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Orthogonal Array Application and Response Surface Method Approach for Optimal Product Values: An Application for Oil Blending Process

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Abstract**—**This paper presents a methodical approach for designing and optimizing process parameters in oil blending industries. Twenty seven replicated experiments were conducted for production of A-Z crown super oil (SAE20W/50) employing L9 orthogonal array to establish process response parameters. Power law model was fitted to experimental data and the obtained model was optimized applying the central composite design (CCD) of response surface methodology (RSM). Quadratic model was found to be significant for production of A-Z crown supper oil. The study recognized and specified four new lubricant formulations that conform to ISO oil standard in the course of analyzing the batch productions of A-Z crown supper oil as: L1: KV = 21.8293Cst, BS200 = 9430.00Litres, Ad102=11024.00Litres, PVI = 2520 Litres, L2: KV = 22.513Cst, BS200 = 12430.00 Litres, Ad102 = 11024.00 Litres, PVI = 2520 Litres, L3: KV = 22.1671Cst, BS200 = 9430.00 Litres, Ad102 = 10481.00 Litres, PVI= 2520 Litres, L4: KV = 22.8605Cst, BS200 = 12430.00 Litres, Ad102 = 10481.00 Litres, PVI = 2520 Litres. The analysis of variance showed that quadratic model is significant for kinematic viscosity production while the R-sq value statistic of 0.99936 showed that the variation of kinematic viscosity is due to its relationship with the control factors. This study therefore resulted to appropriate blending proportions of lubricants base oil and additives and recommends the optimal kinematic viscosity of A-Z crown super oil (SAE20W/50) to be 22.86Cst.

Keywords**—**Additives, control factors, kinematic viscosity, lubricant, orthogonal array, process parameter.

I. INTRODUCTION

EXPERIMENTAL design (DOE), Taguchi robust design and response surface method (RSM) are usually employed to achieve optimum manufacturing which is usually aimed at establishing the best setting of control process parameters for optimum process response. Reference [1] used Taguchi robust design to establish optimal process parameters for an oil blending process. It is intended by this study to use RSM to optimize oil process variables for enhanced industrial revenue. Reference [2] defines RSM as a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables. It can also be defined as collection of statistical and mathematical techniques for developing, improving and optimizing processes. More information on RSM is found in [2]-[7].

In the 1980s, Genichi Taguchi introduced robust design on quality engineering through the statistical design of experiments as discussed in [8] and [9]. The concepts of robust design and its realization methods are significant contributions to modern quality and process improvement. Lubricant production industries in Nigeria have shown little or no concern in application of quality engineering in their production process. This is because the process of making a lubricant involves blending of base oil and additives to obtain a certain kinematic viscosity range for each blend of oil [1]. At the end of the process, if achieved viscosity falls within the defined specifications, then it is accepted for packaging and sales. Less emphases is laid on the variations from the target point as well as the number of times the process is repeated to achieve the required viscosity.

To solve a blending problem of optimum process parameters we have applied RSM over experimental data obtained using Taguchi experimental design to fit response surface model over the region of interest, optimize the quality characteristics (kinematic viscosity) of the process and to select the manufacturing conditions to meet specifications or customer requirements.

ISO standard for production of oil like (SAE20W/50) is that the Kinematic viscosity must be maintained within the range 19.50Cst to 22.55Cst. The applied RSM of this study achieved the kinematic viscosity range of 19.00Cst to 22.8605Cst.

II. METHODOLOGY AND MATERIALS

A. Experimental Design and Manufacture of A-Z Crown Supper Oil

The L9 (3^4) standard orthogonal for design of experiments was used to plan experimental manufacture of A-Z crown supper oil to establish optimal kinematic viscosity [1, 10, 11]. The L_9 (3^4) array is shown in Table I and the control factor levels established from previous productions are shown in

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Table II. Batch production for this oil is considered at 23,000litres per batch with the following materials of manufacture: Base Oil: BS 200 (9,430litres), Additives: 102(10,481litres), PVI (1,702 litres), 1222(691 kg), 1240(691 kg) and 5907(5kg). The mean values of kinematic viscosity experiment for nine experimental runs of [12] at three trials are shown in Table III.

TABLE I
$L_9(3^4)$ ARRAY APPLIED FOR A-Z CROWN SUPPER OIL MANUFACTURE

	FACTORS			
Runs	**A**	**B**	**C**	**D**
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

TABLE II
CONTROL FACTORS AND LEVELS FOR MANUFACTURE

		Levels		
Label	**Factors**	**Low**	**Medium**	**High**
A	BS 200	9430	10930	12430
B	Ad 102	10481	10752	11024
C	PVI	1702	2111	2520

TABLE III
EXPERIMENTAL DESIGN FOR KINEMATIC VISCOSITY

	Factor Values			**Response Values of KV**			
Runs	Factor A	Factor B	Factor C	Trial 1	Trial 2	Trial 3	Average viscosity
1	9430	10481	1702	19.34	18.25	18.90	18.83
2	9430	10752	2111	20.12	19.95	20.15	20.07
3	9430	11024	2520	22.52	21.24	22.00	21.92
4	10930	10481	2111	21.23	20.41	20.90	20.85
5	10930	10752	2520	22.89	22.40	23.21	22.83
6	10930	11024	1702	18.46	18.20	15.84	17.50
7	12430	10481	2502	20.46	22.52	21.99	21.66
8	12430	10752	1702	19.33	20.40	19.86	19.86
9	12430	11024	2111	20.60	20.77	22.10	21.16

B. Curve Fitting and Modeling Experimental Data with Power Law Model

In most cases linear functional responses depend on more than one variable that multiple linear regression model expressed as $y = b_o + b_1x_1 + b_2x_2 + \cdots . + b_mx_m + e$ is usually employed for response modeling and analysis. But in many cases functional relationship with independent factors are never linear that nonlinear regression is employed in modeling and analysis of experimental data response as presented in [12] and [13]. A second order and third order regression models with independent variables to detect nonlinearity and second order effects are given in [1] as

$$y = b_o + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{12}x_1x_2 + b_{22}x_2^2 \quad \text{and}$$

$$y = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 \tag{1}$$

These nonlinear models are needed to detect nonlinearity and second order effects within population of data but however it needs great computational effort to fit them to data. The nonlinear response data can be transformed by employing the classical power law model,

$$y = \mathrm{a}_o\, x_1^{\mathrm{a}_1}\, x_2^{\mathrm{a}_2}\, x_3^{\mathrm{a}_3} \ldots x_m^{\mathrm{a}_m} \tag{2}$$

Multiple linear regression has additional educational utility in the derivation of power equation of (2). To fit the power law model of (2) to experimental data using linear regression approach, the power law equation (2) is transformed by taking its logarithm to yield,

$$\mathrm{Log}\, \mathrm{y} = \mathrm{Loga}_o + \mathrm{a}_1 \mathrm{Log}\, x_1 + \mathrm{a}_2 Log x_2 + \cdots + \mathrm{a}_m \mathrm{Log}\, x_m$$

where the value of $\mathrm{a}_o = \text{Antilog } a_o$

By having three independent factors, A(BS200), B(Ad102), and C(PVI) and one dependent factor kinematic viscosity KV = y , transformation of (2) gives

$\mathrm{Log\,KV} = \mathrm{Loga}_o + \mathrm{a}_1 \mathrm{Log}\, A + \mathrm{a}_2 Log B + \mathrm{a}_3\, \mathrm{Log}\, C$, so that equation (2) becomes

$$KV = \mathrm{a}_o\, A^{\mathrm{a}_1}\, B^{\mathrm{a}_2}\, C^{\mathrm{a}_3} \tag{3}$$

Table III was used to establish a power law model after transformation of (3) by taking logarithm of both sides of (3). The resulting power law model becomes after putting values in (3),

$$K = 4.570881896 \times BS200^{0.111} \times AD102^{0.303} \times PVI^{\ 0.430} \tag{4}$$

It is good to mention at this stage that the major objective of response surface methodology (RSM) is to fit nonlinear model of at least second order to capture the main effects of factors, interaction effects of factors and higher order effects of factors.

III. OPTIMIZATION OF KINEMATIC VISCOSITY RESPONSE WITH RSM

This entails optimization of equation (4) using RSM. Equation (4) is optimized following using design expert8

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software central composite design (CCD) option. The CCD design matrix generated for the power law model of equation (4) using Table II for low and high control factor limits is presented in Table IV. The responses of Table IV are predictions of equation (4) with factor combinations of Table V generated with design expert8 software.

TABLE IV
CENTRAL COMPOSITE DESIGN MATRIX FOR OPTIMIZATION OF NONLINEAR RESPONSE MODEL OF PRODUCT PARAMETER (KINEMATIC VISCOSITY)

KV=4.570881896*BS200^(0.111)*AD102^(-0.303) *PVI^(0.430).

		Factor 1	Factor 2	Factor 3	Response 1
Std	Run	A:BS 200	B:Ad 102	C:PVI	KV
3	1	9430	11024	1702	18.4355
12	2	10930	11209.11	2111	20.4549
2	3	12430	10481	1702	19.3026
13	4	10930	10752.5	1423.15	17.4839
20	5	10930	10752.5	2111	20.7143
18	6	10930	10752.5	2111	20.7143
17	7	10930	10752.5	2111	20.7143
1	8	9430	10481	1702	18.7198
4	9	12430	11024	1702	19.0095
9	10	8407.31	10752.5	2111	20.1196
10	11	13452.69	10752.5	2111	21.1973
11	12	10930	10295.89	2111	20.9884
15	13	10930	10752.5	2111	20.7143
8	14	12430	11024	2520	22.504
7	15	9430	11024	2520	21.8245
14	16	10930	10752.5	2798.85	23.3852
16	17	10930	10752.5	2111	20.7143
6	18	12430	10481	2520	22.8511
19	19	10930	10752.5	2111	20.7143
5	20	9430	10481	2520	22.1611

IV. DISCUSSION OF RESULTS

The response surface method applied with desgn expert8 software yielded two quadratic models of (5) and (6) for the production of A-Z supper crown elite oil. Two quadratic models are produced through CCD of RSM in terms of coded and in terms of actual factors. The two equations show highest main effect for factor C (PVI) followed by factor B (Ad102). The interaction effects of factors reveal that BC interaction effect is highest followed by AC and AB respectively. As usual with blending process it is the additives that influence the quality of the finished product.

Final Equation in Terms of Coded Factors:

$$KV = +20.71 + 0.32 * A - 0.16 * B + 1.74 * C - 2.416E - 003 * A * B + 0.027 * A * C - 0.01 * B * C - 0.019 * A^2 + 3.195E - 003 * B^2 - 0.098 * C^2 \tag{5}$$

Final Equation in Terms of Actual Factors:

$$KV = +14.63061 + 3.70149E - 004 * BS\,200 - 1.19725E - 003 * Ad\,102 + 7.55338E - 003 * PVI - 5.93366E - 009 * BS\,200 * Ad\,102 + 4.33294E - 008 * BS\,200 * PVI - 1.19504E - 007 * Ad\,102 * PVI - 8.50945E - 009 * BS\,200^2 + 4.33500E - 008 * Ad\,102^2 - 5.87741E - 007 * PVI^2 \tag{6}$$

The nonlinear power equation is now transformed to a third order nonlinear model of the form

$$y = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}\,x_1^2 + b_{22}\,x_2^2 + b_{33}x_3^2$$

This model establishes the major effects of factors, the interaction effects of factors and finally the higher other effects of factors accounting for the nonlinear response of of kinematic viscosity. The goodness of fit of the model as depicted in table VI with R-sq value of 0.999966 means that about 100% variation of the kinematic viscosity is explained by the model and that the variation of the response is due to its relationship with the control independent variables. Also explained by Fig.1 with the standard error of design 0.60 is that the predicted value of the kinematic viscosity is permitted to vary with 0.60Cst within the line of best fit. This is explained in Fig. 1 where plots of predicted and actual value of kinematic viscosity fall within the regression line.

The standard error of design describes the spread within the regression or line of best fit while the standard deviation value of 0.01211Cst presented in table VI describes the spread around the mean of the predicted kinematic viscosity values. The F-value = 32834.56 and P-value = 0.0001 of table V show that the model and all the model terms are significant while table VI suggested quadratic model to be the best fit with R-sq value of 0.99966, this means that about 100% variation of kinematic viscosity is due to the three factors considered. The closeness of R-sq value (0.999936), Adjusted R-sq value (0.999936) and Predicted R-sq value (0.999743) show that the model is well fitted and the model will predict new observation accurately.

TABLE V
ANOVA FOR QUADRATIC RESPONSE OF KINEMATIC VISCOSITY

	Sum of Squares	Df	Mean Square	F Value	p-value Prob >
Source Model	43.3408	9	4.8155	32834.56	< 0.0001
A	1.378415	1	1.378415	9398.679	< 0.0001
B	0.341142	1	0.341142	2326.068	< 0.0001
C	41.46976	1	41.46976	282760.2	< 0.0001
AB	4.67E-05	1	4.67E-05	0.318526	0.5849
AC	0.005653	1	0.005653	38.54519	0.0001
BC	0.001409	1	0.001409	9.605628	0.0113
A^2	0.005283	1	0.005283	36.02117	0.0001
B^2	0.000147	1	0.000147	1.00334	0.3401
C^2	0.139306	1	0.139306	949.8504	< 0.0001
Residual	0.001467	10	0.000147		
Lack of Fit	0.001467	5	0.000293		
Pure Error	0	5	0		
Cor Total	43.34127	19			

The model statistics plots of Fig. 1, Fig. 2 and Fig. 3 show that quadratic model fits the data properly. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data;

otherwise, a non-linear model is more appropriate. This is the case of this study where the plot of residual is not randomly distributed and the quadratic model is confirmed.

Fig. 4 shows design points at the corners of the contour plot with maximum value of kinematic viscosity as 22.8605Cst and that KV increases as the quantity of Base oil increases. The cube plot of Fig. 5 clearly depicts possible blending factors levels of oil that will give oil products that will still conform to ISO standard for production of oil. The possible lubricants of this study from Fig. 5 are of the following specifications:

a. L1:KV = 21.8293Cst, BS200 = 9430.00Litres, Ad102 = 11024.00Litres, PVI= 2520Litres
b. L2:KV = 22.513Cst, BS200 = 12430.00Litres, Ad102=11024.00Litres, PVI= 2520Litres
c. L3:KV = 22.1671Cst, BS200 = 9430.00Litres, Ad102=10481.00Litres, PVI= 2520Litres
d. L4:KV = 22.8605Cst, BS200 = 12430.00Litres, Ad102=10481.00Litres, PVI= 2520Litres

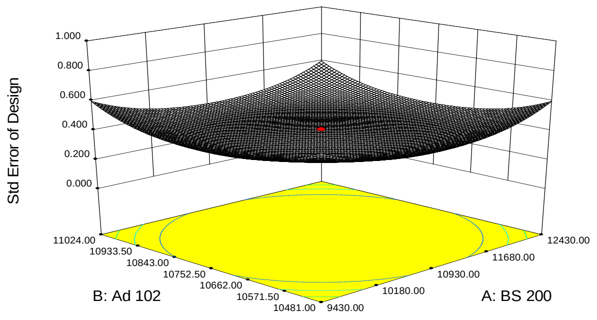

Fig. 1 Standard error of design at PVI = 2111.00 liters

TABLE VI
MODEL SUMMARY STATISTICS

Source	Std. Dev.	R-Sq	AdjR-Sq	PredR-Sq	PRESS	
Linear	0.097455	0.996494	0.995836	0.994216	0.250675	
2FI	0.105557	0.996658	0.995115	0.993063	0.300658	
Quadratic	0.01211	0.999966	0.999936	0.999743	0.011141	Suggested
Cubic	0.002024	0.999999	0.999998	0.999875	0.005417	Aliased

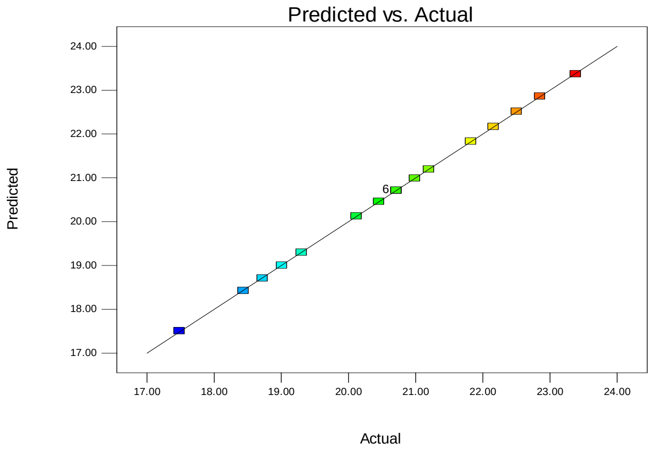

Fig. 2 Predicted vs Actual value of kinematic viscosity

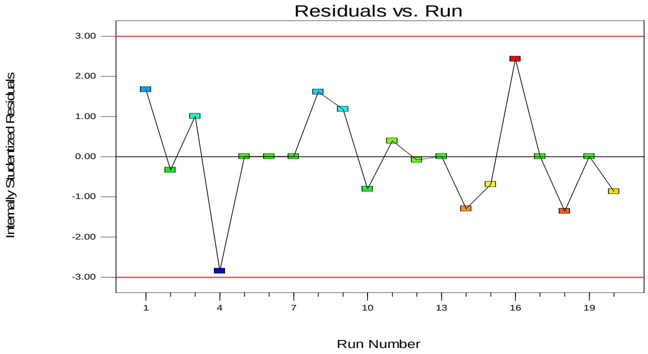

Fig. 3 Residual vs experimental number

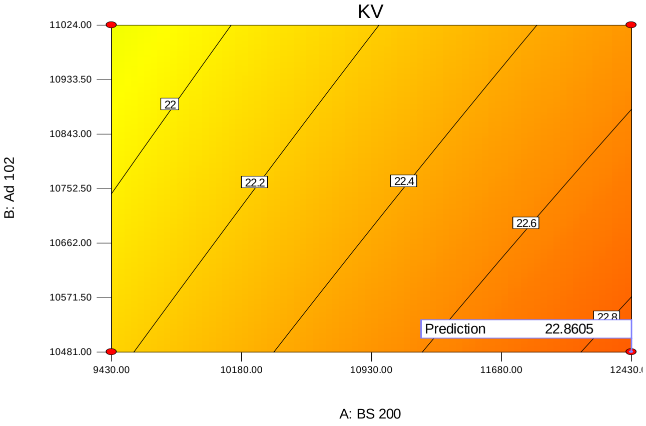

Fig. 4 Design expert8 contour plot depiction of optimum (maximum) KV

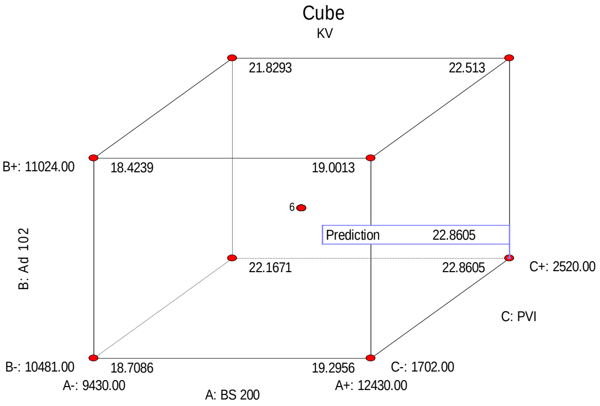

Fig. 5 Design expert8 cube depiction of viscosity at different design points

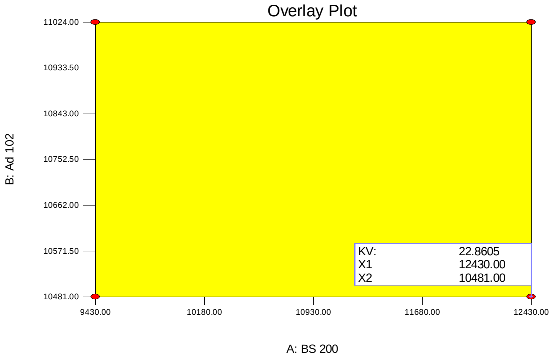

Fig. 6 Design expert8 overlay plot depiction of maximum KV when PV1 = 2520 litres

V. CONCLUSION

The following conclusions were drawn from study:

- Lubricant blending factors were successfully modeled and optimized
- The optimal value of kinematic viscosity of oil was established as 22.86Cst
- Quadratic model was found to be significant for production of A-Z supper crown oil
- The study recognized and specified four new lubricant formulations that conform to ISO oil standard in the course of analyzing the batch productions of A-Z crown supper oil as: L1: KV = 21.8293Cst, BS200 = 9430.00Litres, Ad102=11024.00Litres, PVI= 2520Litres, L2: KV = 22.513Cst, BS200 = 12430.00Litres, Ad102=11024.00Litres, PVI= 2520Litres, L3:KV = 22.1671Cst, BS200 = 9430.00Litres, Ad102=10481.00Litres, PVI= 2520Litres, L4:KV = 22.8605Cst, BS200 = 12430.00Litres, Ad102=10481.00Litres, PVI= 2520Litres.
- This study resulted to appropriate blending proportions of lubricants base oil and additives and recommends the optimal kinematic viscosity of A-Z crown super oil (SAE20W/50) to be 22.86Cst.

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