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Calculation of the Ceramics Weibull Parameters

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***Abstract*—**The paper deals with calculation of the parameters of ceramic material from a set of destruction tests of ceramic heads of total hip joint endoprosthesis. The standard way of calculation of the material parameters consists in carrying out a set of 3 or 4 point bending tests of specimens cut out from parts of the ceramic material to be analysed. In case of ceramic heads, it is not possible to cut out specimens of required dimensions because the heads are too small (if the cut out specimens were smaller than the normalised ones, the material parameters derived from them would exhibit higher strength values than those which the given ceramic material really has). On that score, a special testing jig was made, in which 40 heads were destructed. From the measured values of circumferential strains of the head's external spherical surface under destruction, the state of stress in the head under destruction was established using the final elements method (FEM). From the values obtained, the sought for parameters of the ceramic material were calculated using Weibull's weakest-link theory.

***Keywords*—**Hip joint endoprosthesis, ceramic head, FEM analysis, Weibull's weakest-link theory, failure probability, material parameters

I. INTRODUCTION

A problem that is being solved is the destruction of the ceramic heads of total hip joint endoprostheses in vivo, that had occurred in a series of Czech hospitals (Fig. 1). The ceramic heads are made of Al_2O_3 and put on conical stem made of austenitic steel. The implant's failure of the „ceramic head destruction“ type“ has always traumatic consequences for the patient, since a part of or even the whole endoprosthesis has to be re-operated, after which must again follow reconvalescence and rehabilitation. Hence, it is desired to reduce the number of implant re-operations to the minimum. The reliability of the ceramic component is based on Weibull weakest-link theory [1] and the failure probability depends on three Weibull's parameters. These parameters are obtained from the statistical analysis of the set of the destructed specimens which are subjected to 3- or 4- point bending.

In case of ceramic heads of endoprosthesis, the snag is that the head's dimensions are too small and do not allow to cut out from them a specimen that would comply with the standard for 3- or 4-point bending. One of the possible solutions to this problem is not to cut out the specimens from the heads, but to calculate the material parameters directly from the destruction of whole ceramic heads.

Fig. 1 In vivo destructed ceramic heads

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II. Generally About The Failure Probability Of Ceramic Material

For computational modelling of failure probability, two computation models are used which differ from each other in that which stresses are included into the computation. The first more simple model includes into the computation only the first principal stress - σ_1 (see relation (1)), whereas the second model includes into the computation all three principal stresses. This model is suitable for cases where the second and the third principal stress take marked tensile values nearing to the values of the first principal stress [2, 4]. In case of a ceramic head of total hip joint endoprosthesis, the probability of its failure can be computed using a more simple model, since the maximum values of the second principal stress take at most 30 % of the value of the first principal stress in the head [3]. For computation of the ceramic head failure probability, the following relation is used, that is based on Weibull's weakest-link theory [1]:

$$P_f = 1 - e^{-\sum_{i=1}^{n}\left(\frac{\sigma_i - \sigma_u}{\sigma_o}\right)^m \Delta V_i}, \qquad \sigma_i \geq \sigma_u, \qquad (1)$$

where : n – number of elements into which the analysed head is divided, using the finite elements method, ΔV_i – volume of the i-th element, σ_i – first principal stress (σ_1) acting in volume ΔV_i, σ_u- stress, beneath the value of which material failure does not occur, σ_o – normalised material strength of a volume unit of material, m - Weibull modulus (connected with the scatter of measured values). These parameters are derived from processing the destruction forces obtained from a set of 3- or 4- point bending tests. This way of assessing material parameters is universally applicable to any ceramic product, from which specimens are made for the mentioned bending test. The number of specimens must exceed 35, if the Weibull statistical method of the weakest link is at all to be applied. Small components present another problem which applies also to ceramic heads, namely that the cut out specimens are too small. Small specimens can be subjected only to 3-point bending, at which maximum tensile stress is concentrated only in a small area in the middle of the specimen, in contrast to 4-point bending at which maximum tensile stresses concentrate in two areas between two supports. The material characteristics obtained from 4-point bending tests show a greater plausibility than those from 3-point bending tests; therefore 4-point bending test is to be preferred.

Fig. 2 Dismounted jig for the head destructions

III. Experimental Destructions Of The Heads

For carrying out the destruction of a set of ceramic heads a special testing jig (Fig. 2) has been made in which the heads are subjected to compressive load that acts only on the surface of the head's opening, as shown in Fig. 3. In the course of the test, circumferential strains are measured on the head's external surface. On each head, two gauges were stuck (at 180°) which, in addition, provided information on whether the deformations in the head are distributed evenly on the circumference – the data of the gauges served for judging the reliability of the results of experiments. The compression load acting on the head was developed by the testing device ZWICK Z 020-TND (Fig. 4) whose head loaded the jig's piston which, in turn, acted on rubber NBR 90 inside the jig, which, subsequently, acted on the ceramic head. The destructed head is shown in Fig. 5.

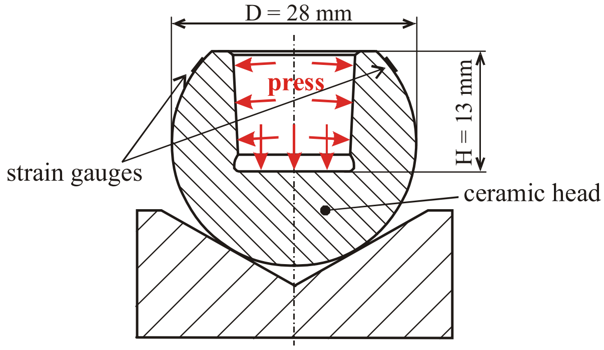

Fig. 3 Scheme of the head's load and bounds

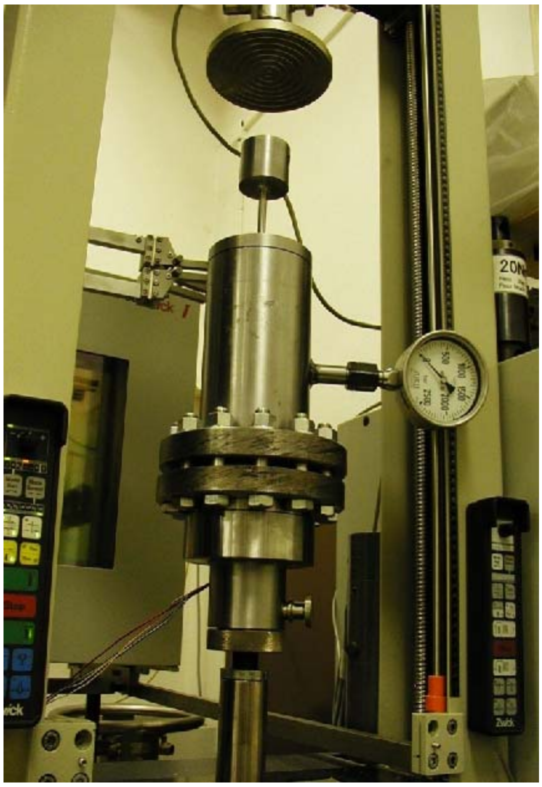

Fig. 4 Experimental jig fixed in the ZWICK testing machine

Fig. 5 Destructed head

The result of experiments was a set of the strain values established under the destruction of the individual heads – Fig. 6. The highest value of the difference of the measured values was 8.7 % (head No. 27 in Fig. 6 and Fig. 7). In two ceramic heads the gauges were destructed before the destruction of the head (heads No. 32 and 33 in Fig. 6). The values of the difference of the measured strains on the ceramic heads are show in Fig. 7 – only 6 measured stains have the large difference than 5 %, so the pressure loading of the heads were equable and the results are suitable for the calculation of the material parameters of the ceramic material.

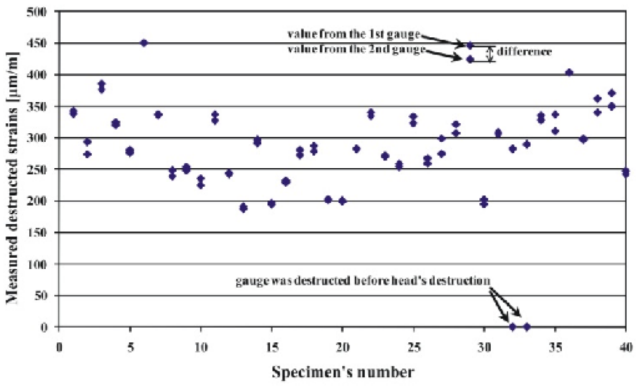

Fig. 6 Circumferential strain values under heads' destruction

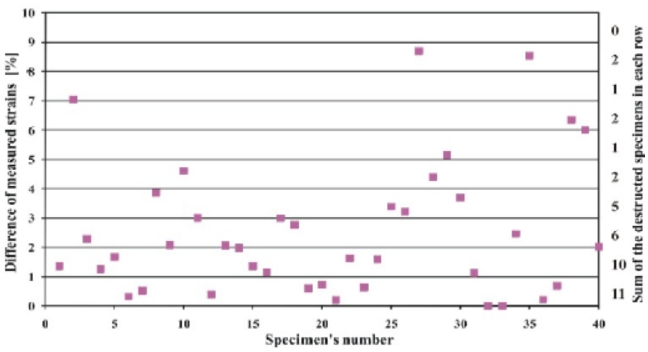

Fig. 7 Difference of the measured strains for destructed heads

IV. Calculation Of The Material Parameters

In the first phase, from the strains established under the heads' destruction, the stress field produced by the destruction had to be defined. To this purpose, computational modelling (FEM method) was used. The circumferential strain field and the first principal stress field for compressive load p = 100 MPa (modelling the stress field in the sorted specimen No. 32 in the Table I) are shown in Fig. 8 and 9. The destruction strain $\varepsilon_{des}t$ = 332 μm/m (Fig. 8) corresponds to the maximum value of the first principal stress $\sigma_{1max\text{-}dest}$ = 371 MPa in the point of the conical opening's transition into the bottom of the head's opening (the transition radius gives rise to the concentration of stress), see Fig. 9. Since a linear task is the matter, even the remaining values ε_{dest} can be recounted in a linear way to $\sigma_{1max\text{-}dest}$. Thus we obtain a set of values of destruction stresses that have to be arranged in descending order for further analysis. Each *j-th* destructed head is assigned the probability of its failure (see Table I and Fig. 10), e.g. from the relation $P_f(j) = j/(r+1)$, where j is the serial number of the arranged head and r is the total number of destructed heads, in our case r = 40.

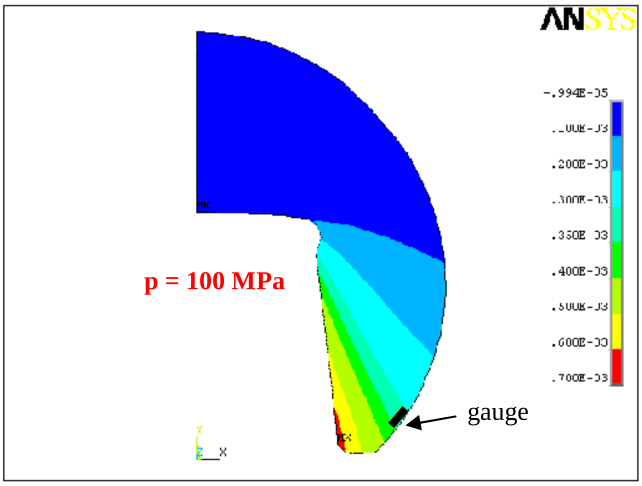

Fig. 8 Circumferential strain in the head for load p = 100 MPa

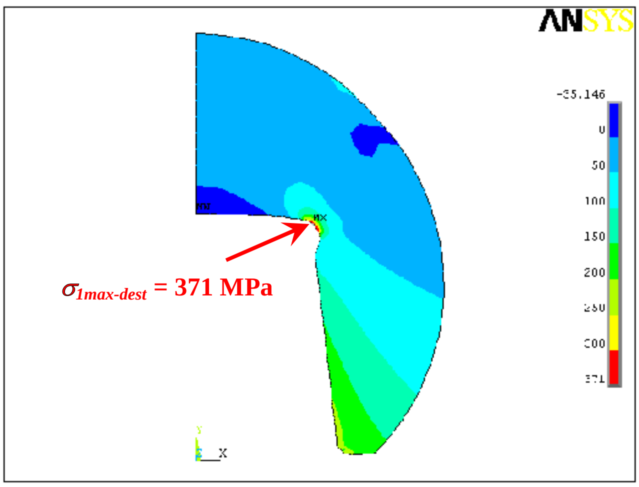

Fig. 9 The first principal stress (σ_1) in the head for load p = 100 MPa

TABLE I
VALUES $\sigma_{1MAX\text{-}DEST}$ ARRANGED IN DESCENDING ORDER AND VALUE P_F

j	$\sigma_{1max\text{-}dest}$ [MPa]	P_f	j	$\sigma_{1max\text{-}dest}$ [MPa]	P_f
1	208.4	0.024	21	314.8	0.512
2	216.4	0.049	22	322.8	0.537
3	217.1	0.073	23	324.5	0.561
4	221.8	0.098	24	330.5	0.585
5	224.5	0.122	25	340.8	0.610
6	250.8	0.146	26	343.0	0.634
7	255.6	0.171	27	346.2	0.659
8	266.6	0.195	28	357.2	0.683
9	270.4	0.220	29	360.1	0.707
10	270.6	0.244	30	364.7	0.732
11	277.2	0.268	31	365.2	0.756
12	283.6	0.293	32	371.2	0.780
13	289.1	0.317	33	374.5	0.805
14	301.3	0.341	34	377.1	0.829
15	303.9	0.366	35	379.3	0.854
16	305.5	0.390	36	390.2	0.878
17	306.4	0.415	37	420.1	0.902
18	307.7	0.439	38	449.3	0.927
19	311.1	0.463	39	473.4	0.951
20	314.4	0.488	40	501.7	0.976

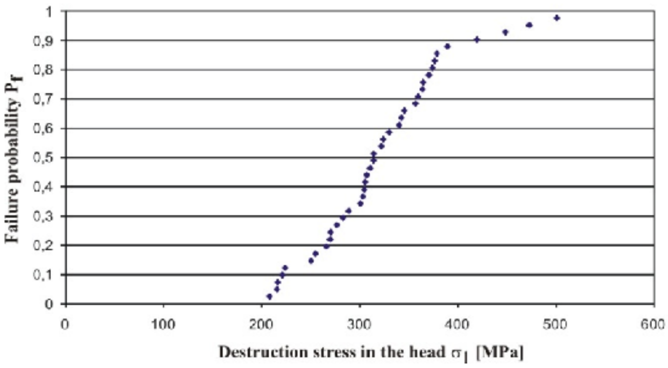

Fig. 10 Data from the Table I in the graphic form

The first of Weibull parameters is stress σ_u, which must be lower than the minimum value $\sigma_{1max\text{-}dest}$. ($\sigma_{1max\text{-}dest\text{-}min}$ = 208.4 MPa – see Table I or Fig. 10), so the stress σ_u was assumed for the following values - 0, 50, 100, 150 and 200 MPa. Conservative approach considers σ_u = 0 MPa, then all tensile stresses influence the body's destruction (thus Weibull 3-parameter analysis changes to a 2-parameter analysis).

The second parameter (Weibull modulus m) is connected with the dispersion of experimentally established values and it is determined as gradient of a line interlaid with experimentally established data from the Table I. Weibull modulus for above assumed values of the stress σ_u is show in the Fig. 11. The values of the correlation between the blue experimental data and the red line in Fig. 11 are shown in Fig. 12. The highest value of the correlation is for stress σ_u = 160 MPa – Fig. 13, so this is the first Weibull material parameter and the Weibull modulus m = 2.4166 (Figs. 12 and 13).

The last of the parameters (normalised volume strength σ_o) is calculated from the following equation

$$\sigma_o = \sqrt[m]{\sum_{i=1}^{n} (\sigma_i - \sigma_u)^m \Delta V_i}, \ \sigma_i \geq \sigma_u, \qquad (2)$$

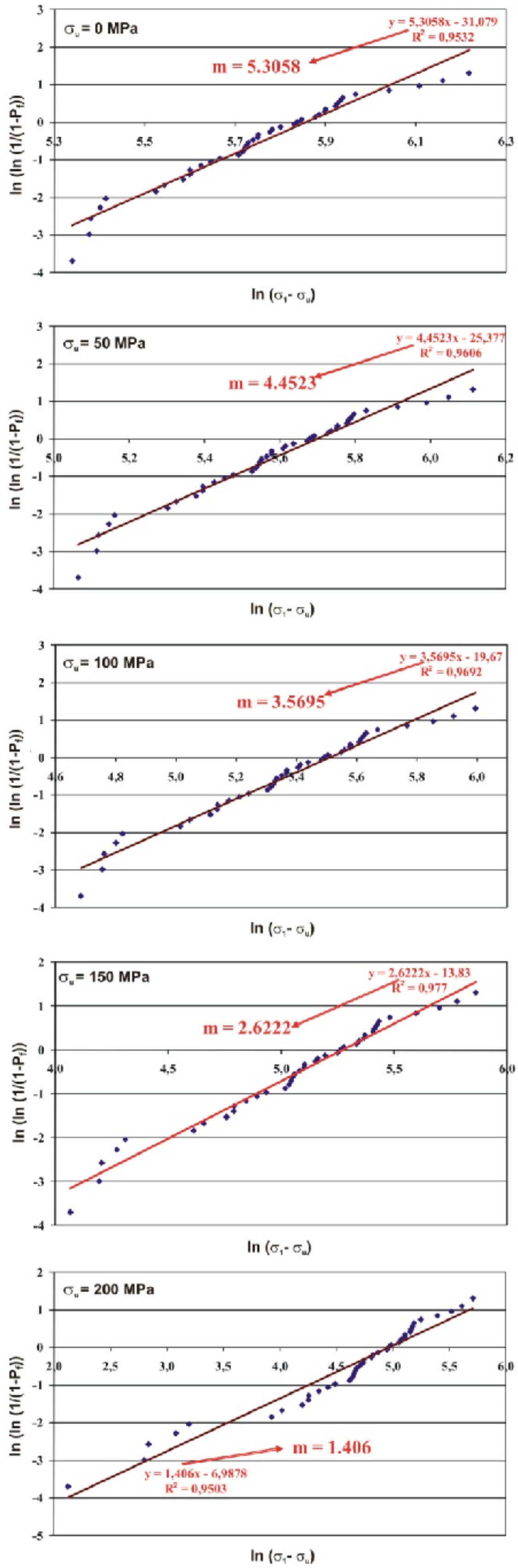

Fig. 11 Weibull plot of the normalized modulus of rupture data for different value of the stress σ_u

which was derived by modification of the equation (1) for P_f = 1-1/e = 0.63212 (failure probability for the experimental data and for the material parameters will be the same – 0.63212 – see fitted point if Fig. 13). As values σ_i are used the set of the values σ_1 calculated by the finite elements method in the head's all elements, for which it holds that $\sigma_1 > \sigma_u$.

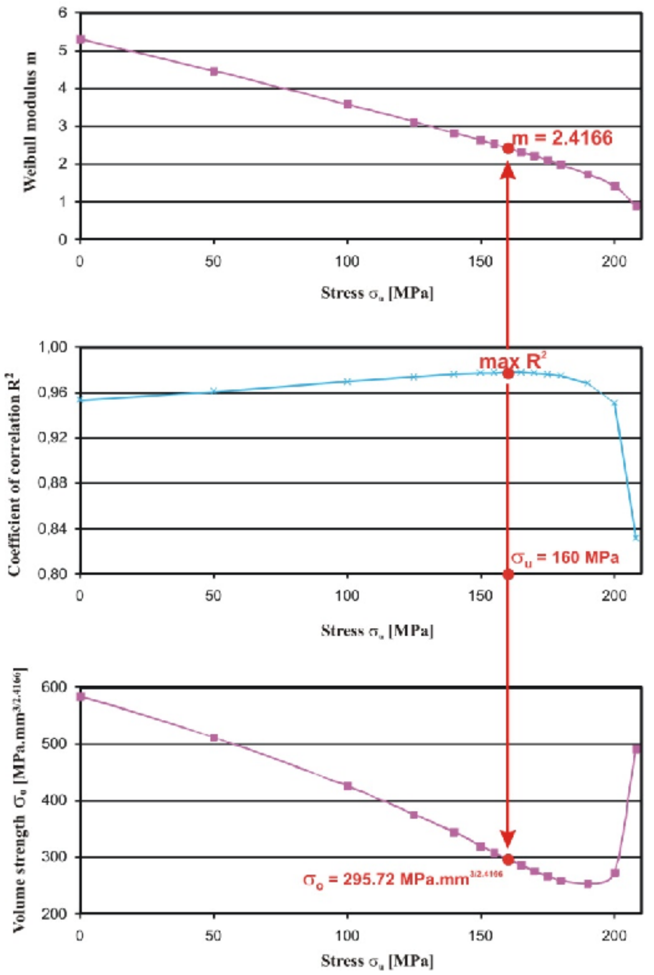

Fig. 12 Determination of the optional stress σ_u, Weibull modulus m and volume strength

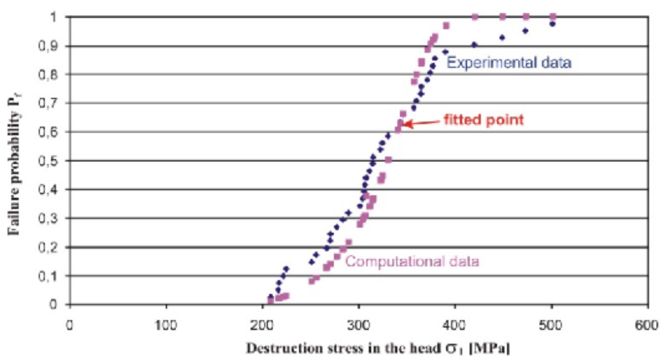

Fig. 13 Comparison of the experimental and computational data for 3-parameter Weibull material parameters with σ_u = 160 MPa

The same way is used to calculation of the material parameters for the conservative approach (σ_u = 0 MPa) and the comparison of experimental and computational data is show in Fig. 14. The material parameters for both models (2-parameter and 3-parameter) are show in Table II.

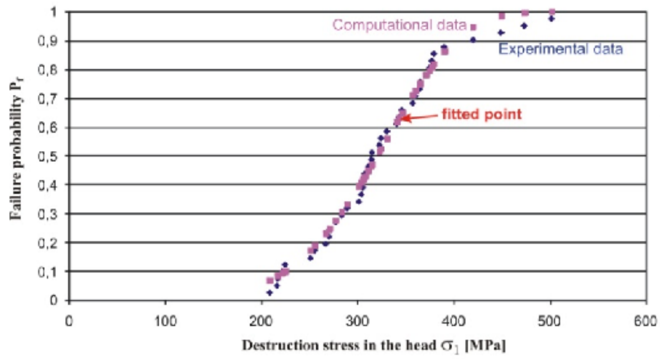

Fig. 14 Comparison of the experimental and computational data for 2-parameter Weibull material parameters with σ_u = 0 MPa

TABLE II
CALCULATED MATERIAL PARAMETERS

	2-parameters	3-parameters
σ_u	0 MPa	160 MPa
m	5.3058	2.4166
σ_o	583.7	295.7

From the comparison of the experimental and computational data in Figs. 13 and 14 implies the following results:

- 2-parameter Weibull material parameters better fit the experimental data thank 3-paramerter parameters but for the low values of the failure probability is large difference (the bioimplants have to reach the very low failure probability so this model have to be improved – for example fitting point will be moved from P_f = 0.63212 to value 0.024 (the first sample in the Table I)),
- 3-parameter Weibull material parameters is not conservative because the low values of the failure probability is lower than experimental data (in the reality is failure probability higher than calculated).

V. CONCLUSION

The material parameters of ceramic material used for manufacture of heads of total hip joint endoprosthesis have been determined on the basis of a set of head destruction tests. The analysis of the calculated material parameters show that both assumed models have the lacks but the 2-parameters Weibull material parameters are usable than 3-parameters ones.

The reliability of the parameters thus obtained is higher than those obtained from specimens cut out from the heads, because their dimensions are markedly smaller than those required by the standard. The presented approach is general and applicable to any brittle material whose strength characteristics exhibit Weibull probability distribution.

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