A New Analytical Approach to Reconstruct Residual Stresses Due to Turning Process

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Abstract—A thin layer on the component surface can be found with high tensile residual stresses, due to turning operations, which can dangerously affect the fatigue performance of the component. In this paper an analytical approach is presented to reconstruct the residual stress field from a limited incomplete set of measurements. Airy stress function is used as the primary unknown to directly solve the equilibrium equations and satisfying the boundary conditions. In this new method there exists the flexibility to impose the physical conditions that govern the behavior of residual stress to achieve a meaningful complete stress field. The analysis is also coupled to a least squares approximation and a regularization method to provide stability of the inverse problem. The power of this new method is then demonstrated by analyzing some experimental measurements and achieving a good agreement between the model prediction and the results obtained from residual stress measurement.

Keywords—Residual stress, Limited measurements, Inverse problems, Turning process.

I. INTRODUCTION

RESIDUAL stresses on the surface of engineering workpiece induced by machining processes like turning can greatly affect its ability to withstand several functional aspects such as fatigue lifetime, stress corrosion, wear resistance and cracking [1]. These residual stresses resulting from metal removal processes have been studied for several decades [2]. In most cases after machining operations, a thin layer with remarkable tensile residual stress state can be found on the machined surface [3]. Research in the field of metal cutting has usually been based on experimentation and prototyping, which is expensive and rather slow. Most studies on the surface integrity and more specifically on residual stresses, found in literature, are performed experimentally [4]. Recently, more FEM-based studies are introduced in literature and nowadays it seems that, it is in all probability the only tool to provide models capable of fully predicting all the relevant variables in metal cutting [5,6]. However those methods present numerous numerical problems and hardly respect the physical laws [7] and the application of such a method to get quantitative results is questionable [8].

Reconstruction of residual stresses has been attempted previously by Smith *et al.* [9] where a simple analysis for determining the multiaxial distribution from a limited set of measurements in steel cylindrical bars that had been hot forged and then shot blasted. Korsunsky [10] used a finite element based formulation for determining the distribution of eigenstrains in some practical cases from a set of experimental measurements of residual stresses. Qian *et al.* [11] used the boundary element method to reconstruct residual stress field for some typical cases.

In contrast to earlier studies, this research represents a new analytical method to reconstruct the residual stress field that requires neither numerical tools such as the finite element or boundary element methods nor an assumed eigenstrain distribution. The complete residual stress field is determined utilizing an incomplete set of data by solving the stress equilibrium equations directly. The method is an analytical approach that uses an Airy stress function as the primary unknown while satisfying the equilibrium equations and traction free boundary conditions. The method is described in the next section and its strength is illustrated by examining experimental results from M'Saoubi *et al.* [3].

II. ANALYTICAL RECONSTRUCTION OF RESIDUAL STRESSES

A. Governing equations

Residual stresses are defined as the stresses supported in a body in a fixed reference configuration where there is the absence of external forces and thermal gradients [12]. In the following we consider a residual stress field for axisymmetric conditions. This is similar to the approach adopted in earlier work [13] to examine residual stresses in autofrettaged thick-walled tubes. The equilibrium equations are [14],

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0$$

$$\frac{d\sigma_{r\theta}}{dr} + \frac{2}{r}\sigma_{r\theta} = 0, \qquad \frac{d\sigma_{rz}}{dr} + \frac{1}{r}\sigma_{rz} = 0 \qquad (1)$$

The boundary conditions including zero traction on the outer surface are,

$$at \ \ r = R, \quad \sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = 0 \qquad (2)$$

where R is the radius of the bar. Solving the two last equations of (1) for $\sigma_{r\theta}, \sigma_{rz}$ together with employing the boundary conditions of (2) results in,

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$$\sigma_{r\theta} = \sigma_{rz} = 0 \qquad \text{every where} \tag{3}$$

Another condition for the axisymmetric problem is that the hoop stresses sum to zero when integrated over the volume, so that

$$\int_0^R \sigma_{\theta\theta}(r)\, dr = 0 \tag{4}$$

It is now appropriate to introduce an Airy stress function, $\varphi(r)$ that satisfies the equilibrium equations (1),

$$\sigma_{rr}(r) = \frac{1}{r}\varphi(r), \qquad \sigma_{\theta\theta}(r) = \frac{d\varphi(r)}{dr} \tag{5}$$

To satisfy the traction free boundary conditions and (4), let us introduce the Airy stress function in the form,

$$\varphi(r) = r(r - R)f(r) \tag{6}$$

where $f(r)$ is an arbitrary smooth analytical function which should have at least continuous derivatives on the whole domain . This choice of function not only satisfies all of requirements but also produces a smooth nonsingular stress field. Also it is important to note that due to the presence of the plastic strain field, in contrast to linear elasticity theory, the Airy stress function does not need to be biharmonic.

By introducing an Airy stress function of the form given by (6) the smooth nonsingular forms of the radial and hoop residual stresses are given by,

$$\sigma_{rr}(r) = (r - R)f(r)$$
$$\sigma_{\theta\theta}(r) = (2r - R)f(r) + r(r - R)\frac{df(r)}{dr} \tag{7}$$

To control the behavior of the solution and to ensure the existence of the approximate solution and its uniqueness an arbitrary function $f(r)$ with the following asymptotic expansion is considered,

$$f(r) = \sum_{k=0}^{M} c_k f_k(r) \tag{8}$$

where c_k are the unknown real coefficients to be determined later. Consequently, the Airy stress function and stress field are rewritten as,

$$\varphi(r) = r(r - R)\left(\sum_{k=0}^{M} c_k f_k(r)\right)$$
$$\sigma_{rr}(r) = \sum_{k=0}^{M} c_k \Phi_k(r) = \sum_{k=0}^{M} c_k \left[(r - R)f_k(r)\right] \tag{9}$$
$$\sigma_{\theta\theta}(r) = \sum_{k=0}^{M} c_k \Psi_k(r) = \sum_{k=0}^{M} c_k \left[(2r - R)f_k(r) + r(r - R)\frac{df_k(r)}{dr}\right]$$

To choose the best form of the base functions $f_k(r)$, it is important to note that for the case of turning, the level of stresses decreased continuously with depth down to a minimum compressive stress value and then increased and stabilized at a level corresponding to the state of the material before machining, which means that the stress field smoothly vanishes as r approaches the centre of the bar [3]. So similar to wavelet theory [15], the function $\Gamma(r)$ is selected as a candidate of a modulation function. We use a family of shape functions derived from $\Gamma(r)$ by translation and dilation. The

function $f_k(r)$ is,

$$f_k(r) = \Gamma\left(\frac{r - a_k}{b_k}\right) \tag{10}$$

where $\Gamma(r)$ is a smooth analytical nonlinear function. To ensure that $\Gamma(r)$ satisfies the physical conditions, an exponential function is selected, where

$$\Gamma(r) = \exp[-\gamma r] \tag{11}$$

It should be noted that b_k is a positive strictly decreasing sequence that converges to zero and γ is a positive real constant that governs the rate of convergence of the solution which may be found numerically to depend on the sequence b_k. It is assumed that the sequence of a_k is equal to the radius of the bar.

B. Least squares approximation and Stabilization

To achieve the best values for the coefficients of c_k that appear in the asymptotic expansion (9), a least squares approximation analysis is developed similar to that developed earlier [13]. We assume that only limited measurements of the hoop stress are made at known radial locations. Evaluating $f_k(r)$ at each q-measurement point at coordinate r_q results in predicted values $f_{kq} = f_k(r_q)$. The corresponding values of the hoop stress at r_q from the measurements are denoted by T_q. To assign our confidence in the results a weight function $w(r)$ at each q-measurement is introduced. Therefore the application of weight function results in predicted values $w_q = w(r_q)$. To apply a least square analysis to obtain the coefficients c_k associated with the hoop stresses the following error function is introduced,

$$J = \sum_{q=1}^{N} w_q \left(\sum_{k=1}^{M} c_k f_{kq} - T_q\right)^2 \tag{12}$$

where N is the number of measurement points and M is the number of truncated series used to approximate the residual stress field. The form of weight function $w(r)$ can be selected on the basis of the accuracy of measurement at different points. For simplicity it is assumed that $w(r) = 1$.

It can be shown [13] that unique values of the coefficients c_k can be found using,

$$\{c_m\} = \left[\tilde{f}_{km}\right]^{-1}\{\tilde{T}_k\} \tag{13}$$

where

$$\tilde{f}_{km} = \sum_{q=1}^{N} w_q f_{kq} f_{mq}, \qquad \tilde{T}_k = \sum_{q=1}^{N} w_q f_{kq} T_q \tag{14}$$

Inverse problems are generally ill-posed and therefore regularity of the approximate solution, i.e. whether the approximate solution depends smoothly on the unknown parameters, has to be guaranteed. Similar to earlier work [13] a Tikhonov-Morozov regularization method [16] has been adopted. Knowledge of the bound of the measurement error

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or noise level is essential in utilizing Tikhonov-Morozov regularization method and the noise level for the regularization was assumed to be 50MPa based on the resolution of the X-ray diffraction method [3].

III. ANALYSIS OF EXPERIMENTAL RESULTS AND DISCUSSION

In this section we consider three set of results obtained from experimental measurements of residual stresses in an austenitic stainless steel machined bar. In all cases it has been assumed that only the magnitude of the hoop residual stress is known within a thin layer near to the machined surfaces. The complete residual stress field is then reconstructed and compared to experimental measurements.

M'Saoubi *et al.* [3] studied an austenitic stainless steel (AISI 316L steel) bar with diameter of 150mm. Orthogonal cutting tests were conducted with tungsten carbide tools. Residual stress measurements were performed using X-ray diffraction method. More details of the original experimental measurements can be found in [3].

Fig. 1 shows the hoop residual stresses profiles measured by M'Saoubi *et al.* [3] on three specimens cut at three different feed rates of 0.1, 0.2 and 0.25 mm/rev, here called specimens 1, 2 and 3 respectively.
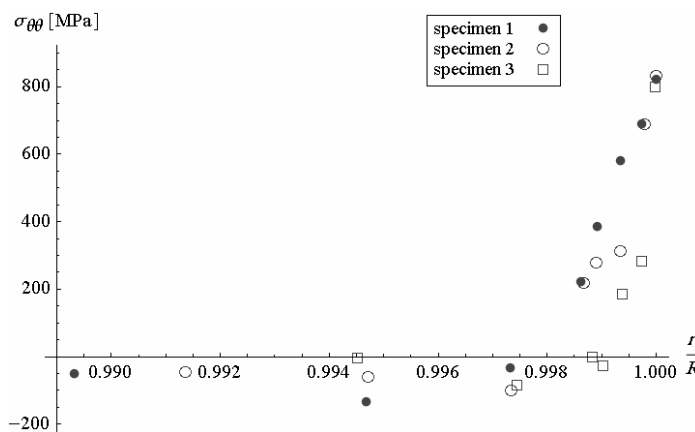

Fig. 1 Measured hoop residual stresses using the X-ray diffraction method in machined austenitic steel [3]

It is assumed that the generated residual stresses were axisymmetric and the residual stresses were relatively short range and confined to the near surface. These results are used here to reconstruct the complete residual stress fields using the analysis developed in the previous section.

For each set of results shown in Fig. 1 the coefficients c_k were determined and regularized and consequently the residual stress field distribution given by (9) was reconstructed in the complete section. The unknown parameters appearing in the definition of the base functions $f_k(r)$, (10) and (11), together with the number of terms M used in (9) are given in Table I.

Fig. 2 compares experimental results with the reconstructed hoop stress profile as a function of the normalized distance from the centre for specimens 1. Similar results were obtained

for specimens 2 and 3 and are presented in Fig. 3 and Fig. 4, respectively. Also Fig. 5 shows the reconstructed radial stress

TABLE I
VALUES OF THE RECONSTRUCTION PARAMETERS AND THE NUMBER OF TERMS USED IN (9)

	b_k	γ	M
Specimen 1	3^{-k}	6	8
Specimen 2	3^{-k}	7	8
Specimen 3	4^{-k}	12	7

profile along the normalized distance from the centre for specimens 1, 2 and 3.

The analysis provided an optimal agreement between the experimental measurements and model predictions. Also note that the reconstructed stress field satisfies all of the continuum mechanics requirements.
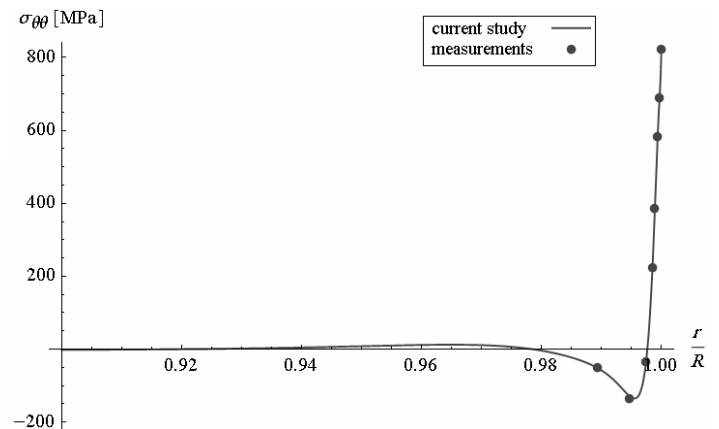

Fig. 2 Reconstructed hoop residual stresses compared with X-ray diffraction results for specimen 1
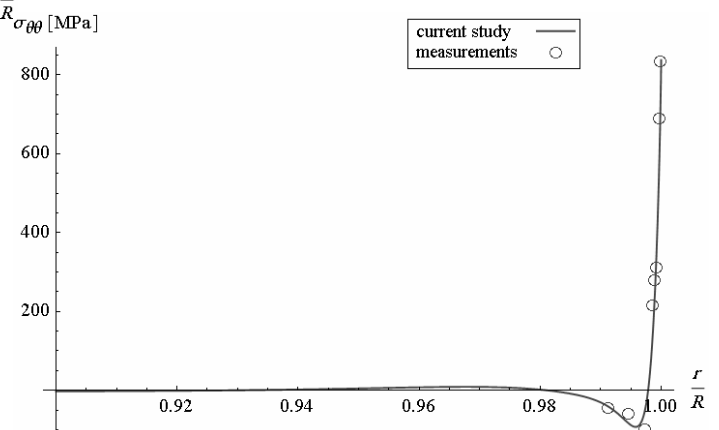

Fig. 3 Reconstructed hoop residual stresses compared with X-ray diffraction results for specimen 2

M'Saoubi *et al.* [3] indicated in their research that based on a triaxial stress analysis conducted on the workpiece, they found that the radial and shear stress components were negligible, when compared to the high values found in the

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circumferential (hoop) directions. Note that in this analytical approach the same results were found for the shear stress components based on (3) and as was expected large values for radial residual stresses were not found and values were less than the resolution of the X-ray diffraction method.
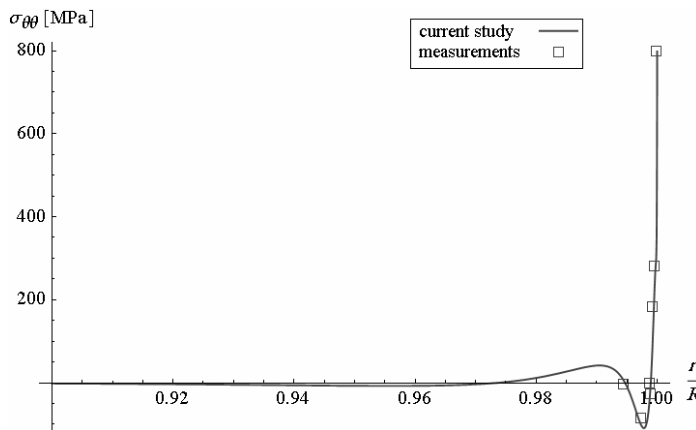

Fig. 4 Reconstructed hoop residual stresses compared with X-ray diffraction results for specimen 3
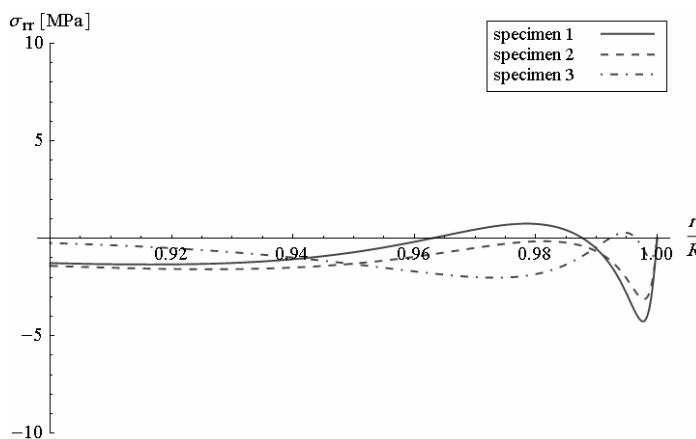

Fig. 5 Reconstructed radial residual stresses for specimen 1, 2 and 3

The choice of function is however not arbitrary and prior knowledge of the expected distribution lead to the choice of appropriate functions. For each continuous function there is a corresponding formal expansion, i.e. (9), which has the property that its partial sums are the best approximations to the function in the least squares sense. It can be shown that this partial sum in the least squares sense will always converge to the function under the conditions of linear independency of the shape functions which are already satisfied here (For more mathematical details see [13].) However it is desirable to study the rate and radius of convergence. As indicated earlier, the real constant γ was introduced to control the rate of convergence and its values for each reconstructed results is given in Table I. Choosing a good value for the positive real constant γ results in having a truncated series with less than 10 terms and therefore provided rapid convergence.

A key feature in this analysis and reconstruction of the residual stress throughout the domain is that it not only interpolates between measured values but also provides additional information. Furthermore, the possible fields included in the analysis are only those that satisfy all of the requirements given by the governing equations. This results in very significant additional constraints being placed on data interpretation. Provided the analysis of the experimental data is carried out using the above procedure, all the predicted stress fields necessarily conform to these constraints, furnishing additional insight into the residual stress field being studied. A further significant feature of this work is that the analysis not only enables minimization of the error on one stress component and determines an expansion for that component the technique also provides expansions for the other components of the residual stress field.

IV. CONCLUSION

A new analytical approach has been developed to reconstruct the residual stress distributions in axisymmetric components containing axisymmetric residual stress fields. The results and analysis provided in this research demonstrate that limited measurements, together with an assumed Airy stress function, provide a method of determining a complete residual stress field. The analysis has been applied to three examples where the residual stresses were introduced by turning process. The approach thus provides a useful method for experimental residual stress analysis, where usually the complete determination of the stress state at every point is often difficult, expensive and time consuming.

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