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Synthesis of Wavelet Filters using Wavelet Neural Networks

Wajdi Bellil, Chokri Ben Amar, and Adel M. Alimi

Abstract—An application of Beta wavelet networks to synthesize pass-high and pass-low wavelet filters is investigated in this work. A Beta wavelet network is constructed using a parametric function called Beta function in order to resolve some nonlinear approximation problem. We combine the filter design theory with wavelet network approximation to synthesize perfect filter reconstruction. The order filter is given by the number of neurons in the hidden layer of the neural network. In this paper we use only the first derivative of Beta function to illustrate the proposed design procedures and exhibit its performance.

Keywords—Beta wavelets, Wavenet, multiresolution analysis, perfect filter reconstruction, salient point detect, repeatability.

I. Introduction

RECENTLY, the subject of wavelet analysis has attracted much attention from both mathematicians and engineers alike. Wavelets have been applied successfully to multiscale analysis and synthesis, time-frequency signal analysis in signal processing, function approximation, approximation in solving partial differential equations. Wavelets are well suited to depicting functions with local nonlinearities and fast variations because of their intrinsic properties of finite support and self-similarity.

The relationship between the scaling function and the wavelet function is now clear. The scaling function provides a set of basis function to approximate a signal at a certain resolution and the wavelet provides a set of basis functions for the detail signal. When the detail signal is added to the signal approximation, the result is the signal approximation at the next higher level of resolution. For a general continuous time signal f(t), these successive additions of detail signals to create the next higher resolution approximation must continue forever to get an accurate representation of f(t). This problem is neatly fixed when dealing with discrete time signals as they are already defined with finite time resolution and can be accurately represented in some subspace V_k where $k < +\propto$.

In this paper, we propose a novel method to generate wavelet filters using Beta Wavelet Neural Network (BWNN).

Wajdi Bellil is with University of Sfax, National Engineering School of Sfax, B.P. W, 3038, Sfax, Tunisia (phone: 216-98212934; e-mail: Wajdi.bellil@ieee.org).

Chokri Ben Amar is with University of Sfax, National Engineering School of Sfax, B.P. W, 3038, Sfax, Tunisia (phone: 216-98638417; e-mail: chokri.benamar@ieee.org).

Adel M. Alimi is with University of Sfax, National Engineering School of Sfax, B.P. W, 3038, Sfax, Tunisia (phone: 216-98667682; e-mail: adel.alimi@ieee.org).

The advantage of the proposed method is demonstrated by computer simulations. This paper is organized as follows. Section 2 presents the theory of Beta wavelet. Section 3 shows the discrete wavelet transform, MRA and filter implementation. Section 4 illustrates the reason why a WNN is needed to synthesis wavelet filters. Section 5 demonstrates the simulation results on Beta wavelet filters and some others. Section 6 concludes this paper.

II. A Novol Beta Wavelet Family

The Beta function [1, 2] is defined as:

$$\beta(x;p,q,x_0,x_1)=\begin{cases}\left(\frac{x-x_0}{x_c-x_0}\right)^p\left(\frac{x_1-x}{x_1-x_c}\right)^q & if \quad x\in[x_0,x_1]\\ 0 & else\end{cases} \tag{1}$$

$$with \quad p,q,x_0<x_1\in\Re \qquad and \qquad x_c=\frac{px_1+qx_0}{p+q}$$

We have proved in [1, 2] that all derivatives of Beta function $\in L^2(IR)$ and are of class $C\infty$, so they have the property of universal approximation. The general form of the nth derivative of Beta function is:

$$\Psi_n(x) = \frac{d^{n+1}\beta\ (x)}{dx^{n+1}}\ P_{n+1}(x)\beta(x) \tag{2}$$

$$= \left[(-1)^n\ \frac{n!\,p}{(x-x_0)^{n+1}}-\frac{n!\,q}{(x_1-x)^{n+1}}\right]\beta(x)+P_n(x)P_1\beta(x)+$$
$$\sum_{i=1}^{n-1}\left[C_n^i(-1)^{n-i}\ \frac{(n-i)!\,p}{(x-x_0)^{n+1-i}}-\frac{(n-i)!q}{(x_1-x)^{n+1-i}}\right]P_i(x)\beta(x)$$

$$Where\ P_1(x) = \frac{p}{x-x_0} - \frac{q}{x_1-x} \tag{3}$$
$$and\ P_n(x)=P_{n-1}(x)P_1(x)+P_{n-1}^{'}(x)\quad \forall\, n>1$$

III. Discrete Wavelet Transform, MRA and Filter Implementation

The DWT will transform a discrete time signal to a discrete wavelet representation [3]. The first step is to discretize the wavelet parameters. This is commonly done with the dyadic sampling grid, defined by:

$$\Psi_{m,n}(t)=2^{m/2}\Psi\left(2^m t-n\right),\ m,n\in Z \tag{4}$$

This reduces the previously continuous set to a now discrete, orthogonal set. The analysis formula becomes

$$W_{m,n}=\left\langle f(t),\Psi_{m,n}(t)\right\rangle,\ m,n\in Z \tag{5}$$

With the reconstruction formula

$$f(t)=\sum_{m}\sum_{n}W_{m,n}\Psi_{m,n}(t) \quad (6)$$

Next, consider fixing the scale factor m, so that we have ψ(t) Since the mother wavelet has compact frequency support, this wavelet set represents a discrete set of temporally translated wavelets with fixed frequency localization. As the inner product operation can also be interpreted as a filtering operation, the projection onto this set of wavelets can be considered a set of temporally translated band-pass filters of fixed frequency response. If the scale factor m is reinserted, the discrete wavelet series wm,n can now be considered the result of applying a set of temporally and spectrally translated band-pass filters.

This tells us that the scaling function at one resolution level can always be expressed as a linear combination of the translated scaling functions at higher resolutions. Thus, we can write:

$$\Phi(t)=\sum_{k}h_k\sqrt{2}\Phi(2t-k) \quad (7)$$

This is known as the multiresolution analysis (MRA) equation.

Because of the ease with which digital filters can be implemented, most wavelet decomposition and synthesis schemes are designed by creating the equivalent filters as opposed to the wavelet and scaling functions themselves. This usually begins by resigning the low-pass filter, which then predetermines the matching high-pass filter. Of course, there are specific restrictions (omitted here) placed on the filters so that they do indeed correspond to a wavelet transform. Since the entire signal information is captured when both filters are applied at a specific resolution level. They correspond to a perfect reconstruction mirror filter bank shown in Fig. 1.

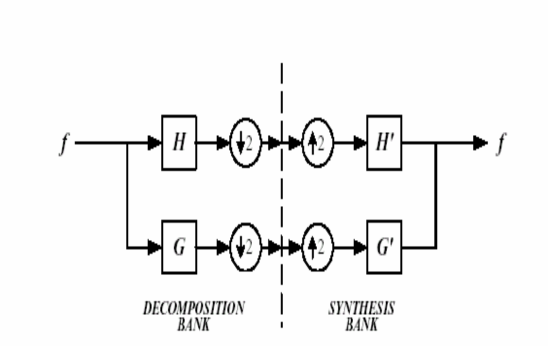


Fig. 1 Perfect Reconstruction Filter Bank

H and G correspond to the low and high-pass filters respectively. The down-sampling procedure is possible due to the perfect, non-redundant two-channel decomposition. The reconstruction filters H' and G' are, in most instances, just the filters H and G reversed.

IV. Wavelet Neural Network (WNN)

Given an n-element training set, the overall response of a WNN is:

$$\hat{y}(x)=\sum_{i=1}^{N_p}w_i\ \Psi_i\left(\frac{x-t_i}{a_i}\right) \quad (8)$$

Where N_p is the number of wavelet nodes in the hidden layer and w_i is the synaptic weight of WNN. A WNN can be regarded as a function approximator which estimates an unknown functional mapping:

$$y = f(x) + \varepsilon \quad (9)$$

Where f is the regression function and the error term ε is a zero-mean random variable of disturbance. There are a number of approaches for WNN construction [4-5], we pay special attention on the model proposed by Zhang [6].

If we choose $y(x)=\Phi(x)$ the output of the network is:

$$\hat{\Phi}(x)=\sum_{i=1}^{N_p}w_i\ \Psi_i\left(\frac{x-t_i}{a_i}\right) \quad (10)$$

The pass low filter is given by:

$$\Psi(x)=2\sum_{k}g_k\,\Phi(2x-k) \quad (11)$$

We demonstrate that:

$$\Phi\left(\frac{x}{a_i}\right)=\frac{1}{a_i}\Phi(x) \quad (12)$$

From (10) and (7) and for a choice of $a_i=\frac{1}{\sqrt{2}}$ we deduce that $h_i = w_i$

We can calculate the pass-high coefficients filter using a wavelet neural network, the size of the filter is given by the number of wavelet used in the hidden layer of the network.

V. Coffecients Filters and Simulation Result

The originality of this work is to calculate the Beta coefficients filters using an iterative method based on Beta wavelet neural network. The output function of a wavelet neural network is given by the equation (10), the perfect reconstruction filters should satisfies the equation (12) which can be seen as a wavelet neural network with three hidden layer. In this work we calculate the pass-high and pass-low filters of Beta wavelet for different length.

A. Associate Filters for Beta Wavelet

We construct a Beta wavelet neural network, the transfer function of the neurons is the first derivative of Beta function (BW 1).

For ∀ (p, q) ∈ $\Re^2$, if the number of neuron N_p in hidden layer is equal to 2, the pass-high and pass-low filters are the same as Haar coefficients filters.

VI. Application: Salient Point Detection

A. Results for Repeatability

In this section we will compare the performance of Beta wavelets in the case of wavelet-based salient point detect. We briefly present the outline of the algorithm and we show some examples of detected salient points.

Before we can measure the repeatability of a particular detector we first had to consider typical image alterations such as image rotation and image scaling. In both cases, for each image we extracted the salient points and then we computed the average repeatability rate over the database for each detector. In the case of image rotation, the rotation angle varied between 0° and 180°. The repeatability rate in a ε=1 neighborhood for the rotation sequence is displayed in Fig. 2.

TABLE I
Beta Wavelet Filters

Filter order	Pass-low filter h	Pass-high filter g
2	0.50000000000000	0.50000000000000
	0.50000000000000	-0.50000000000000
4	0.49523195168333	-0.01123780429743
	0.51546157224974	-0.00054428036434
	0.00054428036434	0.51546157224974
	-0.01123780429743	-0.49523195168333
6	0.49523195168333	-0.01123780429743
	0.62758229715635	0.12627540899907
	0.62758229715635	-0.12627540899907
	-0.00130688815728	0.00130688815728
	-0.00130688815728	-0.00130688815728
	-0.12627540899907	-0.62758229715635
	-0.12627540899907	0.62758229715635

The detectors using Beta wavelet transform, applied on cameraman image, give better results compared with the other ones (Haar and Daubechies 4). Note that the results for all detectors are not very dependent on image rotation. The best results are provided by Daubechies 4 detector.

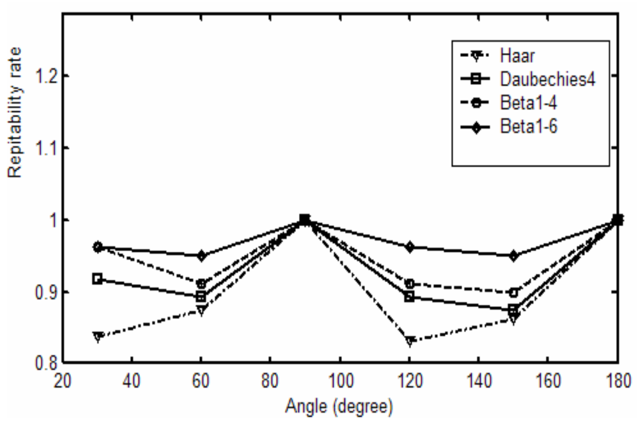


Fig. 2 Repeatability rate for image rotation $\varepsilon = 1$

In the case of scale changes, for each image we considered a sequence of images obtained from the original image by reducing the image size so that the image was aspect-ratio preserved.

The repeatability rate for scale changes [10-12], applied on cameraman image, is presented in Fig. 3. All detectors are very sensitive to scale changes. The repeatability is low for a scale factor above 3 especially for Haar detectors. The detectors based on Beta 1-6 wavelet transform provide better results compared with the other ones.

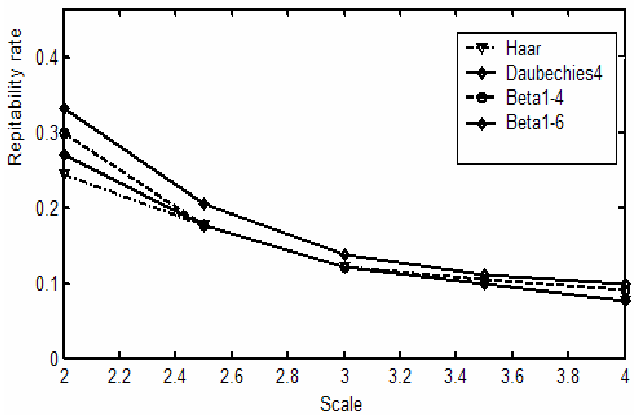


Fig. 3 Repeatability rate for image scale change $\varepsilon = 1$

B. Results for Information Content

In the specifications of our point detector, points shouldn't gather in small regions. The aim is that the extracted points represent different parts and patterns of the image. We introduce the entropy to evaluate how much the extracted points are spread in the image. Of course this criterion doesn't assure the set of points is relevant for indexing, but we believe it is necessary to describe different parts of the image [7-10].

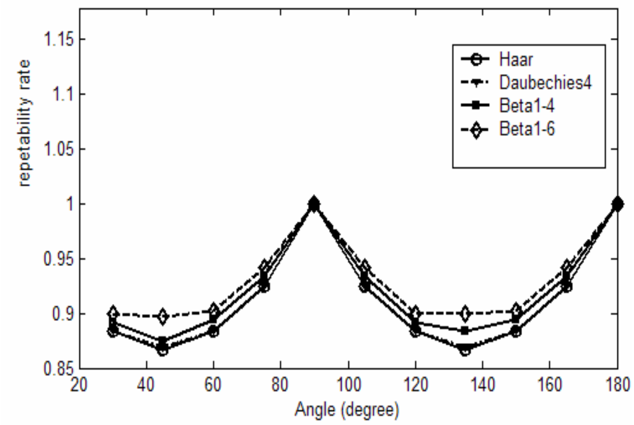
The idea is to compare the entropy of different sets of points extracted with different detectors. The points shouldn't necessarily be a uniform repartition in the image. If there is "nothing" in some parts of the image, there shouldn't be points in these parts. But still some detectors will lead to points more spread than others.

We define a grid on the image. The probability p_i of the point distribution in the cell i is: number of extracted points in this cell / total number of extracted points. The entropy of the extracted point distribution of an image I for a detector d is:

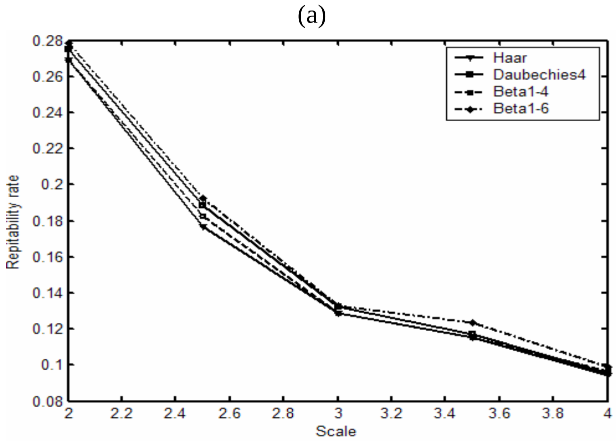
$$entropy(I,d) = -\sum_{i=1}^{N_C} p_i \log p_i \tag{13}$$

With N_C the number of cells.

The mean repeatability rate for image rotation (a) and scale change (b) is summarized in Fig. 4.



(a)



(b)

Fig. 4 The mean repeatability rate for image rotation and scale change

In summary, the most "interesting" salient points were detected using the Beta 1-6 detector. These points have the highest information content and proved to be the most robust to rotation and scale changes.

VII. Conclusion

We review the filter-bank implementation of the discrete wavelet transform (DWT) and show how it may be synthesized using Beta wavelet network for processing images and other multi-dimensional signals. We show then that the condition for inversion of the DWT (perfect reconstruction) forces severe shift dependence on gain ratio. In this work we calculate the pass-law and pass-high filter for the first derivative of Beta wavelet. These filters can be optimized if we adjust the support of the wavelet, the order of derivation and the p and q parameters.

In conclusion, the novel contribution of this paper is in showing that the Beta wavelet-based salient points technique are able to capture the local feature information and therefore, they provide a better characterization for the scene content than Haar or Daubechies wavelets since they are more distinctive and invariant.

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