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Design of an Stable GPC for Nonminimum Phase LTI Systems

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***Abstract*—**The current methods of predictive controllers are utilized for those processes in which the rate of output variations is not high. For such processes, therefore, stability can be achieved by implementing the constrained predictive controller or applying infinite prediction horizon. When the rate of the output growth is high (e.g. for unstable nonminimum phase process) the stabilization seems to be problematic. In order to avoid this, it is suggested to change the method in the way that: first, the prediction error growth should be decreased at the early stage of the prediction horizon, and second, the rate of the error variation should be penalized. The growth of the error is decreased through adjusting its weighting coefficients in the cost function. Reduction in the error variation is possible by adding the first order derivate of the error into the cost function. By studying different examples it is shown that using these two remedies together, the closed-loop stability of unstable nonminimum phase process can be achieved.

***Keywords*—**GPC, Stability, Varying Weighting Coefficients.

I. Introduction

MODEL predictive or receding horizon controllers have received a great deal of attention and receive an ever growing interest for applications in industrial process control. Nevertheless the stability of model predictive control schemes is difficult to analyze and few results are known which guarantees stability of the model predictive controllers.

In particular, four main approaches can be distinguished in addressing the issues of the closed-loop stability. The first approach is to use an infinite prediction horizon with a finite control horizon [1]. This method is not practical for some processes. Another approach is to add an equality constraint (hard constraint) in the objective function [5, 6]. Usage of this method may cause in saturation on the input and/or infeasibility of the optimization problem. In the third approach, hard constraint substituted with a penalty on the end point state in the cost function [2-4]. In the fourth method, the process is first stabilized and then controller is designed.

We first show that for some processes, stability doesn't occur by implementation of these methods and then propose a new approach to solve the problem.

The paper is organized as follows. In Section II, the prediction model structure is introduced. In Section III, the proposed cost function is discussed. Discussion on error growth avoidance is given in Section IV. Numerical simulations are carried out in Section V for illustration and verification of the presented methodology. Finally some concluding remarks are given in Section VI.

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II. Process Model and Control Strategy

Generalized predictive control (GPC) has been widely used in process control engineering, because of its good tracking performance and ability to manipulate constraint. Furthermore, it is basically output feedback control and for this reason we discuss model and prediction issues on the transfer function basis.

Consider the ARIMAX (Auto-Regressive Integrated Moving-Average eXogenous) model given by:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})}{\Delta}e(t), \quad \Delta = 1 - q^{-1} \tag{1}$$

where q is the shift operator, $u(t)$ and $y(t)$ are the input and the output of the system, and $e(t)$ is the unpredictable disturbance in the system. For Convenience, $C(q^{-1})$ is assumed to be one, which means that the disturbance sequences are uncorrelated.

In order to derive a GPC formulation, the future outputs of the system should be predicted based on the future inputs and past inputs and outputs. To do this consider:

$$\begin{aligned} &y(t+j) = G_j \Delta u(t+j-1) + f(t+j), \quad j = 1, 2, \ldots, P \\ &f(t+j) = H_j \Delta u(t-1) + F_j y(t) \\ &1 = (A\Delta).E_j + q^{-j}.F_j \\ &B.E_j = G_j + q^{-j}.H_j \end{aligned} \tag{2}$$

In the vector-matrix form, relation in (2) can be written as:

$$\boldsymbol{y} = \boldsymbol{G}\Delta\boldsymbol{u} + \boldsymbol{f} \tag{3}$$

$$\Delta\boldsymbol{u} = [\Delta u(t) \quad \Delta u(t+1) \cdots \Delta u(t+P-1)]^T$$
$$\boldsymbol{y} = [y(t+1) \quad y(t+2) \cdots y(t+P)]^T$$
$$\boldsymbol{f} = [f(t+1) \quad f(t+2) \cdots f(t+P)]^T$$
$$\boldsymbol{G} = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ g_2 & g_1 & 0 & \cdots & 0 \\ \vdots & & & & \\ g_P & g_{P-1} & \cdots & & g_1 \end{bmatrix}$$

If $r(t)$ is to be as a reference signal and α as a time

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constant for input filter (in which its value determines the way of reaching output to reference signal based on equation (4)), then the original cost function for GPC is considered in the form of equation (5).

$$y_d(t) = \frac{1-\alpha}{1-\alpha q^{-1}} r(t) \tag{4}$$

$$J = \sum_{j=1}^{P} (q_j (y_d(t+j) - y(t+j))^2 + r_j \Delta u(t+j-1)^2) \tag{5}$$

III. New Cost Function

Consider the following cost function with three terms: prediction error, prediction error variation, and input variation.

$$\begin{aligned} J &= \sum_{j=1}^{P} q_j (\varepsilon(j))^2 + s_j (\varepsilon(j) - \varepsilon(j-1))^2 + r_j (\Delta u(t+j-1))^2 \\ \varepsilon(j) &= y_d(t+j) - y(t+j) \end{aligned} \tag{6}$$

The future outputs of the system can be determined as follows:

$$\begin{aligned} &y(t+1) = g_1 \Delta u(t) + f(t+1) \\ &y(t+2) = g_2 \Delta u(t) + g_1 \Delta u(t+1) + f(t+2) \\ &\vdots \\ &y(t+P) = g_P \Delta u(t) + \cdots + g_1 \Delta u(t+P-1) + f(t+P) \end{aligned} \tag{7}$$

Define prediction error variation vector $\bar{\boldsymbol{e}}$ and desired output vector $\boldsymbol{y}_d$ as;

$$\begin{aligned} \bar{\boldsymbol{e}} &= [e(1) - e(0) \quad e(2) - e(1) \quad \cdots \quad e(P) - e(P-1)]^T \\ \boldsymbol{y}_d &= [y_d(t+1) \quad y_d(t+2) \quad \cdots \quad y_d(t+P)]^T \end{aligned} \tag{8}$$

the cost function in (6) can be simplified as;

$$J = (\boldsymbol{y_d} - \boldsymbol{y})^T \boldsymbol{Q} (\boldsymbol{y_d} - \boldsymbol{y}) + \Delta \boldsymbol{u}^T \boldsymbol{R} \Delta \boldsymbol{u} + \bar{\boldsymbol{e}}^T \boldsymbol{S} \bar{\boldsymbol{e}}\,, \tag{9}$$

where

$$\boldsymbol{R} = diag(r_1, r_2, \ldots, r_P),\ \boldsymbol{Q} = diag(q_1, q_2, \ldots, q_P),\ \boldsymbol{S} = diag(s_1, s_2, \cdots, s_P).$$

By substituting from (3) for $\boldsymbol{y}$, (9) is rewritten as;

$$\begin{aligned} J = {} & (\boldsymbol{y_d} - \boldsymbol{G}\Delta \boldsymbol{u} - \boldsymbol{f})^T \boldsymbol{Q} (\boldsymbol{y_d} - \boldsymbol{G}\Delta \boldsymbol{u} - \boldsymbol{f}) + \Delta \boldsymbol{u}^T \boldsymbol{R} \Delta \boldsymbol{u} + \\ & (\bar{\boldsymbol{y}}_d - \bar{\boldsymbol{G}}\Delta \boldsymbol{u} - \bar{\boldsymbol{f}})^T \boldsymbol{S} (\bar{\boldsymbol{y}}_d - \bar{\boldsymbol{G}}\Delta \boldsymbol{u} - \bar{\boldsymbol{f}})) \end{aligned} \tag{10}$$

where bar (-) stands for variation of the corresponding term.

Minimizing of the cost function in (9) by adjusting input variations results in the following performances;

1. Minimizing the difference between the process output and its desired trajectory.
2. Decreasing the variation of the above differences.
3. Minimizing the variation of the future inputs.

The optimal input variations are given as;

$$\Delta \boldsymbol{u} = (\boldsymbol{G}^T \boldsymbol{Q} \boldsymbol{G} + \boldsymbol{R} + \bar{\boldsymbol{G}}^T \boldsymbol{S} \bar{\boldsymbol{G}})^{-1} [\boldsymbol{G}^T \boldsymbol{Q} (\boldsymbol{y_d} - \boldsymbol{f}) + \bar{\boldsymbol{G}}^T \boldsymbol{S} (\bar{\boldsymbol{y}}_d - \bar{\boldsymbol{f}})] \tag{11}$$

Using the first term of $\Delta \boldsymbol{u}(t)$, $u(t)$ is obtained from the following equation and applied to the process.

$$u(t) = \Delta u(t) + u(t-1)$$

Based on the obtained result and the latest measured data the whole procedure is repeated in the next steps.

IV. Avoiding the Error Growth

Consider the cost function of equation (6) to be one that is used in the predictive controller. The weight matrices of $\boldsymbol{S}$, $\boldsymbol{R}$, and $\boldsymbol{Q}$ are diagonal with varying elements respect to the prediction time.

$\boldsymbol{S}$, $\boldsymbol{R}$, and $\boldsymbol{Q}$ matrices are considered to have the following structure.

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & \gamma^1 & 0 & \ldots & 0 \\ 0 & 0 & \gamma^2 & & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \ldots & \gamma^{p-1} \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & \alpha^1 & 0 & \ldots & 0 \\ 0 & 0 & \alpha^2 & \cdots & 0 \\ \vdots & & \cdots & \vdots & \\ 0 & 0 & 0 & \cdots & \alpha^{p-1} \end{bmatrix},$$

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & \beta^1 & 0 & \ldots & 0 \\ 0 & 0 & \beta^2 & & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \ldots & \beta^{p-1} \end{bmatrix}$$

where α, β, and γ have values between 0-1. In this case the weight of the first sentences in the cost function (6) would be more than the final sentences. This matter leads to the point that the trust of first sentences in optimization is greater than the final sentences. This action has two outcomes: first, the error weight of the first predictive step in optimization is considered more than the others, and second, the first terms of predictive input play more roles in optimization. With applying this method in an unstable process, the errors are forced not to be increased.

V. Computer Simulations

For comparing this method with original GPC and as well as stabilizing methods (of predictive controller type) different states of unstable process were studied, where the outcomes are as follows:

A. Process with non-repetitive right hand side poles

To control this kind of process such as the one given in (12), existing methods such as the Constraint Receding Horizon Predictive Control (CRHPC) which stabilizes the closed loop system using the hard constraint and Weighted

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GPC (WGPC) which stabilizes the closed loop system using the soft constraint work properly provided that appropriate control parameters P (predictive horizon), M (control horizon) (e.g. $P = M = 6-8$), and number of final constraint (e.g. $m = 3$) are selected.

$$G(s) = \frac{0.25s + 0.025}{s^2 + 0.45s - 0.025} \tag{12}$$

B. Process with non-repetitive right hand side poles and zeros

Transfer function of (13) represents a system with right hand side pole and zero.

$$G(s) = \frac{-0.25s + 0.025}{s^2 + 0.45s - 0.025} \tag{13}$$

Ordinary GPC is not able to guarantee the closed stability of the system. However, implementation of GRHPC method with $P = M = 6-8$ and final states constraint $m = 3$, stabilizes the closed loop system.

C. Process with repetitive right hand side poles

Process given by (14) has repetitive poles in right hand side.

$$G(s) = \frac{s^2 + 0.2s + 0.26}{s^3 - 0.9s^2 + 0.15s + 0.025} \tag{14}$$

The existing methods of stabilizing GPC such as CRHPC, WGPC, and Mixed Weighting GPC (MWGPC) [7] were implemented in which all relative conditions are considered. Adjusting the control parameters did not help to get a stable closed loop system.

The proposed method of this paper implemented using the following set of control parameters. Simulation results are given in Fig. 1.

$$P = M = 5, \quad \beta = 0.05, \quad \alpha = 0.05, \quad \gamma = 0.05$$

D. Process with repetitive right hand side poles and zeros

Process given by (15) has real repetitive poles and two zeros in right hand side.

$$G(s) = \frac{s^2 - 0.2s + 0.26}{s^3 - 0.9s^2 + 0.15s + 0.025} \tag{15}$$

The existing methods of stabilization were implemented. None of them was able to stabilize the closed loop system although all given conditions of each method were considered. Using our proposed method, the stability is achieved with the following control parameters. Simulation results in this case are shown in Fig. 2.

$$P = M = 5, \quad \beta = 0.1, \quad \alpha = 0.1, \quad \gamma = 0.1$$

VI. Conclusion

Here securing the closed loop stability is formed by the idea of avoiding the error growth. To fulfill this mean, first step in predictive more attention than next ones in cost function. Among general advantages of this method the followings may be of considerable ones.

1. Since constraint is not added to the optimizing problem in the direct way, therefore those related problems to existing constraint such as offset and feasibility are avoided.
2. The optimal value of α, β, and γ can be determined offline, and then the controller can be used in online form.
3. In studying different processes, it becomes appear that by selecting aforementioned optimized parameters; the system can be controlled with minimum control and predictive horizon.
4. Since the determining of α, β, and γ parameters are accomplished in the form of offline, therefore, within the operation one can use the broad and strong searching methods such as simplex search or genetic algorithm in determining the optimized value.

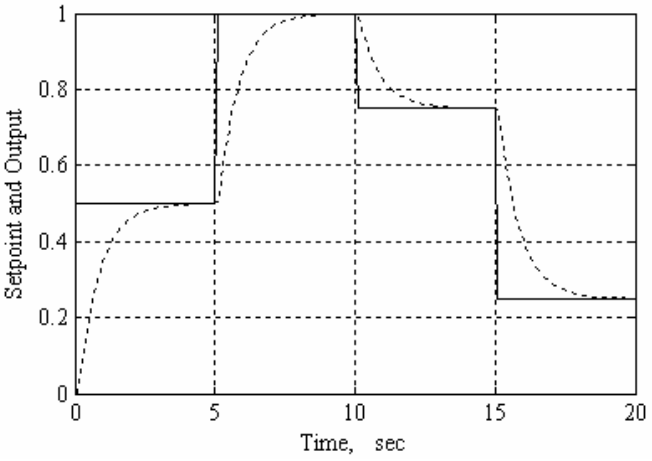


Fig. 1 Simulation results for system in (15)

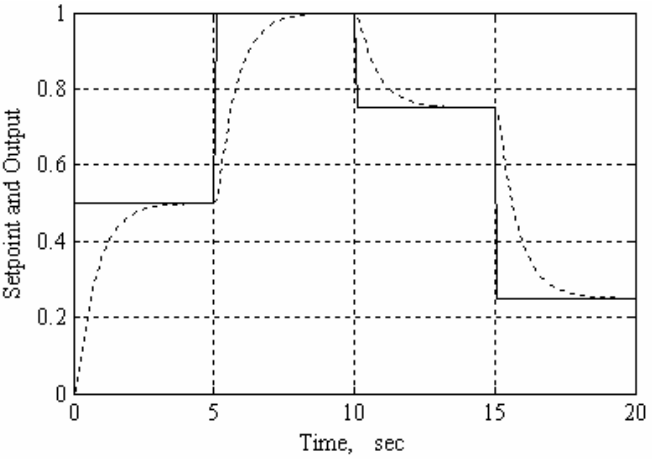


Fig. 2 Simulation results for system in (16)

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