Model Predictive Fuzzy Control of Air-ratio for Automotive Engines

Hang-cheong Wong, Pak-kin Wong, Chi-man Vong, Zhengchao Xie, and Shaojia Huang

Abstract—Automotive engine air-ratio plays an important role of emissions and fuel consumption reduction while maintains satisfactory engine power among all of the engine control variables. In order to effectively control the air-ratio, this paper presents a model predictive fuzzy control algorithm based on online least-squares support vector machines prediction model and fuzzy logic optimizer. The proposed control algorithm was also implemented on a real car for testing and the results are highly satisfactory. Experimental results show that the proposed control algorithm can regulate the engine air-ratio to the stoichiometric value, 1.0, under external disturbance with less than 5% tolerance.

Keywords—Air-ratio, Fuzzy logic, online least-squares support vector machine, model predictive control.

I. INTRODUCTION

VEHICULAR emissions are the major source of air pollution, especially in urban area, which impacts the public health and environment significantly. Studies show that hundred thousand mortalities every year all over the world relate to vehicular emissions and the mortalities cause billions economic lost [1,2]. Besides, non-renewable resources scarcity promotes the essence of fuel consumption reduction. Air-ratio, which is also called lambda, relates closely to emissions, engine power and fuel consumption among all of the engine control variables. Air-ratio indicates the amount that the actual available air-fuel mixture differs from the stoichiometric ratio, e.g. 14.7:1 for gasoline, of the fuel being used [3]. Air-ratio is measured after combustion, and the reading is taken by using a lambda sensor located in the exhaust pipe.

Reference [4] mentioned that when an air-ratio deviates more than 2% from its stoichiometric ratio, the conversion efficiencies of three-way catalytic converter for converting carbon monoxide, hydrocarbons and nitrogen oxides drops more than 50%. Modern automotive engines are controlled by the electronic control unit (ECU) which usually uses look-up tables with compensation of a proportional-integral-derivative (PID) closed-loop controller for air-ratio control. However, the engine combustion nature is multivariable, time-variant, time-delay and chaotic [5].

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Such control method cannot produce desirable accurate control performance [6].

References [7] proposed an emerging and effective feedback control strategy for engine air-ratio control, model predictive control (MPC) which bases on prediction model and receding horizon optimization. For this kind of control strategy, the prediction model is a crucial component because the essence of MPC is to optimize the forecast of process behavior, and the forecast is accomplished with the prediction model. To develop successful MPC for real-time engine air-ratio control, an accurate and reliable air-ratio model with online update ability is necessary. However, the chaotic nature of engine combustion [5] makes conventional linear prediction methods inapplicable for air-ratio modelling.

Recently, artificial neural network (NN) is widely applied to automotive engine modelling [7,8]. However, the inherent drawbacks of NN, including multiple local minima, user burden on selection of optimal NN structure and over-fitting risk, constrains the application of NN to engine air-ratio control [9]. With an innovative nonlinear modelling technique of online least-squares support vector machines (OLSSVM) [9], [10] which combines the advantages of NN and nonlinear regression as well as its self-update ability, the previous drawbacks from NN are overcome. Therefore, OLSSVM is proposed to build the engine air-ratio model in this paper. Based on the multiple-step-ahead prediction of the air-ratio, an optimal control signal is obtained to regulate the air-ratio to be stoichiometry even when engine speed and load change. In this paper, a new optimization algorithm for MPC, fuzzy logic optimizer, is also proposed for the receding horizon optimization rather than those general optimization techniques, like Newton's method, Brent's method, sequential quadratic programming, etc, due to its fast computation, robustness, low cost and insensitive to noise.

II. ONLINE LEAST-SQUARES SUPPORT VECTOR MACHINES

The objective of air-ratio modelling is to build an air-ratio model for predicting the future air-ratio using the pervious and/or current engine parameters. In this paper, the proposed OLSSVM for air-ratio modelling are derived from classical least-squares support vector machines (LSSVM), so the structures of OLSSVM are presented together with its corresponding algorithms after presenting the classical LSSVM modelling algorithm.

A. Classical Least-Squares Support Vector Machines

The classical LSSVM formulation for the case of nonlinear modelling is derived as follows.

Consider the dataset, $\mathbf{D} = \{(X_1, y_1) \cdots (X_N, y_N)\}$, with input data

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 $X_t \in R^d$ and output $y_t \in R$, where N is the number of training data. LSSVM deals with the following optimization problem in the primal weight space.

min
$$J(\boldsymbol{w}, \boldsymbol{e}) = \frac{1}{2} \boldsymbol{w}^{\mathrm{T}} \boldsymbol{w} + \frac{\gamma}{2} \sum_{t=1}^{N} \boldsymbol{e}_{t}^{2}$$
 (1)

such that

$$\boldsymbol{e}_{t} = \boldsymbol{y}_{t} - \left[\boldsymbol{w}^{\mathrm{T}}\boldsymbol{\varphi}(\boldsymbol{X}_{t}) + \boldsymbol{b}\right] \qquad t = 1 \cdots N$$
(2)

where *w* is the weight vector of the air-ratio model, $\boldsymbol{e} = [e_1 \cdots e_N]$ is the residual vector, and $\varphi(...)$ maps the input data to a higher dimensional feature space, and the bias *b*. The built air-ratio model is expressed as below:

$$Y = M(X) = w^{\mathrm{T}}\varphi(X) + b$$
(3)

However, w may be in very high or even infinite dimensions that cannot be solved directly. In order to resolve the problem, the Lagrangian of (1) is constructed to derive the dual problem and the Lagrangian is as below:

$$L(\boldsymbol{w}, b, \boldsymbol{e}, \boldsymbol{a}) = J(\boldsymbol{w}, \boldsymbol{e}) - \sum_{t=1}^{N} \alpha_t \left[\boldsymbol{w}^T \varphi(\boldsymbol{X}_t) + b + \boldsymbol{e}_t - \boldsymbol{y}_t \right]$$
(4)

where α_t are Lagrange multipliers. After optimizing (4) and eliminating *w* and *e*, the LSSVM dual formulation of air-ratio modelling is then expressed as below [11]:

solve in
$$\boldsymbol{\alpha}, \boldsymbol{b}$$
:

$$\begin{bmatrix} 0 & \mathbf{1}_{\boldsymbol{v}}^{\mathrm{T}} \\ \mathbf{1}_{\boldsymbol{v}} & \boldsymbol{\Omega} + \frac{1}{\gamma} \mathbf{I}_{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix}$$
(5)

where $\mathbf{y} = [y_1 \cdots y_N]^T$, 1_v is an *N*-dimensional vector $= [1 \cdots 1]^T$, $\mathbf{a} = [\alpha_1 \cdots \alpha_N]^T$, and $\gamma \in R$ is a user-defined hyperparameter. The kernel trick is employed as follows:

$$\boldsymbol{\Omega} = \begin{bmatrix} \varphi(\boldsymbol{X}_{1})^{\mathrm{T}} \varphi(\boldsymbol{X}_{1}) & \cdots & \varphi(\boldsymbol{X}_{1})^{\mathrm{T}} \varphi(\boldsymbol{X}_{N}) \\ \vdots & \ddots & \vdots \\ \varphi(\boldsymbol{X}_{N})^{\mathrm{T}} \varphi(\boldsymbol{X}_{1}) & \cdots & \varphi(\boldsymbol{X}_{N})^{\mathrm{T}} \varphi(\boldsymbol{X}_{N}) \end{bmatrix}$$
(6)
$$= \begin{bmatrix} K(\boldsymbol{X}_{1}, \boldsymbol{X}_{1}) & \cdots & K(\boldsymbol{X}_{1}, \boldsymbol{X}_{N}) \\ \vdots & \ddots & \vdots \\ K(\boldsymbol{X}_{N}, \boldsymbol{X}_{1}) & \cdots & K(\boldsymbol{X}_{N}, \boldsymbol{X}_{N}) \end{bmatrix}$$

where $K(X_n, X_n)$ is a predefined kernel function and radial basis function (RBF) is chosen in this paper. The resulting air-ratio model built from classical LSSVM becomes:

$$Y = M(\mathbf{X})$$

$$= \sum_{t=1}^{N} \alpha_{t} \varphi(\mathbf{X}_{t})^{\mathrm{T}} \varphi(\mathbf{X}) + b$$

$$= \sum_{t=1}^{N} \alpha_{t} K(\mathbf{X}_{t}, \mathbf{X}) + b$$

$$= \sum_{t=1}^{N} \alpha_{t} \exp\left(-\frac{\|\mathbf{X}_{t} - \mathbf{X}\|^{2}}{\sigma^{2}}\right) + b$$
(7)

where $\alpha_t, b \in R$ are the solution of (5), X_t is the training input data vector. X is the new input vector for air-ratio prediction in this application. In the RBF kernel function, $K(X_t, X)$, σ^2 specifies the kernel sample variance which is also a user-defined hyperparameter, and $\|\cdot\|$ means Euclidean distance. Based on (7), the air-ratio model Y=M(X) can be obtained. In this paper, leave-one-out cross-validation is chosen to infer the values for the two user-defined hyperparameters (γ, σ) .

B. Online Incremental Procedure

By reformulating (5), the following relation is obtained below:

$$\mathbf{A}_{N}\boldsymbol{\alpha}_{N}=\boldsymbol{Y}_{N} \tag{8}$$

Where

$$\mathbf{A}_{N} = \begin{bmatrix} \mathbf{0} & \mathbf{1}_{v}^{T} \\ \mathbf{1}_{v} & \mathbf{\Omega} + \frac{1}{\gamma} \mathbf{I}_{N} \end{bmatrix}, \quad \boldsymbol{\alpha}_{N} = \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix}, \quad \boldsymbol{Y}_{N} = \begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix}$$

Whenever a new air-ratio sample (X_{N+1} , y_{N+1}) is added to the training dataset, **D**, the updated air-ratio model including N+1 samples can be obtained by the new incremental procedure:

$$\mathbf{A}_{N+1}\boldsymbol{\alpha}_{N+1} = \boldsymbol{Y}_{N+1} \tag{9}$$

where

$$\mathbf{A}_{N+1} = \begin{bmatrix} \mathbf{A}_{N} & \mathbf{a} \\ \mathbf{a}^{T} & \gamma^{-1} + 1 \end{bmatrix}, \quad \mathbf{\alpha}_{N} = \begin{bmatrix} \mathbf{\alpha}_{N} \\ \alpha_{N+1} \end{bmatrix},$$
$$\mathbf{Y}_{N+1} = \begin{bmatrix} \mathbf{Y}_{N} \\ \mathcal{Y}_{N+1} \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} 1 & K(\mathbf{X}_{1}, \mathbf{X}_{N+1}) & \cdots & K(\mathbf{X}_{N}, \mathbf{X}_{N+1}) \end{bmatrix}$$

The online incremental procedure aims to efficiently update \mathbf{A}^{-1}_{N+1} whenever a new air-ratio sample is added without explicit computation of the matrix inverse. Reference [12] showed that the matrix inverse \mathbf{A}^{-1}_{N+1} can efficiently be obtained by:

$$\mathbf{A}_{N+1}^{-1} = \begin{bmatrix} \mathbf{A}_{N}^{-1} & 0\\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \gamma^{-1} + 1 - \boldsymbol{a}^{\mathrm{T}} \mathbf{A}_{N}^{-1} \boldsymbol{a} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{A}_{N}^{-1} \boldsymbol{a}\\ -1 \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{\mathrm{T}} \mathbf{A}_{N}^{-1} & -1 \end{bmatrix}$$
(10)

C. Online Decremental Procedure

The online incremental procedure presented above can update and improve the built air-ratio model continually. However, the online incremental procedure increases the memory length continuously and the sparseness property in classical LSSVM is lost due to the condition $\alpha_t = \gamma e_t$ for optimality [12]. Therefore, the online incremental procedure is inefficient when dealing with online prediction with exceedingly large amount of data because it is costly and there is not enough space to store a lot of continually accumulated coefficients and data.

In view of this limitation, an online decremental procedure [9] for air-ratio model update is employed. This online decremental procedure is done after every online incremental procedure presented in the previous section, by omitting the earliest trained support values. As a result, the built air-ratio model is able to adapt any engine property change by continually update the built air-ratio model while maintaining the simple approximation and efficient implementation of classical LSSVM.

Specifically, the online decremental procedure means that a support value is removed when an earliest training data is removed. Similar to the case of the online incremental procedure, to avoid computing the matrix inverse, \mathbf{A}^{-1}_{N} must be updated from \mathbf{A}^{-1}_{N+1} . Here \mathbf{A}^{-1}_{N} is the matrix without the first row and the first column. Based on the way of the online decremental procedure, when the first sample is pruned from the *N*+1 pairs of the dataset, the update rule is obtained as shown in. (11).

$$\beta_{ij} - \beta_{11}^{-1} \beta_{i1} \beta_{1j} \to \beta_{ij} \tag{11}$$

where $i, j = 2 \cdots N$, β_{ij} stands for the element at the *i*-th row and *j*-th column of \mathbf{A}^{-1}_{N+1} . According to (11), \mathbf{A}^{-1}_N can be efficiently updated from \mathbf{A}^{-1}_{N+1} without explicitly computing the matrix inverse. Then the elements of the air-ratio model can be updated with (10). The presented online decremental procedure prunes the first air-ratio sample only from the *N*+1 pairs of the dataset that can maintain time history continuity of the built air-ratio model. So, the presented OLSSVM makes online learning for the classical LSSVM possible. Furthermore, the sparseness of the air-ratio model can be maintained by gradually pruning of the early trained support vectors.

III. MODEL PREDICTIVE CONTROL

MPC is based on prediction model and receding horizon optimization. Fig. 1 illustrates the proposed online least-squares support vector machines model predictive controller (OLSSVMMPC) which consists of an OLSSVM air-ratio model and a fuzzy logic optimizer. The OLSSVM air-ratio model predicts the engine response over a specified time horizon and the predictions are then used by the fuzzy logic optimizer to determine the optimal control signal, fuel injection time. The optimal fuel injection time is finally fed to fuel injectors of the engine. In addition, the measured air-ratio updates the OLSSVM air-ratio model continually.



Fig. 1 Structure of OLSSVMMPC

A. Fuzzy Logic Optimizer

Fuzzy logic deals with uncertain and imprecise situations. Linguistic variables (rich, stoichiometry, lean, etc.) are used to represent the domain knowledge, with their truth values lying between 0 and 1. Basically, a fuzzy logic optimizer has got the following components [13]:

- 1. A fuzzification interface to scale and map the measured variables to suitable linguistic variables.
- 2. A knowledge base comprising linguistic control rule base.
- 3. A decision making logic to infer the fuzzy logic control action based on the input variables, which is much similar to the human decision making.
- 4. A defuzzification interface to scale and map the linguistic control actions inferred to yield a non-fuzzy control signal to the fuel injectors of the engine.

TABLE I INPUT, OUTPUT, LINGUISTIC VARIABLE AND SCALING FACTOR OF FUZZY LOGIC OPTIMIZER

	0.		
Original Variable	Туре	Linguistic Variables	Scaling Factor
Engine speed (ES)		Idle (I)	
	Input	Cruising (C)	G_{ES}
	1	High speed (H)	
Predicted air-ratio (λ_p)		Very rich (VR)	
		Rich (R)	
	Input	Stoichiometry (S)	$G_{\lambda p}$
		Lean (L)	Ĩ
		Very lean (VL)	
Target air-ratio (λ_r)		Rich (R)	
	Input	Stoichiometry (S)	$G_{\lambda r}$
		Lean (LE)	
Throttle position (TP)	Input	Zero throttle (ZT)	
		Partial throttle (PT)	G_{TP}
		Full throttle (FT)	
Fuel injection time (<i>u</i>)		Low duty (LD)	
		Partial duty (PD)	
	Output	Medium duty (MD)	G_u
		High duty (HD)	
		Full duty (FD)	



As a demonstration, triangular membership functions are used to represent the linguistic variables for all input and output variables. All input and output variables of the fuzzy logic optimizer, the linguistic variables and the corresponding scaling factors for each input and output variable are tabulated in Table I. The scaling factors in Table I are used for mapping the input and output variables to the respective universes of discourses. A rule base developed by heuristics is given in Table II and the membership grade of the fuel injection time, the fuzzy fuel injector duty cycle u_6 is determined by (12).

$$u_{f} = S \times F[\mu(G_{ES}ES), \mu(G_{TP}TP), \mu(G_{\lambda_{p}}\lambda_{p}), \mu(G_{\lambda_{r}}\lambda_{r})] \quad (12)$$

where *F* denotes the fuzzy relation defined in the fuzzy rule base, and *S* is the transfer function of the fuel injector that convert the fuzzy duty cycle to the fuel injection time. The conventional max-min composition rule of inference is used for determining the appropriate fuzzy duty cycle. Here the height method of defuzzification is used that the centroid of each output membership function for each rule is evaluated first and the final optimizer output, *u*, is calculated as multiplying the gain G_u by the average of the individual centroid, weighted by their heights as (13).

$$u = G_u \frac{\sum \mu(u_f) u_f}{\sum \mu(u_f)}$$
(13)

IV. EXPERIMENTAL SETUP

The proposed OLSSVMMPC was implemented and tested on a performance test car, Honda DC5 Type-R with K20A i-VTEC engine connected to a MoTeC M800 programmable ECU and National Instrument (NI) CompactDAQ chassis DAQ-9178. NI DAQ-9178 includes an analog input module NI 9125, an analog output module NI 9263 and a digital input/output module NI 9924. The OLSSVMMPC was implemented using MATLAB. MoTeC M800 is mainly used for engine control whereas NI DAQ-9178 is used for sending the control signal to MoTeC ECU via a LabVIEW interface program according to the OLSSVMMPC implemented in MATLAB. In other words, NI DAQ-9178 serves as an interface between the OLSSVMMPC and MoTeC ECU and the detail signal flow between the test car and the OLSSVMMPC is shown in Fig. 2.

The training data for building the OLSSVM air-ratio model were obtained using a wide-band lambda sensor subject to random throttle condition.



Fig. 2 Detail signal flows between test car and OLSSVMMPC

A. Pilot Test

Fig. 3 shows a test cycle where the throttle position changes from 0 to 50% throttle. This change can be viewed as a disturbance. The air-ratio is needed to control within the $\pm 5\%$ bounds of the stoichiometric value, 1.0. After choosing the sampling time to be 0.01*s*, the effectiveness of the OLSSVMPC can be examined.



Fig. 3 Throttle position in test cycle

B. Results

With the above test cycle, the air-ratio control result and fuel injection time are shown in Fig. 4 and 5. In Fig. 4, the target air-ratio, λ_r =1.0, is 1. An air-ratio greater than 1 indicating fuel lean whereas and an air-ratio less than 1 means fuel rich.

In order to show the advantages of the proposed OLSSVMMPC, its control result is compared with that of a typical PID controller used in the existing automotive ECU. The PID gains of the controller were obtained by the Ziegler-Nichols rules. The best control result and the corresponding fuel injection time of the PID controller are shown in Figs. 6 and 7.



Fig. 4 Air-ratio control result of OLSSVMMPC



Fig. 5 Fuel injection time of OLSSVMMPC



Fig. 6 Air-ratio control result of PID controller



Fig. 7 Fuel injection time of PID controller

Fig. 4 shows that the OLSSVMMPC can control the air-ratio with a significantly less deviation from target air-ratio, $\lambda_r = 1.0$, under disturbance. The control performances of the OLSSVMMPC and the PID controller are evaluated through three measures: mean absolute percentage error (*MAPE*) defined in (14), maximum overshoot and maximum response time which are summarized in Table III.

$$MAPE = \frac{1}{N_t} \sum_{i=1}^{N_t} \left(\frac{|\lambda_i - 1|}{1} \times 100\% \right)$$
(14)

where λ_i and N_t are the air-ratio at the *i*-th time step and the total number of time step respectively.

TABLE III Control Performance of OLSSVMMPC and PID Controller				
Controller	MAPE	Max. Overshoot	Max. Response Time	
OLSSVMMPC	0.67%	0.48	2.90s	
PID	2.44%	0.15	3.48 <i>s</i>	

The *MAPE* of the OLSSVMMPC and PID controller are 0.67% and 2.44% respectively. The control performance of the OLSSVMMPC is 72.54% better than that of the PID controller. Moreover, the OLSSVMMPC can control the overall air-ratio deviation within \pm 5%. Table III also reveals that the maximum response time of OLSSVMMPC are superior to the PID controller. The promising results show that the proposed OLSSVMMPC can really improve the air-ratio control performance.

V.CONCLUSION

This study is the first attempt at applying MPC by combining OLSSVM and fuzzy logic optimizer for engine air-ratio control. In view of the high accuracy of the air-ratio model and its self-update ability, the OLSSVMMPC can perform air-ratio control effectively. The proposed OLSSVMMPC was successfully implemented and tested on a real performance car. Experimental results show that the air-ratio control performance of the OLSSVMMPC is significantly better than that of a conventional PID controller in current automotive ECU. Thus, the OLSSVMMPC is a promising control scheme to replace the PID controller in the automotive ECU for engine air-ratio control.

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