World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:3, No:1, 2009

An Expectation of the Rate of Inflation According to Inflation-Unemployment Interaction in Croatia

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Abstract**—**According to the interaction of inflation and unemployment, expectation of the rate of inflation in Croatia is estimated. The interaction between inflation and unemployment is shown by model based on three first-order differential i.e. difference equations: Phillips relation, adaptive expectations equation and monetary-policy equation. The resulting equation is second order differential i.e. difference equation which describes the time path of inflation. The data of the rate of inflation and the rate of unemployment are used for parameters estimation. On the basis of the estimated time paths, the stability and convergence analysis is done for the rate of inflation.

Keywords**—**Differencing, inflation, time path, unemployment.

I. Introduction

THE main aim of the paper is to estimate the expectation of the rate of inflation in Croatia using mathematical model based on the system of differential i.e. difference equations.

The paper is organized in five sections. After this introductory section, in the second section the mathematical model is presented. The basic framework is taken from [2] where the resulting equations are achieved according to the three differential/difference equations describing inflation-unemployment interaction. In this paper, in discrete form of the model, instead of monetary policy equation based on the rate of inflation from forward periods, the latter equation is based on the current rate of inflation. That implies new form of the resulting second order difference equation in the rate of inflation. The third section uses statistical-econometric methods for the parameters estimation from the previously derived model. All approximations are done using the monthly data of the rate of inflation and the rate of unemployment in Croatia in period from 1998. to 2008. The presented and used parameter results satisfy all appropriate tests, what is clearly exposed in this part of the paper. In the fourth section, the presented model with incorporated estimated parameters is applied on Croatian economy. The convergent time path of the rate of inflation with a stationary intertemporal equilibrium is obtained. Finally, some concluding remarks are given.

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II. The Model

A. The Expectations-Augmented Phillips Relation

The most widely used concept in analyzing the problem of inflation and unemployment is the Phillips relation which, in its original formulation, depicts an empirically based negative relation between the rate of growth of money wage and the rate of unemployment [3]:

$$w = f(U),\ f'(U) < 0. \tag{1}$$

where w denotes the rate of growth of money wage W ($w = W'/W$) and U is the rate of unemployment. This relation can be adapted into a function that links the rate of inflation (instead of w) and the rate of unemployment. A positive w, reflecting growing money-wage cost, would carry inflationary implications, and that makes the rate of inflation, like w, a function of U. However, the inflationary pressure of a positive w can be offset by an increase in labor productivity T and the inflationary effect can materialize only to the extent that money wage grows faster than productivity. Thus, for the rate of inflation it can be written:

$$p = w - T, \tag{2}$$

where $p = P'/P$, P is the price level. By adopting the linear version of the function $f(U)$, an adapted Phillips relation follows [5]:

$$p = \alpha - T - \beta U,\ \alpha, \beta > 0. \tag{3}$$

More recently, economists have preferred to use the expectations-augmented version of the Phillips relation [5]:

$$w = f(U) + h\pi,\ 0 < h \le 1, \tag{4}$$

where π denotes the expected rate of inflation. If an inflationary trend has been in effect long enough, people are apt to form certain inflation expectations which they then attempt to incorporate into their money-wage demands. Thus w should be an increasing function of π. Carried over to (3) this idea results in the equation:

$$p = \alpha - T - \beta U + h\pi,\ 0 < h \le 1. \tag{5}$$

With the introduction of a new variable to denote the expected rate of inflation, it becomes necessary to hypothesize how inflation expectations are specifically formed. Here the adaptive expectations hypothesis is adopted:

$$\frac{d\pi}{dt} = j(p - \pi),\ 0 < j \le 1. \tag{6}$$

This equation describes the pattern of change over time of the expected rate of inflation. If the actual rate of inflation p turns out to exceed the expected rate π, the latter, having now been proven to be too low, is revised upward ($d\pi/dt > 0$). Conversely, if p falls short of π, then π is revised in the downward direction.

B. The Feedback from Inflation to Unemployment

Equation (5) tells how U affects p - largely from the supply side of the economy. But p surely can affect U in many ways. For simplicity, it will be only taken into consideration the feedback through the conduct of monetary policy. Denoting the nominal money balance by M and its rate of growth by $m = M'/M$, let us postulate that:

$$\frac{dU}{dt} = -k(m - p),\ k > 0. \tag{7}$$

It is easy to see that the expression (m-p) represents the rate of growth of real money:

$$m - p = \frac{M'}{M} - \frac{P'}{P} = \frac{M'P - MP'}{MP} = \frac{\dfrac{M'P - MP'}{P^2}}{\dfrac{M}{P}} = \frac{\left(\dfrac{M}{P}\right)'}{\dfrac{M}{P}}.$$

Thus (7) stipulates that dU/dt is negatively related to the rate of growth of real-money balance. As the variable p now enters into the determination of dU/dt, the model now contains a feedback from inflation to unemployment.

Together, (5), (6) and (7) constitute a closed model in the three variables π, p, and U. By eliminating two of the three variables, the model can be condensed into a single (second-order) differential equation in a single variable.

C. Model in Discrete Time

Previously discussed model in continuous-time framework can be reformulated as a difference-equation model:

Phillips Relation:

$$p_t = \alpha - T - \beta U_t + h\pi_t,\ \alpha, \beta > 0; 0 < h \le 1 \tag{8}$$

Adaptive - Expectations Equation:

$$\pi_{t+1} - \pi_t = j(p_t - \pi_t),\ 0 < j \le 1 \tag{9}$$

Monetary - Policy Equation:

$$U_{t+1} - U_t = -k(m - p_t),\ k > 0. \tag{10}$$

In continuation of the analysis, the model is condensed into a single equation in a single variable, firstly p. Since equation (8), unlike the other two equations, doesn't by itself describe a pattern of change, it is accomplished by differencing p_t, i.e., by taking the first difference of p_t, $\Delta p_t = p_{t+1} - p_t$. We shift the time subscripts in (8) in forward one period:

$$p_{t+1} = \alpha - T - \beta U_{t+1} + h\pi_{t+1}.$$

Now, the first difference of p_t that gives the desired pattern of change is:

$$\begin{aligned} p_{t+1} - p_t &= -\beta(U_{t+1} - U_t) + h(\pi_{t+1} - \pi_t) = \\ &= \beta k(m - p_t) + hj(p_t - \pi_t), \end{aligned} \tag{11}$$

according to (9) and (10). The π_t term needs to be eliminated from the above equation. We make use of the fact that

$$h\pi_t = p_t - (\alpha - T) + \beta U_t, \tag{12}$$

by (8). Substituting this into (11) and collecting terms, it is obtained:

$$p_{t+1} - [1 - \beta k - j(1-h)]p_t + j\beta U_t = \beta km + j(\alpha - T). \tag{13}$$

To eliminate a U_t term, it is necessary to difference (13) to get a $U_{t+1} - U_t$ term and then use (10) to eliminate the latter. Now, the desired difference equation in the rate of inflation is of the form:

$$p_{t+2} - [2 - \beta k - j(1-h)]p_{t+1} + [(1-j)(1-\beta k) + hj]p_t = j\beta km. \tag{14}$$

We may write

$$p_{t+2} + a_1 p_{t+1} + a_2 p_t = c, \tag{14a}$$

where

$a_1 = -[2 - \beta k - j(1-h)]$, $a_2 = (1-j)(1-\beta k) + hj$,
$c = j\beta km$.

III. Parameter Estimation

The expected rate of inflation π is estimated according to monthly data of the rate of inflation in Croatia, from January 1998. to June 2008. Inflation VAR model (Vector Autoregression model) [4] is estimated and presented in Table I. The expected rate of inflation suits to the VAR model with two time lags. It is more appropriate than lag one according to all econometric indicators. Namely, model with two lags has bigger R-squared and smaller Akaike criterion and Schwartz criterion values.

TABLE I
INFLATION VAR MODEL

Sample(adjusted): 3 126	
Included observations: 124 after adjusting endpoints	
Standard errors & t-statistics in parentheses	
	RI
RI(-1)	1.222022 (0.08909) (13.7172)
RI(-2)	-0.217128 (0.08971) (-2.42036)
C	-0.249591 (0.45636) (-0.54691)
R-squared	0.997290
Adj. R-squared	0.997245
Sum sq. resids	22.49034
S.E. equation	0.431127
F-statistic	22260.67
Log likelihood	-70.10223
Akaike AIC	1.179068
Schwarz SC	1.247301
Mean dependent	95.06129
S.D. dependent	8.213460

Source: According to data from Croatian National Bank

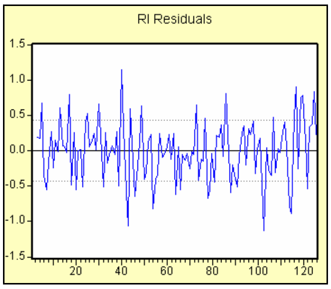

Fig. 1 Inflation VAR model residuals

Table II presents inflation VAR model residuals correlogram. It can be seen that Ljung-Box Q-test values have large p-probabilities confirming that residuals are stationary. Model random residuals are presented in Fig. 1.

In Table III, final Phillips relation parameters are presented. Phillips relation model is satisfying according to all econometric criterions. R-squared is high, Akaike info criterion and Schwartz criterion are low. Durbin-Watson statistic shows that there is no residuals autocorrelation. Variance inflation factor (VIF) is 1,486 which confirms that there is no multicolinearity problem in presented model. The parameter of unemployment variable doesn't show significance.

Omitted redundant variable testing [5] is made, but model without unemployment variable is not possible to be estimated because of near singular matrix. For this reason all appropriate variables are involved in Phillips model ([3] and [6]).

TABLE II
INFLATION VAR MODEL RESIDUALS CORRELOGRAM

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.022	0.022	0.0633	0.801
		2	-0.109	-0.109	1.5798	0.454
		3	-0.033	-0.028	1.7213	0.632
		4	0.170	0.161	5.4720	0.242
		5	-0.143	-0.164	8.1690	0.147
		6	-0.152	-0.116	11.243	0.081
		7	-0.070	-0.083	11.901	0.104
		8	0.113	0.059	13.618	0.092
		9	0.032	0.057	13.756	0.131
		10	-0.021	0.005	13.815	0.182
		11	0.082	0.086	14.738	0.195
		12	0.205	0.150	20.579	0.057

Source: According to data from Croatian National Bank.

TABLE III
PHILLIPS RELATION

Dependent Variable: INFLATION				
Method: Least Squares				
Sample: 1 124				
Included observations: 124				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.386816	0.789397	0.490015	0.6250
UNEMPLOYMENT	-0.010729	0.017961	-0.597385	0.5514
EXPECT_INFL	0.998029	0.005769	173.0110	0.0000
R-squared	0.997298	Mean dependent var		95.06129
Adjusted R-squared	0.997253	S.D. dependent var		8.213460
S.E. of regression	0.430493	Akaike info criterion		1.176123
Sum squared resid	22.42420	Schwarz criterion		1.244356
Log likelihood	-69.91964	F-statistic		22326.50
Durbin-Watson stat	1.956204	Prob(F-statistic)		0.000000

Source: According to data from Croatian National Bank

From Fig. 2 it can be seen that Phillips model residuals are normally distributed with almost zero mean and constant standard deviation.

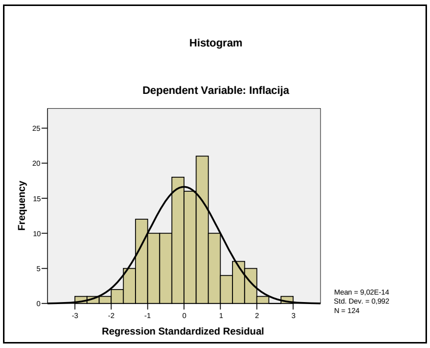

Fig. 2 Phillips model residuals

Model of adaptive-expectations is expressed by equation (9) $\pi_{t+1} - \pi_t = j(p_t - \pi_t)$ and after econometric analysis it is confirmed that parameter j equals 1. On Fig. 3 there are model residuals with almost zero mean and constant standard deviation by which normal distribution assumption can be accepted.

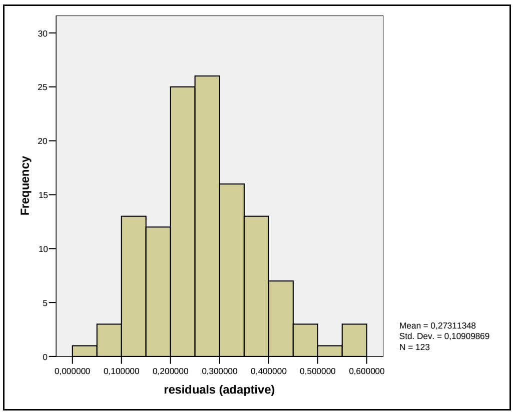

Fig. 3 Adaptive-expectations model residuals

In Table IV, monetary policy equation parameters are presented. Because of strong seasonal component of the unemployment rate variable, dummy variables and appropriate lag variables are included in the model. All representative indicators are satisfying.

TABLE IV
MONETARY POLICY EQUATION

Dependent Variable: UNEMPLOYMENT RATE
Method: Least Squares
Sample(adjusted): 13 113
Included observations: 101 after adjusting endpoints
DUT=C(1)+C(2)*MPT_1+C(3)*KVART_1+C(4)*KVART_2+C(5)*KVART_3+C(6)*DUT_LAG_12+C(7)*TREND+C(8)*DUT_1+C(9)*DUT_2

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.825689	1.676580	2.281841	0.0248
C(2)	0.046700	0.020630	2.263685	0.0259
C(3)	-0.069887	0.055451	-1.260329	0.2107
C(4)	-0.121087	0.086098	-1.406395	0.1630
C(5)	0.119930	0.075025	1.598523	0.1134
C(6)	0.530163	0.077540	6.837299	0.0000
C(7)	0.009299	0.004304	2.160676	0.0333
C(8)	0.301543	0.076134	3.960688	0.0001
C(9)	0.090012	0.083592	1.076804	0.2844

R-squared	0.820752	Mean dependent var	-0.046123
Adjusted R-squared	0.805165	S.D. dependent var	0.406236
S.E. of regression	0.179313	Akaike info criterion	-0.514480
Sum squared resid	2.958095	Schwarz criterion	-0.281450
Log likelihood	34.98125	Durbin-Watson stat	1.512137

Source: According to data from Croatian National Bank.

Namely, unemployment in Croatia has strong seasonal character. There is very strong influence of tourist seasons in Croatian macroeconomy and unemployment is always low in third quarter in all observed period. So, qualitative quarter dummy variables are included in model. VAR analysis confirms two time lags effects of exogenous variable too.

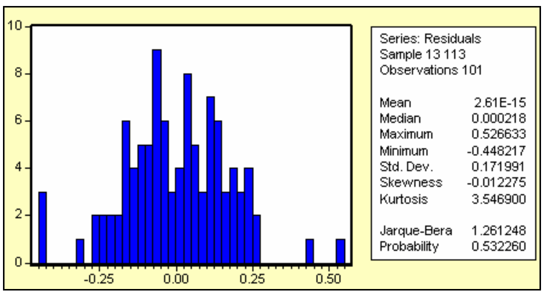

Fig. 4 Monetary policy equation residuals

On Fig. 4 monetary policy equation residuals are shown. Jarque-Bera test confirmes normallity assumption.

IV. THE TIME PATH OF THE RATE OF INFLATION

The previous analysis has shown parameters values: $\beta = 0{,}011;\ h = 0{,}998;\ j = 1$ and $k = 0{,}0467$.

By taking m as an average rate of growth of nominal money M in Croatia in observed period, the coefficients of the difference equation (14a) are calculated:

$$a_1 = -[2 - \beta k - j(1-h)] = -1{,}9974863;$$
$$a_2 = (1-j)(1-\beta k) + hj = 0{,}998;$$
$$c = j\beta km = 0{,}0005137m,$$

and now the equation is of the form:

$$p_{t+2} - 1{,}9974863\, p_{t+1} + 0{,}998\, p_t = 0{,}0005137m. \qquad (15)$$

The solution of (15) has two components: a particular integral $\overline{p}$ representing the intertemporal equilibrium level of p, and a complementary function p_c specifying for any time period, the deviation from the equilibrium.

The particular integral, defined as any solution of the equation, can be found as a solution of the form $\overline{p} = k_1$. In that case from (14) follows:

$$\overline{p} = m. \qquad (16)$$

Therefore, the equilibrium rate of inflation is exactly equal to the rate of monetary expansion.

To find the complementary function we must solve the characteristic (quadratic) equation of the reduced equation (15):

$$r^2 + a_1 r + a_2 = 0,$$

$$r^2 - 1{,}9974863r + 0{,}998 = 0. \qquad (17)$$

The characteristic roots of (17) are complex roots:

$$r_{1,2} = a \pm bi = 0{,}99874315 \pm 0{,}022630075i. \qquad (18)$$

The complementary function thus becomes [1]:

$$p_c = A_1 r_1^t + A_2 r_2^t = A_1(a+bi)^t + A_2(a-bi)^t, \qquad (19)$$

and thanks to De Moivre's theorem the complementary function can be transformed into trigonometric form:

$$(a \pm bi)^t = R^t(\cos\Theta t \pm i\sin\Theta t), \qquad (20)$$

where the value of R is:

$$R=\sqrt{a^2+b^2}=0{,}998999499, \qquad (21)$$

and Θ is the radian measure of the angle in the interval $[0{,}2\pi)$:

$$cos\Theta=\frac{a}{R}=0{,}999743394;\ sin\Theta=\frac{b}{R}=0{,}022652739. \qquad (22)$$

Finally, the complementary function (19) can be transformed as follows:

$$p_c=0{,}9989995^t(A_3 cos1{,}298017t+A_4 sin1{,}298017t), \qquad (23)$$

where the symbols: $A_3=A_1+A_2$ and $A_4=(A_1-A_2)i$ are adopted.

The general solution of the equation (15) is:

$$\begin{aligned} p_t &= p_c+\overline{p}= \\ &=0{,}998995^t(A_3 cos1{,}298017t+A_4 sin1{,}298017t) \qquad (24) \\ &+100{,}097566, \end{aligned}$$

where the rate of monetary expansion m is expressed as a geometric mean of the monthly indices.

To define arbitrary constants A_3 and A_4, two initial conditions are necessary. From the data of Croatian rate of inflation follow initial conditions $p_0=80$ and $p_1=80{,}4$.

By substituting $t=0$ and $t=1$ successively in (24), values $A_3=-20{,}097566$ and $A_4=16{,}555526$ are calculated.

The definite solution then can finally be written as:

$$\begin{aligned} p_t &= p_c+\overline{p}=0{,}998995^t\cdot \\ &\cdot(-20{,}097566\cdot cos1{,}298017t+ \qquad (24a) \\ &+16{,}555526\,sin1{,}298017t)+100{,}097566. \end{aligned}$$

The particular integral is $\overline{p}=100{,}097566$ and so there is a stationary equilibrium. The convergence of the time path dependence solely on whether complementary function p_c tends toward zero as $t\to\infty$. From (23) it can be seen that the key factor for the convergence is R^t. It is $R<1$ and hence $\lim\limits_{t\to\infty} R^t=0 \Rightarrow \lim\limits_{t\to\infty} p_c=0$, therefore the time path convergence to the stationary equilibrium $\overline{p}=m$ (Fig. 5).

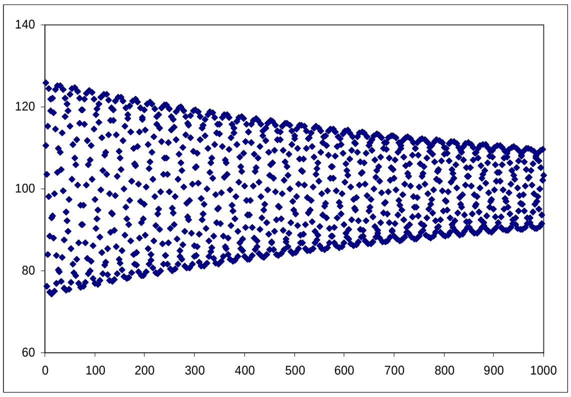

Fig. 5 The time path of the rate of inflation

It is obvious that the resulting path displays a sort of stepped fluctuation.

V. Conclusion

The model based on inflation-unemployment interaction is applied on Croatian economy to estimate the time path of the rate of inflation. Similarly, with the same mathematical-statistical apparatus, the time paths of all variables in model can be obtained.

Estimated convergent time path of the rate of inflation with stable intertemporal equilibrium is real indicator of inflation trend in Croatia but without additional external shocks in economy.

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