New Technologies for Modeling of Gas Turbine Cooled Blades

1-5 Azerbaijan National Academy of Aviation
AZ-1045, Bina, 25th km, Baku, Azerbaijan
Phone: (99412) 497-28-29, 497-28-38; Fax: (99412) 497-28-29
e-mail: sadixov@mail.ru

Abstract - In contrast to existing methods which do not take into account multiconnectivity in a broad sense of this term, we develop mathematical models and highly effective combination (BEM and FDM) numerical methods of calculation of stationary and evanstationary temperature field of a profile part of a blade with convective cooling (from the point of view of realization on PC). The theoretical substantiation of these methods is proved by appropriate theorems. For it, converging quadrature processes have been developed and the estimations of errors in the terms of A.Zigmund continuity modules have been received.

For visualization of profiles are used: the method of the least squares with automatic conjecture, device spline, smooth replenishment and neural nets.

Boundary conditions of heat exchange are determined from the solution of the corresponding integral equations and empirical relationships. The reliability of designed methods is proved by calculation and experimental investigations heat and hydraulic characteristics of the gas turbine 1st stage nozzle blade.

Keywords - multiconnected systems, method of the boundary integrated equations, splines, neural networks.

1. INTRODUCTION

The development of aviation gas turbine engines (AGTE) at the present stage is reached, in main, by assimilation of high values of gas temperature in front of the turbine ($T_\text{lim}$). Despite of the fact, that the activities on temperature increase are conducted in several directions. However, assimilation of high $T_\text{lim}$ in AGTE is reached by refinement of cooling systems of turbine hot details, and first of all, nozzle and working blades.

It is especially necessary to underline, that with $\rho_\text{c}$ increase the requirement to accuracy of eventual results will increase. In other words, at allowed (permissible) in AGTE metal temperature ($T_\text{lim}$) = (1100...1300 K), $\rho_0$, the absolute error of temperature calculation should be in limits (20-30 K), that is no more than 2-3%.

It is difficult to achieve. In skew fields of complicated shape with various cooling channels, quantity and arrangement have a complex configuration, that is in multiply connected areas with variables in time and coordinates by boundary conditions such problem solving requires application of modern and perfect mathematical device.

2. PROBLEM FORMULATION

In classical statement a heat conduction differential equation circumscribing in common case non-stationary process of distribution of heat in many - dimensional area (an equation of Fourier-Kirhgof) has a kind [1]:

$$\frac{\partial (\rho C_v T)}{\partial t} = \text{div} (\lambda \nabla T) + q_v,$$  

(1)

where $C_v$ - accordingly material density, thermal capacity and heat conduction, $q_v$ - internal source or drain of heat, and $T$-required temperature.

By results of researches, it has been established [2], that the temperature condition of a profile part of a blade with radial cooling channels can with a sufficient degree of accuracy be determined, as two-dimensional. Besides if to suppose a constancy of physical properties, absence of internal sources (drains) of heat, then the temperature field under fixed conditions will depend only on the shape of a skew field and on distribution of temperature on skew field boundaries.

In this case, equation (1) will look like:

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$  

(2)

When determining particular temperature fields in gas turbine elements are more often set boundary conditions of the third kind describing heat exchange between a skew field and an environment on the basis of a hypothesis of a Newton-Riemann. In that case, these boundary conditions will be recorded as follows:

$$\alpha_0 (T_\text{lim} - T_\infty) = \lambda \frac{\partial T}{\partial n}$$  

(3)

Characterizes quantity of heat transmitted by convection from gas to unit of a surface of a blade and assigned by heat conduction in a skew field of a blade:

$$-\lambda \frac{\partial T}{\partial n} = \alpha_0 (T_\text{lim} - T_\text{air})$$  

(4)

Characterizes quantity of a heats assigned by a convection of the chiller, which is transmitted by heat conduction of the material of a blade to the surface of cooling channels; where $T_\text{lim}$, $T_\text{air}$ - temperature of environment at $i=0$; $T_\text{air}$ - temperature of environment at $i = 1, M$ (temperature of the chiller), where $M$ - quantity of outlines; $T_\text{lim}$ - temperature on an outline at $i = 0$, (outside outline of blade); $T_\text{air}$ - temperature on an outline at $i = 1, M$ (outline of cooling channels); $\rho_0$ - heat transfer factor from gas to a surface of a blade (at $i = 0$) - heat transfer factor from a blade to the cooling air at $i = 1, M$; $\lambda$ - thermal conductivity of the material of a blade; $n$-external normal on an outline of researched area.

3. PROBLEM SOLUTION

At present for the solution of this boundary value problem (2)-(4) most broad distribution have received four numerical methods – methods of final differences (MFD), finite element method (FEM), probabilistic method (or method Monte-Carlo) and method of boundary integral equations (BIEM) (or its discrete analog - a method of boundary element (BEM)).

Let’s consider BIEM application for the solution of problem (2)-(4) [1, 3, 14].

3.1. In contrast to [4] we offer to decide the given boundary value problem (2)-(4) as follows [1-3, 14]. We suppose, that distribution of temperature $T(x,y)$ we locate as follows:

$$T(x,y) = \int_0^\rho /nR^4 d\sigma ,$$  

(5)
where $\Gamma = \bigcup_{i=0}^{M} \gamma_i$ - smooth closed Jordan curve; $M$-quantity of cooled channels; $\rho = \bigcup_{i=0}^{M} \rho_i$ - density of a logarithmic potential uniformly distributed on $\gamma_i$, $S = \bigcup_{i=0}^{M} S_i$.

Thus curve $\Gamma = \bigcup_{i=0}^{M} \gamma_i$ are positively oriented and are given in a parametric kind: $x = x(s), y = y(s), s \in [0, L]$.

Using BIEM and expression (5) we shall put problem (2)-(4) to the following system of boundary integral equations:

$$
\rho(s) - \frac{1}{2\pi i} \int_{\gamma_i} \left[ (\rho(x) - \rho(\xi)) \frac{d}{dn} \ln R(s, \xi) \right] d\xi = 0,
$$

(6) where

$$
R(s, \xi) = \left( (x(s) - x(\xi))^2 + (y(s) - y(\xi))^2 \right)^{1/2}.
$$

For an evaluation of the singular integral operators who are included in the discrete operators of a logarithmic potential of simple and double layer (6) are investigated, their connection is shown and the evaluations in term of modules of a continuity (evaluation such as assessments by A. Zigmound are obtained).

**Theorem (main)**

Let

$$
\int_0^\infty \frac{e^{-x}}{x} dx < +\infty
$$

and let the equation (12) have the solution $\tilde{\mathcal{G}} \in C_{\bar{E}}$ (the set of continuous functions on $\Gamma$). Then $\exists N \in \mathbb{N}$ such that $\forall N > N_0$ the discrete system, obtained by using the discrete double layer potential operator (its properties has been studied), has unique solution $w = W(z, \bar{x})$.

3.2. The given technique of calculation of a temperature field of a blade can be applied and to blades with the plug – in split by blade and also conditions of interfaces between segments of a partition of an outline as equalities of temperatures and heat flows

$$
T_C(x, y) = T_{s1}(x, y),
$$

(7)

$$
\frac{\partial T_C(x, y)}{\partial n} = \frac{\partial T_{s1}(x, y)}{\partial n},
$$

(8)

where $T_C$ - number of segments of a partition of an outline of section of a blade; $x, y$ - coordinates. At finding of best values $T$ (chiller). It is necessary to decide a return problem of heat conduction. For it is necessary to find at first solution of a direct problem of heat conduction under boundary condition of the III kind from a leg of gas and boundary conditions 1 kinds from a leg of a cooling air

$$
T_v(x, y) = T_{b0},
$$

(9)
Were $T_b$ - unknown optimum temperature of a wall of a blade from a leg of a cooling air.

3.3. The developed technique for the numerical decision of stationary task heat conduction in cooled blades can be spread also on evavistant case.

Let’s consider a third regional task for evavistent lines of the heat conduction equation:

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right) = 0$$

(10)

$$\alpha_1(T_e - T_p) - \lambda(T)\frac{\partial T}{\partial n} = 0$$

(11)

For lines of a task (10) - (11) we shall take advantage of substitution Kirhgof:

$$A = \int_0^T \lambda(T) d\xi$$

(12)

Then the equation (10) is transformed to the following equation Laplas:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0$$

(13)

For preservation converging composed lines in a regional condition (11) we shall accept in initial approximation constant

$$\lambda(T) = \gamma.$$ Then from (12) we have

$$T = \lambda(T)$$

And the regional condition (11) will be transformed as follows:

$$\alpha_1(T_e - A_p / \lambda(T)) - \frac{\partial A}{\partial n} = 0$$

(15)

So, the stationary task (13), (15) is decided by method of the boundary integrated equations . If the decision L (x, y) in a point (x, y) of a linear third regional task (13), (15) for the equation Laplas to substitute in (12) and after integration to decide(solve) the appropriate algebraic equation, which degree is higher than a degree of function $\lambda_1(T)$, we shall receive meaning of temperature $T(x, y)$ in the same point. Thus in radicals the algebraic equation of a degree above fourth is decided

$$a_0T^4 + a_1T^3 + a_2T^2 + a_3T + a_4 = 0,$$

(16)

That corresponds to the task $\lambda(T)$ as the multinem of a degree above third. In result the temperature field will be determined as a constant first approximation, as the boundary condition $\lambda(T)$ took into account constant meaning heat conduction lc in convective thermal flows. According to it we shall designate this decision $T^{(1)}$ (accordingly $T^{(2)}$).

For definition subsequent and $A^{(2)}$ (accordingly $T^{(2)}$) the function An $(T)$ is displayed in a Taylor number a vicinity $T^{(1)}$ and the linear members are left in it only. In result is received a third regional task for the equation Laplas concerning function An $A^{(2)}$. Temperature $T^{(2)}$ is determined by the decision of the equation (14).

3.4. The repeated computing experiments with the use of BIEM on calculation of temperature fields nozzle and working blades with various quantity and arrangement of cooling channels having a complex configuration have shown, that for practical calculations in the approach, offered by us, the discretization of areas of an integration can be conducted with rather smaller quantity of discrete points. Thus the reactivity of the developed algorithms and accuracy of evaluations is increased. The accuracy of calculation of temperatures, required consumption of a cooling air, heat flows, losses from cooling margins of safety etc. essentially depends on reliability of boundary conditions, included in calculation, of heat exchange.

3.5. Piece-polynomial smoothing of cooled gas-turbine blade structures with automatic conjecture is considered: the method of the least squares, device spline, smooth replenishment and neural nets is used. The new approach to identification of the mathematical models parameters is considered. This approach is based on artificial neural networks (Soft computing) [7-9].

Let’s consider the regression equations:

$$Y_i = \sum_{j=1}^n a_{ij} \otimes \alpha_{ij}: i = \frac{1}{m},$$

(17)

$$Y_i = \sum_{r,s} a_{rs} \otimes x_{r} \otimes x_{s}: r = 0, l, s = 0, l; r + s \leq l,$$

(18)

where $a_{rs}$ - required parameters (regression coefficients). The definition task of values $a_{ij}$ and $a_{rs}$ parameters of the equation (17) and equations (18) is put based on the statistical experimental data of process, that is input $x_j$ and $x_{1}, x_{2}, \ldots, x_{n}$, output coordinates $Y$ of model.

Neural network (NN) consists from connected between their sets neurons. At use NN for the definition (17) and (18) input signals of the network are accordingly values of variable $X = (x_1, x_2, \ldots, x_n)$, $X = (x_1, x_2)$ and output $Y$ .

As parameters of the network are values of parameters $a_{ij}$ and $a_{rs}$.

At the decision of the identification task of parameters $a_{ij}$ and $a_{rs}$ for the equations (17) and (18) with using NN, the basic problem is training the last.

We allow, there are statistical data received based on experiments. On the basis of these input and output data is made training pairs $(X, T)$ for training a network. For construction of process model on input

NN input signals, move and outputs are compared with reference output signals . After comparison the deviation value is calculated

$$E = \frac{1}{2} \sum_{j=1}^m (Y_j - T_j)^2$$

If for all training pairs, deviation value less given then training (correction) parameters of a network comes to end (fig.1). In opposite case it continues until value will not reach minimum. Correction of network parameters for left and right part is carried out as follows:

$$a_{rs}^{n} = a_{rs}^{n-1} - \frac{\partial E}{\partial a_{rs}},$$

where $a_{rs}^{n}, a_{rs}^{n-1}$ - old and new values of NN parameters, $-training speed$.

The structure of NN for identification the equation (17) parameters are given on fig. 2.

3.6. For studying of AGTE blades temperature fields, the task of turbine cascade flow by the gas stream is considered. The decision is based on the numerical realization of the boundary integrated equation Fredholm II with feature.
Distribution of speeds along the profile contour can be determined by solving integral equation for current function 
\[ \psi = \psi_0 + \frac{1}{2\pi} \int \left( \frac{2\pi}{t} (x - x_0) \sin \frac{\pi}{t} (y - y_0) \right) \Delta y. \]

3.6.2. Distribution of speed along the profile contour can be determined by solving integral equation for current function 
\[ \psi' = \psi'_0 + \frac{1}{2\pi} \int \left( \frac{2\pi}{t} (x - x_0) \sin \frac{\pi}{t} (y - y_0) \right) \Delta y. \]

3.7. The data of speed distribution along the profile contour are primary for determining outer boundary heat exchange conditions. For designing local coefficient heat transfer at known geometry of the cooling scheme, for definition of the convective heat exchange local coefficients of the cooler, the task of determining inner boundary heat exchange conditions is necessary. For example, for calculation of heat transfer in the cooling system equivalent hydraulic scheme is built. The construction of the equivalent hydraulic tract circuit of the vane cooling is connected with the description of the cooled vane design. The whole passage of coolant flow is divided in some definite interconnected sections, the so-called typical elements, and every one has the possibility of identical definition of hydraulic resistance. The points of connection of typical elements are changed by node points in which the streams region or division of flows by proposal is taking places without pressure change. All the typical elements and node points are connected in the same sequence and order as the tract sites of the cooled vane.

To describe the coolant flow at every inner node is used the 1st low by Kirghof:
\[ f_1 = \sum_{j=1}^{m} G_{j} = \sum_{j=1}^{m} \text{sign}(\Delta p_j) \sqrt{\Delta p_j}; \quad i = 1, 2, 3, \ldots n \] (21)

where \( G_j \) - discharge of coolant on the element, \( i \), \( j \), \( m \) - the number of typical elements connected to \( i \) node of the circuit, \( n \) - the number of inner nodes of hydraulic circuit, \( \Delta p_j \) - losses of total pressure of the coolant on element \( i \) \( j \). In this formula the coefficient of hydraulic conductivity of the circuit element \( i \) \( j \) is defined as:
\[ k_j = \sqrt{2 f_j \cdot p_j \cdot \xi_j}, \]

where \( f_j \), \( p_j \), \( \xi_j \) - mean area of the cross-section passage of elements \( \{i \ j\} \), density of coolant flow in the element, and coefficient of hydraulic resistance of this element correspondingly. During the flow along the tract the coolant heats that leads to the flow speed achieves the meanings of local sound speed with the results of the shock is this channel.

The system of nonlinear algebraic equations (21) is solved by Zeidel method with acceleration, taken from:
\[ p_{i+1} = p_i - \frac{1}{\sum_{k=1}^{n} f_k \cdot p_k \cdot \xi_k}, \]

where \( p_i \) is the iteration number, \( p_{i+1} \) - coolant pressure in \( i \) node of the hydraulic circuit. The coefficients of hydraulic resistance \( \xi_j \) used in (22) are defined by analytical dependencies, which are in the literature available at present [12].

For example to calculate a part of the cooling tract, which includes the area of deflector perforation coefficients of hydraulic resistance in spray:
\[ \xi_{ce} = \left( \frac{G_c}{G_k} \right)^{m}; \]
\[ m = \frac{1}{2 \left( \frac{f_c}{f_k} \right) - 0.23}, \]

and in general channel:
\[ \xi_{ce} = 1.1 \left( \frac{G_c}{G_k} \right)^{n}; \quad n = 0.381 \ln \left( \frac{f_c}{f_k} \right) - 0.3 \] flows, which were obtained while generalizing experimental data [13]. In formulas \( G_c \), \( G_k \) - are cooling air consumption in the spray stream through the perforation deflector holes and slot channel between the deflector and vanes wall. \( f_c \), \( f_k \) - are the areas of the flow area.

4. RESULTS

The developed techniques of profiling, calculation of temperature fields and parameters of the cooler in cooling systems are approved at research of the gas turbine 1st stage nozzle blades thermal condition. Thus the following geometrical and regime parameters of the stage are used:
- step of the cascade \( t = 41.5 \text{MM} \), inlet gas speed to cascade \( V_1 = 156 \text{m/s} \), outlet gas speed from cascade \( V_2 = 512 \text{m/s} \),
- inlet gas speed vector angle \( \alpha_1 = 0.7^\circ \),
- gas flow temperature and pressure: on the entrance to the stage \( T_{i1} = 1333 \text{ K} \), \( p_i = 1.2095 \cdot 10^5 \text{Pa} \), on the exit from stage \( T_{e1} = 1005 \text{ K} \).
The geometrical model of the nozzle blades (fig.3), diagrams of speed distributions \( V \) and convective heat exchange local coefficients of gas along profile contour (fig.4) are received. The geometrical model (fig.5) and the cooling tract equivalent hydraulic scheme (fig.6) are developed. Cooler basics parameters in the cooling system and temperature field of blade cross section (fig.7) are determined.

5. CONCLUSIONS

The reliability of the methods was proved by experimental investigations of blades heat and hydraulic characteristics in "Turbine construction" laboratory (St. Petersburg, Russia). Geometric model, equivalent hydraulic schemes of cooling tracts have been obtained, cooler parameters and temperature field of "Turbo machinery Plant" enterprise gas turbine nozzle blade of the 1st stage have been determined (Yekaterinburg, Russia). Methods have demonstrated high efficiency at repeated and polivariant calculations, on the basis of which the way of blade cooling system modernization has been offered.

The application of perfect methods of calculation of temperature fields of elements of gas turbines is one from actual problems of an air engine building. The efficiency of these methods in the total renders direct influence to operational manufacturability and reliability of elements of designs, and also on acceleration characteristics of the engine.

REFERENCES


APPENDIX
Fig. 3. The cascade of profiles of the nozzle cooled blade.

Fig. 4. Distribution of the relative speeds (1) and of gas convective heat exchange coefficients (2) along the periphery of the profile contour.

Fig. 5. Geometrical model with foliation of design points of contour (1-78) and equivalent hydraulic schemes reference sections (1-50) of the experimental nozzle blade.

Fig. 6. The equivalent hydraulic scheme of experimental nozzle blade cooling system.

Fig. 7. Distribution of temperature along outside (▲) and internal (■) contours of the cooled nozzle blade.