

# Scheduling a Project to Minimize Costs of Material Requirements

Amir Abbas Najafi<sup>1</sup>, Nima Zoraghi<sup>2</sup>, Fatemeh Azimi<sup>3</sup>

**Abstract**—Traditionally, project scheduling and material planning have been treated independently. In this research, a mixed integer programming model is presented to integrate project scheduling and materials ordering problems. The goal is to minimize the total material holding and ordering costs. In addition, an efficient metaheuristic algorithm is proposed to solve the model. The proposed algorithm is computationally tested, the results are analyzed, and conclusions are given.

**Keywords**—Project scheduling; metaheuristic; material ordering, optimization

## I. INTRODUCTION

PROJECT planning plays an important role in project management. Project scheduling and material planning are two main elements of a project plan. It is the responsibility of a project manager to make sure that the employees have the material required to perform their activities on time. Traditionally, these two issues have been treated independently. Following this strategies, usually the trade-offs between cost elements such as, material ordering cost and inventory holding cost are complexities, therefore the total cost of project are increased. While this approach usually results in a non-optimal solution for the material planning, the best solution is obtained when the project scheduling and the material ordering are determined simultaneously.

In 1980, Aquilano and Smith [1] introduced the incorporated problem of project scheduling and material ordering (PSMO). They developed a model by critical path method and material requirement planning in which it's consisting of materials and inventory levels scheduling. Smith-Daniels and Smith-Daniels [2] investigated the problem with fixed activity duration and showed that the latest starting time schedule provides an optimal solution to the problem. They proof that once the activities schedule is determined, the optimal ordering plan for each of the materials required is obtained by the economic lot size model of Wagner and Whitin. Dodin and Elimam [3] extended Smith-Daniels and Smith-Daniels's work to include variable activity duration, reward and penalty policy for the project and showed that, the variability in activity duration gives more flexibility to project

scheduling, resulting in more cost reduction and allowing for saving on the complete activities and material ordering cost.

Sajadieh et al. [4] proposed a genetic algorithm to solve the PSMO problem where the ordering times of materials for each activity were determined independently.

In this paper, the PSMO problem is extended in a way that the ordering times of materials are corporately determined by considering all activity requirements. This approach reduces the total cost significantly. In addition, an efficient hybrid algorithm is proposed to solve this problem, especially for large-scale problems.

The rest of the paper is organized as follows: In section two, the problem is precisely defined. In section three, we propose a metaheuristic algorithm to solve the problem. We measure the performance of the proposed method in section four, and finally the conclusion comes in section five.

## II. PROBLEM FORMULATION

A project is given with a set of  $n$  activities indexed from 1 to  $n$ . Activities 1 and  $n$ , are dummies and represent the project start and completion respectively. The project network is imposed zero-lag finish-to-start precedence constraints on the sequencing of activities and shown by an activity on node network with no loop. Activity  $i$  has a fixed duration  $d_i$ . Duration  $d_i$  is given where activity  $i$  is started and it runs  $d_i$  time without preemption.

The execution of the  $i^{\text{th}}$  activity requires  $f$ , ( $f = 1, 2, \dots, F$ ) types of materials (non-renewable) over its duration. The resource usage over an activity is taken uniform and a typical activity  $i$  uses  $u_{if}$  units of material  $f$  per period. In addition,  $A_f$  and  $H_f$  denote the ordering cost and the holding cost per unit of the  $f^{\text{th}}$  material per unit time, respectively. Further, the type and the quantity of all materials must be determined at the beginning of each period.

The activities are to be scheduled such that the makespan of the project does not exceed a given due date ( $DD$ ). Furthermore, we assume that the lead time is assumed inappreciable.

We can now formulate the PSMO as follows:

$$\text{Min } z : \sum_{f=1}^F \sum_{t=0}^{DD-1} A_f \lambda_{ft} + \sum_{f=1}^F \sum_{t=0}^{DD} H_f I_{ft} \quad (1)$$

Subject to:

<sup>1</sup> Faculty of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran (Phone: +98 2184063378; Fax: +98 2188674858; Email: aanajafi@kntu.ac.ir).

<sup>2</sup> Faculty of Industrial and Mechanical Engineering, Islamic Azad University, Qazvin Branch, Qazvin, Iran (Phone: +98 912 3386402; Email: Nima.Zoraghi@gmail.com).

<sup>3</sup> Department of Mathematics, Islamic Azad University, Qazvin Branch, Qazvin, Iran (Phone: +98 281 3665275; Email: ff\_azimi@yahoo.com).

$$\sum_{t=ES_j}^{LS_j} t.X_{jt} + d_j \leq \sum_{t=ES_i}^{LS_i} t.X_{it} ; \quad \forall j \in P_i \quad (2)$$

$$\sum_{t=ES_i}^{LS_i} X_{it} = 1 ; \quad i = 1, 2, \dots, n \quad (3)$$

$$\sum_{t=ES_n}^{LS_n} tX_{nt} \leq DD \quad (4)$$

$$I_{ft} = I_{f(t-1)} + Q_{ft} - \sum_{i=1}^n \sum_{w=Max\{t-d_i+1, ES_i\}}^{Min\{t, LS_i\}} u_{if} \times X_{iw} ;$$

$$f=1, 2, \dots, F, \quad t=1, \dots, DD \quad (5)$$

$$I_{f0} = Q_{f0} \quad (6)$$

$$Q_{ft} \leq \lambda_{ft} \times M \quad (7)$$

$$X_{it} = \{0, 1\}; \quad i = 1, 2, \dots, n, \quad t = ES_i, \dots, LS_i \quad (8)$$

$$\lambda_{ft} = \{0, 1\}; \quad f = 1, 2, \dots, F, \quad t = 0, 1, \dots, DD-1 \quad (9)$$

$$Q_{ft} \geq 0 ; \quad f = 1, 2, \dots, F, \quad t = 0, 1, \dots, DD-1 \quad (10)$$

$$I_{ft} \geq 0 ; \quad f = 1, 2, \dots, F, \quad t = 0, 1, \dots, DD \quad (11)$$

Where the decision variables are defined as follows:

$X_{it}$  : A binary variable where it is one if activity  $i$  is started in period  $t$  and zero otherwise.

$\lambda_{ft}$  : A binary variable where it is one if material  $f$  is ordered in period  $t$  and zero otherwise.

$Q_{ft}$  : The ordered quantity of material  $f$  in period  $t$ .

$I_{ft}$  : The inventory level of material  $f$  in period  $t$ .

The objective function (1) minimizes the total costs of the project. Inequality (2) enforces the precedence relations between activities. Constraint (3) states that every activity must be started only once. Inequality (4) ensures that the project ends by the latest allowable completion time. Constraints (5) and (6) balance the levels of the materials over the project execution. Inequality (7) denotes relationship between  $\lambda_{ft}$  and  $Q_{ft}$ . Where,  $ES_i$  is the earliest starting time,  $LS_i$  is the latest start time of activity  $i$  and  $M$  represents a large number. Sets of constraint (8)-(11) denote the domains of the variables.

### III. A SOLUTION PROCEDURE

In this section, a meta-heuristic algorithm is proposed to solve the model. The detailed framework of the solution procedure is presented as follows.

#### A. Proposed Simulated Annealing Algorithm

Simulated annealing (SA) is one of the best meta-heuristics that were initially presented by Kirkpatrick et al [5]. This algorithm, attempts to solve hard combinatorial optimization problems through a controlled randomization. The proposed hybrid algorithm to solve the extended PSMO model consists of two loops. In the first loop, SA attempts to find a schedule for activities. In the second loop, a genetic algorithm is used to find the best materials ordering policy for the activities schedule such that the minimum total holding and ordering costs of materials is obtained.

#### B. Simulated annealing for PSMO

In the basic scheme, SA starts with generation of an initial solution (one point), in the SA part of the proposed hybrid algorithm of this research, it starts with generation of several initial solutions (multi-point). In this research, the initial solution is generated by the critical path method, by which the earliest start time ( $ES_i$ ) and the latest start time ( $LS_i$ ) of the activities are obtained. Then the results of forward and backward pass computations allow for the calculation of float values of the network activities. In other words, the total float of activity  $i$  is defined as:

$$TF_i = LS_i - ES_i \quad (12)$$

Moreover, the floating time of an activity at a given schedule is equal to the difference between the activity's starting time at the schedule and its earliest starting time, i.e., the floating time of activity  $i$ , at a given schedule  $H$ ,  $F_H(i)$  is obtained as follows [6]:

$$F_H(i) = ST_H(i) - ES_i \quad (13)$$

Where  $ST_H(i)$  denotes the starting time of activity  $i$  at schedule  $H$ . We note that the used floating time of an activity should be less than or equal to its total floating time, i.e.,

$$0 \leq F_H(i) \leq TF_i \quad (14)$$

An initial solution is denoted by a vector of  $n-2$  elements  $F = (F(2), F(3), \dots, F(n-1))$ , where the position of each element corresponds to the number of a non-dummy activity and its value denotes the floating time of the activity. As a result, initial solutions can be generated randomly from the feasible region of vector  $F$ . In order to evaluate the objective function for a given feasible solution, both the starting times of all activities and the order quantities of the required materials are needed. Based on the  $F$  vector, the schedule vector  $ST = (ST(2), ST(3), \dots, ST(n-1))$  that contains the start time of the activities is obtained by adding up the elements of the vector  $F$  and the earliest starting times of the activities. Then, its near optimal material requirement planning is determined using a genetic algorithm to evaluate a

schedule.

The SA continues by generating neighborhoods of initial solutions. The roulette wheel procedure is applied as a neighborhood search structure to generate new feasible solutions. Through this mechanism, each activity can move backward or forward to new position based on its floating time. More specifically, a uniform random number is first generated in the interval  $[2, n - 1]$  for each activity. Then, the activities can move to their new positions based on their corresponding floating times and the generated random number. To avoid infeasible solutions, the floating time of the activities given in equation (14) must be met.

As an example, the neighborhood structure of the proposed SA for generating new feasible solutions is illustrated in Fig.1, in which an activity can shift forward (positive numbers in the second row) and backward (negative numbers in the second row) based on the generated uniform random number.

Random number generated at interval $[2, N-1]$	[0,0,1]	[0,1,0.5]	[0.5,0.9]	[0.9,1]
Move activity (Shift forward or backward)	-2	-1	1	-2

Fig. 1 Neighborhood structure of the proposed SA

When SA proceeds, the temperature is gradually lowered under a certain mechanism called cooling schedule. In this paper, a linear cooling scheme is applied to decrease the temperature.

The details of the proposed GA in finding the order quantities of materials and determining the minimum total ordering and holding costs are described in the next subsection.

### C. Genetic Algorithm for PSMO

Genetic Algorithms (GA) have been originally developed by John Holland as artificial adaptive systems simulating natural evolution and have proven themselves as powerful search algorithms [7]. In this paper, we propose a GA approach to find the best material ordering policy of a given activity schedule. The GA starts by generation of an initial population, i.e., the first generation. The initial population is randomly generated according to demand profiles of the materials in the activity schedule. In order to create the next generation, after computing the fitness values of the individuals, two operations are employed; crossover and mutation. Finally, the algorithm stops if a specified number of generation, denote by Gen, are created. The best individual of the last generation is the best ordering policy of the GA for the given activity schedule. In this research, a real mode is used to code the search points of the solution based on the ordered quantities of the materials in each period. Each individual chromosome,  $Q$ , is a matrix of

$F$  rows (for  $F$  types of materials) and  $(DD - 1)$  columns (for periods 0 to  $(DD - 1)$ ), where an element (gene)  $Q_{ft}$  represents the ordered quantity of material  $f$  in period  $t$ . Fig.2 presents the general form of a chromosome.

$$\begin{bmatrix} Q_{10} & Q_{11} & \dots & Q_{1,DD-1} \\ \vdots & \vdots & \vdots & \vdots \\ Q_{F0} & Q_{F1} & \dots & Q_{F,DD-1} \end{bmatrix}_{Q_{ft}}$$

Fig. 2 A chromosome of the interior GA

In crossover operation, two parents are selected by the roulette wheel strategy to create two children. The Uniform continuous crossover operator is employed in this research. Consider two individuals  $P^1$  and  $P^2$  selected for a crossover operation. Therefore, draw a vector  $\alpha = [\alpha_t]_{1 \times DD}$  randomly based on uniform distribution on the interval  $[0,1]$ , where  $\alpha_t$ , used as the crossing point. Then, each gene of the two children  $CH^1$  and  $CH^2$  is obtained as follows:

$$\begin{aligned} CH_{ft}^1 &= P_{ft}^1(\alpha_t) + P_{ft}^2(1 - \alpha_t) \\ CH_{ft}^2 &= P_{ft}^2(1 - \alpha_t) + P_{ft}^1(\alpha_t) \end{aligned} \quad (15)$$

To describe mutation operation, let  $Q$  be the chromosome that is selected for mutation. First, an integer random number,  $R$ , is generated in the interval  $[1, F]$  to select one type of material. Then, two random numbers,  $r_1$  and  $r_2$  such that  $r_1 < r_2$ , are generated in the interval  $[0, DD - 1]$ . Hence,  $(Q_{R,r_1}, \dots, Q_{R,r_2})$  are the genes of the child considered for mutation. Next, the amount of each gene in the new chromosome ( $Q^M$ ) is obtained as follows:

$$\begin{aligned} Q_{R,r_1}^M &= \sum_{j=r_1}^{r_2} Q_{R,j} \\ Q_{R,t}^M &= 0 \quad ; \quad t = r_1 + 1, \dots, r_2 \\ Q_{ft}^M &= Q_{ft} \quad ; \quad otherwise \end{aligned} \quad (16)$$

## IV. EXPERIMENT AND COMPARISONS

In this section, we evaluate the performance of the proposed HSA introduced in the previous sections. To evaluate the performance of the GA we need some good solutions. Since there is no other existing procedure to solve the PSMO problem, we solve the mathematical modeling of the test problems by solver software such as LINGO [36]. To get a PSMO instance set, first three collections of RCPSP instances with 10, 20, and 30 non-dummy activities with 1, 2 and 3 resources are generated by PROGEN [9] for the experiments. PROGEN is an instance generator for a broad class of

resource-constrained project scheduling problem by varying some factors. For each combination set of the non-dummy activities and resources, 10 problems are examined; resulting in 90 test problems. Table 1 shows the computational results of the proposed algorithm in which column A and B denotes the number of instances LINGO and HSA were able to find a local optimal solution in 3600 CPU seconds respectively. In addition, column C displays the average of the relative deviation percentage of instances.

TABLE I  
 COMPUTATIONAL RESULTS

No. of activities	No. of problems	A	B	C
10	30	30	30	-0.79%
20	10	18	10	-0.80%
30	10	9	10	-1.10%

The results of the experiments on 90 test problems with different sizes show that for 10, 20, and 30-activity problems the average of proposed HSA has reached almost the same solutions as the ones from. Moreover, for both 20 and 30-activity problems, while there are many instances the LINGO solver is unable to solve, there is a solution by the proposed method. In addition, the amount of CPU time for the proposed method is much less than that of those obtained by LINGO. In summary, the results show that the proposed HSA has reached good solutions in shorter amount of CPU times than LINGO.

#### V. CONCLUSIONS

This research investigated a class of project scheduling problem, called project scheduling problem with material ordering. The problem was formulated into a mixed integer programming model. In addition to solve the model, a hybrid simulated annealing was applied. Based on standard test problems constructed by the PROGEN project generator, a comprehensive computational experiment is performed. The results of the experimentation were quite satisfactory.

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