite differences solution. Fig. 1 illustrates a simple pump - reservoir system. The pipeline is divided to N equal sections of length Δx . The calculations were made at node 1.

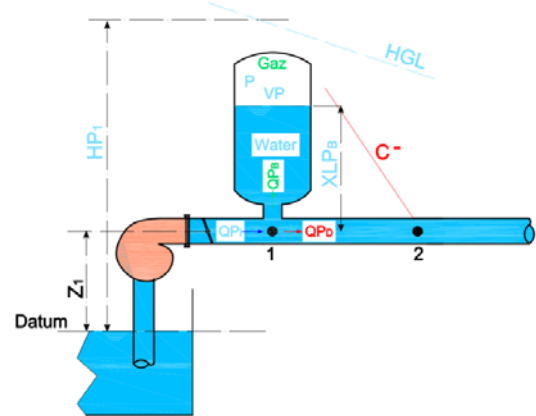


Fig. 1 Air chamber: Notations for head and flow calculations

A transient is generated at time t by pump failure; solution of the equations governing the transient phenomena consists of finding the values of head and flow at each node as the transient progresses. Calculations were made at each Δt time interval, where L is the pipeline length.

$$\Delta t = \frac{L}{aN} \quad (7)$$

In general, to calculate the head and flow at node 1 at time $t_0 + \Delta t$, the head and flow at node 2 at instant time t_0 are assumed to be known before any generated transient.

The unknown head and flow at nodes 1 at time $t_0 + \Delta t$ are labeled HP_1 , and QP_1 can be calculated by integrating (3) and (5). The known head and flow at the previous time step are H_1 and Q_1 . Before the integration, both equations multiplied by adt/g , by changing V to Q , and replacing dt by $dx = adt$. For the C^- equation, the integration was made from node 2 to node 1, therefore (8) is obtained.

$$\int_{H_2}^{H_1} dH + B \int_{Q_2}^{Q_1} dQ + R_1 \int_{x_2}^{x_1} Q|Q|dx = 0 \quad (8)$$

$$HP_1 - H_2 - B(QP_1 - Q_2) + R_1 Q_2 |Q_2| (x_1 - x_2) = 0 \quad (9)$$

$$\text{Let } \Delta x = x_1 - x_2$$

$$HP_1 - H_2 - B(QP_1 - Q_2) - RQ_2 |Q_2| = 0 \quad (10. a)$$

$$\text{where } R = R_1 \Delta x$$

By rearranging the above equation

$$HP_1 - BQP_1 = H_2 - BQ_2 + RQ_2 |Q_2| \quad (10.b)$$

$$\text{Let } CM = H_2 - BQ_2 + RQ_2 |Q_2| \quad (11)$$

Therefore (12) is obtained

$$HP_1 - BQP_1 = CM \quad (12)$$

where H_2 and Q_2 are the head and flow respectively at the nodes 2 at instant time t_0 .

$$\text{With } B = \frac{a}{gA} \text{ and } R = \frac{\lambda \Delta x}{2gDA^2}$$

The equations required to calculate the head and flow at the boundaries after introducing the air chamber at node 1, are illustrated through the following system of equations. In addition to *C-equation* that derived above, the unknown variables in these equations which identified in Fig. 1 are: QP_B , QP_D , HP_1 , VP , XLP_B and P .

$$XLP_B = XL_B + (QP_B + Q_B) \quad (13)$$

$$HP_1 = XLP_B + K_B QP_B |QP_B| + P + Z_1 - H_b \quad (14)$$

$$VP = V - \frac{1}{2}(QP_B + Q_B)\Delta t \quad (15)$$

$$P * VP^\gamma = cte \quad (16)$$

$$QP_i = QP_D + QP_B \quad (17)$$

where $QP_i = 0$ in the sudden pump stopping

$$QP_D = -QP_B \quad (18)$$

where QP_B is the exchanged discharge between the pipe and the air chamber, QP_D is the downstream discharge, QP_i is the pump discharge, HP_1 is the piezometric head at node 1, VP is the air volume inside the air chamber, XLP_B is the water level inside the air chamber, γ is the exponent of the polytropic equation and P is the gas pressure. These equations combined to obtain a single nonlinear equation in QP_B .

$$\left(CM - XL_B - Z_1 - H_b - (QP_B + Q_B) \frac{\Delta t}{2AB} - BQP_B - K_B QP_B |QP_B| \right) \left(V - \frac{1}{2}(QP_B + Q_B)\Delta t \right)^\gamma = cte \quad (19)$$

2. Rigid Column Method

Transients involving changes that occur slowly are called surges. Examples would be an oscillating U-tube, establishment of flow after a valve is opened, and the rise and fall of the water level in a surge tank. The method of surge analysis, called "rigid column theory," usually involves mathematical or numerical solution of simple ordinary differential equations. The compressibility of the fluid and the elasticity of the conduit are ignored and the entire column of fluid is assumed to move as a rigid body [8].

Fig. 2 (a) illustrates a typical closed surge tank. A pump stopping causes the flow variation, which results in the oscillations of the water level in the tank. To simplify the derivation of the dynamic and continuity equations, the following assumption were made:

1. The conduit walls are rigid, and water is incompressible.
2. The inertia of the liquid in the closed surge tank is small compared to that of water in the pipeline and can therefore be neglected.

3. The head losses in the system during the transient state can be computed by using the steady-state formulas for the corresponding flow velocities [1].

The configuration considered in Fig. 2. It consists of a long pipeline in which water is flowing due to the pump effect. For this case study, a simple system is presented in order to illustrate the derivation of equations required in rigid column method for simulating transients.

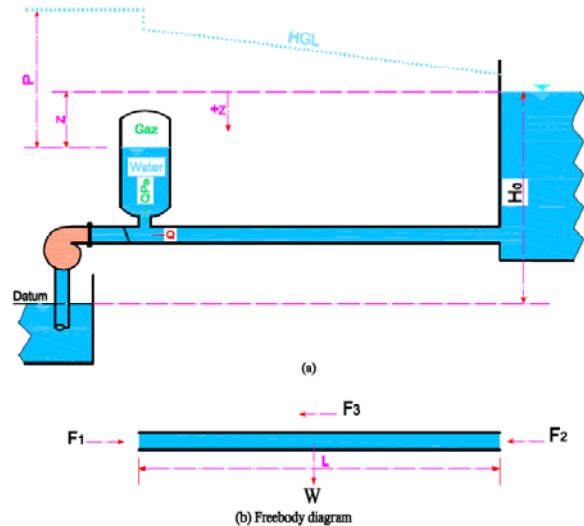


Fig. 2 Air chamber: notations for dynamic equation

Dynamic Equation

A freebody diagram of a horizontal pipeline having constant cross-sectional area is shown in Fig. 2 (b). Forces acting on the liquid are:

$$F_1 = \gamma A(H_0 + P + z) \quad (20)$$

$$F_2 = \gamma A(H_0 + h_v + h_r) \quad (21)$$

$$F_3 = \gamma A h_f \quad (22)$$

where A is the cross-sectional area of the pipeline, H_0 is the static head, γ is the specific weight of liquid, h_v is the velocity head losses at the intake, P is the gas pressure in meter, h_r is the intake head losses, h_f is the friction and form losses in the pipeline between the intake and the closed surge tank, and z is the water level difference between the closed surge tank and the reservoir level (positive downward). Considering the downstream flow direction as positive, the net force acting on the liquid element in the positive direction is

$$\sum F = F_1 - (F_2 + F_3) \quad (23. a)$$

$$\sum F = \gamma A(P + z - h_v - h_r - h_f) \quad (23.b)$$

According to Newton's second law of motion, the rate of change of momentum is equal to the net applied force. Therefore,

$$\frac{\gamma L}{g} \frac{dQ_t}{dt} = \gamma A (P + z - h_v - h_r - h_f) \quad (24)$$

In which $\gamma AL/g$ is the mass of the liquid element, L is the length of the pipeline, g is the acceleration due to gravity, Q is the pipeline flow and t is the time.

By defining the total head losses as: $h = h_v + h_r + h_f = RQ|Q|$ in which R is the flow resistance due to the singular and linear friction, (25) may be written as

$$\frac{dQ_t}{dt} = \frac{gA}{L} (P + z - RQ|Q|) \quad (25)$$

The equations that define the changes in the volume of air depending on the inflow or outflow from the air chamber must add.

$$(P + P_a)V^\gamma = (P_0 + P_a)V_0^\gamma \quad (26)$$

$$V = V_0 - Qdt \quad (27)$$

where: P_0 is the initial gas pressure (actual), V_0 is the initial gas volume at an absolute pressure $P_0 + P_a$, P_a is the atmospheric pressure and V is the volume of air at an absolute pressure $P + P_a$.

Before solving the ordinary differential equations of motion and continuity simultaneously, P must be eliminated from the equation of motion by using the last two algebraic equations. Eliminating Volume V between the two above equations, it follows that:

$$P = (P_0 + P_a) \left(\frac{V_0}{V_0 - Qdt} \right)^\gamma - P_a \quad (28)$$

By replacing P in the dynamic equation (25) by its expression above (28):

$$\frac{dQ_t}{dt} = \frac{gA}{L} \left((P_0 + P_a) \left(\frac{V_0}{V_0 - Qdt} \right)^\gamma - P_a + z - RQ|Q| \right) \quad (29)$$

Continuity Equation

Referring to Fig. 2 (a), the continuity equation for the junction of the pipeline and the closed surge tank may be written as

$$Q_p = Q_B + Q \quad (30)$$

In which Q_B is the flow into the surge tank (positive into the tank), Q_p is the pump flow, and Q is the flow in pipeline. Where $Q_p = 0$ in instantaneous pump stopping.

Oscillation in the closed tank water level is defined by the continuity equation which is written:

$$A_B dz = Q_B dt \quad (31.a)$$

$$\frac{dz}{dt} = \frac{Q_B}{A_B} \quad (31.b)$$

The flow into the air chamber being opposite sign of the fluid in the pipe, the continuity equation according to the flow Q is reduced to:

$$\frac{dz}{dt} = -\frac{Q}{A_B} \quad (32)$$

where z is the difference in water level in the tank compared to the water level in the reservoir.

The dynamic and continuity (29) and (32) constitute the system of differential equations governing the unsteady flow between the surge tank and the reservoir after pumping stops.

There is no need for spatial derivatives of the dynamic and continuity equations, because the pipeline and the liquid were assumed to be rigid, and the flow and the liquid level in the tank vary with respect to time only. If the losses are not neglected, the second order differential equation that describes the oscillations of the water level in the air chamber, being non-linear due to the presence of the term $RQ|Q|$, and can be solved analytically. The only alternative is to solve numerically, using for example the iterative methods. The fourth-order Runge-Kutta method has been used, which ensure sufficiently accurate results that are of the same order of accuracy as that of the input data. Moreover, it's simple to program than other methods.

III. SIMULATION RESULTS

In order to demonstrate the use of the elastic and rigid column method for transient, a pump feeds a reservoir at upstream end is considered. The foregoing case study illustrates a typical concept to consider when analyzing hydraulic transients.

Case study: A pump feeds a reservoir as it shown in Fig. 3, where the water level elevation $H_R = 30\text{m}$, through a conduit having the following characteristics, $L = 1500\text{m}$, $D = 0.4\text{m}$, $\lambda = 0.02$, $E = 2.10 \times 10^{11}\text{Pa}$, $\mu = 0.3$, $D/e = 70$, $c = 1$, $P = 40.33\text{m}$ and $a = 1100\text{ m/s}$. At a given moment the pump is stopped after a power outage. In order to simulate transients for this case, instantaneous pump stopping option had been chosen. A closed surge tank installed just immediately downstream of the pumping station. The surge tank has 6m^3 initial volume and 5m^2 -cross-sectional area and its entrance diameter is 0.15m .

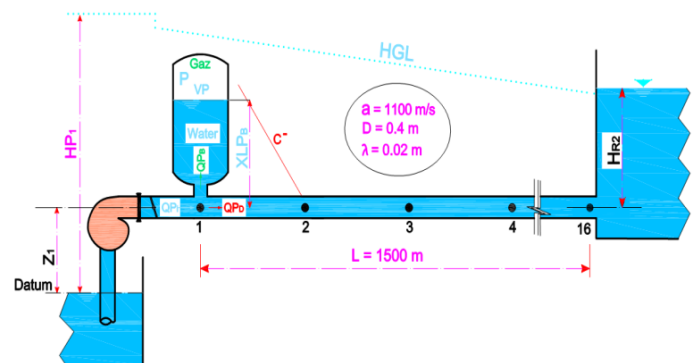


Fig. 3 Simple pump - reservoir system combined with closed surge tank

A. Elastic Method

This case study demonstrates the capability of the developed program to simulate the water hammer effect by simulating the sudden pump stopping at the upstream of a long pipe in which water is flowing. The model takes into account the fluid and pipe wall elasticity. For this case study, a simple system is presented in order to best illustrate the water hammer simulation capability of the developed program. The simulation results for the unprotected pipeline are presented in the following figures for each case.

B. Instantaneous Pump Stopping without Including Protective Devices

A typical starting point of a transient study is to estimate the worst-case (instantaneous stopping) events in the pipeline systems. If the protection strategy is well designed, the combination of various transient events that creates the pressure force will dissipate.

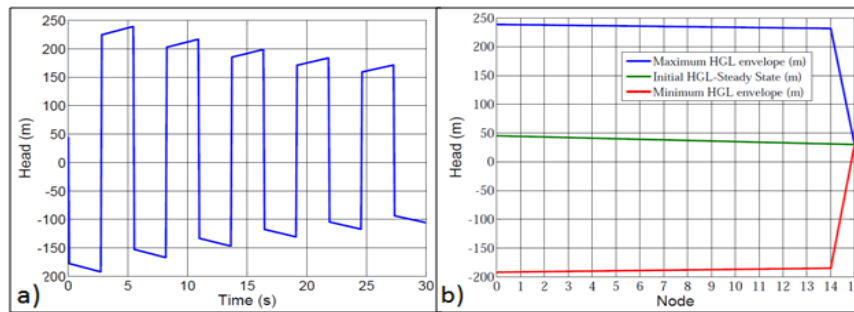


Fig. 4 Transients in a pumping system (a) Head change versus time at the pump and (b) Hydraulic grade lines (without Protection)

Fig. 4 shows that the maximum and minimum pressure occurred at the times 5.454 and 2.727 second respectively, and the pressure head amplitude become weaker from one cycle to another till it is vanish due to head losses, the maximum and

the minimum pressure envelopes for the unprotected pipeline along the entire pipe length are 238.75m and -192.06m respectively, while in the steady state before generating any transient are 45.13m and 30m respectively.

C. Instantaneous Pump Stopping with Closed Surge Tank Included

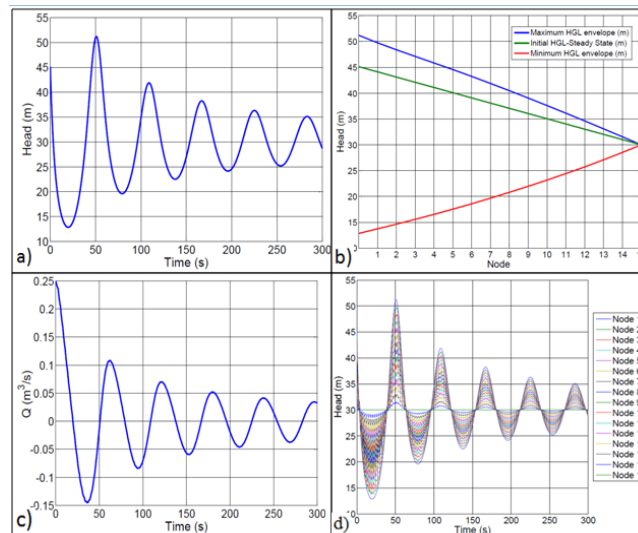


Fig. 5 Transients in a pumping system (a) Head change versus time at the pump (b) Hydraulic grade lines (with including the closed surge (c) Flow change versus time at the pump and (d) Variation of head at each node Vs time (with including the Closed surge tank)

Fig. 5 Shows that the maximum pressure and minimum pressure occurred at the times 50.813 and 19.816 second respectively, and the pressure head amplitude become weaker from one cycle to another till it is vanish due to head losses, the maximum and the minimum pressure envelopes for this case are reduced to 51.19m and 12.84m respectively, the exchanged discharge between the pipe and the air chamber begins from 0.25m³/s and it reduced from a cycle to another,

and the pressure head at the nodes further from the source of transient are less amplitude and become more less whenever it goes further from it.

D. Rigid Column Method

The same previous case study has been considered for rigid column method, the simulation results are presented in the following figures:

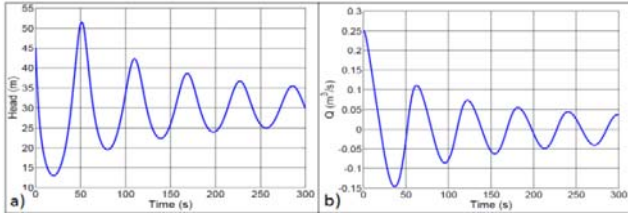


Fig. 6 Transients in a pumping system (a) Head change versus time at the pump and (b) Flow variation versus time at the pump

E. Comparison between the Two Methods

The comparison between the two methods has been done and represented through the following figures:

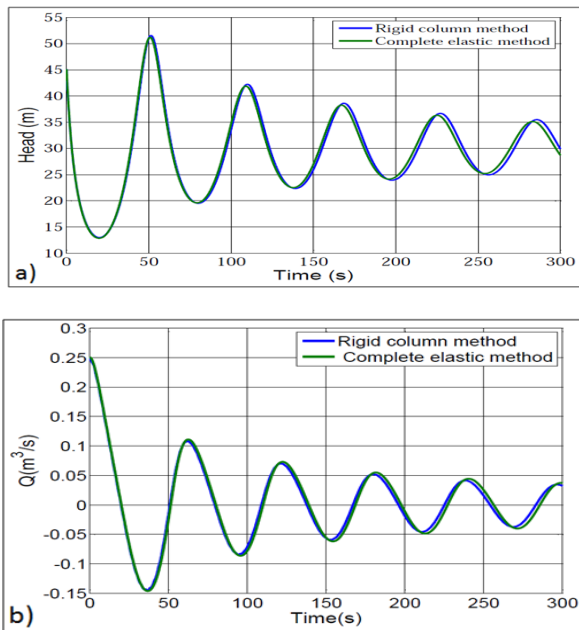


Fig. 7 Transients in a pumping system (a) Head change versus time at the pump and (b) Flow variation versus time at the pump

The general expression for the wave speed is

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + C \frac{K D}{E e}}} \quad (33)$$

where K and ρ are the bulk modulus of elasticity and density of the fluid, D and e are the inner diameter and the thickness of the pipe respectively, E is the Young modulus (modulus of elasticity) of the pipe material, and C is a coefficient that accounts for the pipe support conditions:

$c = 1 - 0.5\mu$, the pipeline is anchored only at the upstream.

$c = 1 - \mu^2$, the pipeline is anchored against longitudinal movement.

$c = 1$, the pipeline has expansion joints throughout.

where μ is the Poisson's ratio for the pipe material.

As fluid becomes more rigid, K increases and, if the medium is assumed to be incompressible, this hypothetical case would correspond to an infinite wave speed. This is not, strictly speaking, possible. No fluid is really incompressible and no pipe is totally rigid, but at times it is a useful approximation when the speed of propagation is much greater than the speed at which boundary conditions respond.

Approximately an identical result has been obtained by using these two methods, due to the fact that the speed of propagation is much greater than the speed at which boundary conditions responds, a transient is generated. The surge tank acts as a reservoir and prevents most of the transient pressures. Infinite wave speed means rigid pipe and incompressible fluid, the pipe which considered in the elastic analysis is rigid enough where the elastic modulus of the pipe material is $E = 2.10 \times 10^{11} \text{ GP}$, and also the fluid bulk modulus is assumed to be high value $K \approx 2.07 \times 10^9 \text{ Pa}$, which considered incompressible fluid, so these assumptions can explain obtaining nearly the same results of the rigid column theory. Moreover, the current case study is a simple pipeline system is presented in order to best illustrate the capability of the developed program to simulate the flow transients.

IV. CONCLUSION

Clearly, flow control actions can be extremely important, and they have implications not only for the design of the hydraulic system but also for other aspects of system operation and protection. Air chambers are effective in controlling both positive and negative transient pressures and are widely used in protecting main transmission pipelines.

Rigid column method effectively avoids the interpolation error occurs in the characteristics method and reducing its calculations complexity. Moreover, this method provides nearly the same simulation results as the full elastic method for some cases, but not always suitable; However, In general, to be in the safe side, the full elastic method should be used for transient analysis; because it takes into consideration all factors that play an important agent on the transient. Numerical simulation model is a helpful tool for the engineers in charge to decide among different technical and economic solutions regarding to the adverse and dangerous effect occurs in the flow transient state.

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