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On the Analysis of Localization Accuracy of Wireless Indoor Positioning Systems using Cramer's Rule

Kriangkrai Maneerat and Chutima Prommak*

Abstract**—**This paper presents an analysis of the localization accuracy of indoor positioning systems using Cramer's rule via IEEE 802.15.4 wireless sensor networks. The objective is to study the impact of the methods used to convert the received signal strength into the distance that is used to compute the object location in the wireless indoor positioning system. Various methods were tested and the localization accuracy was analyzed. The experimental results show that the method based on the empirical data measured in the non line-of-sight (NLOS) environment yield the highest localization accuracy; with the minimum error distance less than 3 m.

Keywords**—***I*ndoor positioning systems, Localization accuracy, Wireless networks, Cramer's rule

I. INTRODUCTION

ACCURACY to define object's location is the main concern of the wireless indoor positioning systems (WIPS). Although Global Positioning System (GPS) have been deployed widely and could provide large service coverage, it could not use in the indoor environment because there is no line-of-sight path between the indoor receivers and the satellites and the satellite signal cannot penetrate the building's walls [1]. Comparing with the outdoor environments, the indoor structure is more complex due to varieties of obstacles such as walls and furniture which cause the multipath effects on the wireless signal. In addition, the human mobility and interference from other wireless networks in the building could impair the signal quality [2]. These indoor issues bring challenges to the deployment of the WIPs.

Deploying positioning systems in the indoor environment, one needs to install wireless networks that generate the referencing signal used to define the object location. Several wireless technologies have been adopted such as the IEEE 802.11 wireless LAN and the IEEE 802.15.4 wireless sensor networks.

The effects of indoor environment on the performance of WIPS have received little attention in the research study. Most of the existing works in literature focused on developing techniques to define the object positions. In [6], the authors proposed the indoor positioning systems that estimate the position of the objects by analyzing the received signal strengths in which the service area is divided into zones and sub-zones. In [7], the authors proposed the algorithm based on the decent gradient iteration to define the object position. In [8], the RSS-based techniques were used to estimate the distance between the object and the referencing nodes and the triangulation techniques were used to define the object position. In [9] and [10], the 3-D positioning systems were simulated. To compute the object position, the Angle of Arrival algorithms was used in [9] whereas the Time Difference of Arrival algorithm (TDOA) was used in [10].

From the literature review, most works in literature focused mainly on developing techniques to define the object positions whereas he effects of indoor environment on the performance of WIPS have received little attention. In order to improve the localization accuracy of the WIPS, we need a suitable method, which consider the effects of indoor environment, for converting the received signal strength to the distance that is used to compute the object location in WIPS. Therefore, in this paper we present the study and comparison of using three methods which are developed by three different ways, including the method based on the empirical data measured in the line-of-sight (LOS) environment, the method based on the non line-of-sight (NLOS) empirical data and the method based on the path-loss model using the empirical path-loss exponent. Specifically, we apply these methods to the WIPS using Cramer's rule.

The rest of the paper is organized as followed. Section II presents the Cramer's rule approach. Experimental designs are explained in Section III. Section IV shows the empirical data measured in various environments and the analysis of the location accuracy of WIPS. Finally, we conclude the paper in section V.

II. CRAMER'S RULE APPROACH

Cramer's rule has been widely applied in the localization applications. It uses the principle concept of the linear equation systems of which the number of equations is equal to the number of variables and formats the linear equations systems in the form of matrix. Then, the determinant is applied and the variables' values are derived [11]. Fig. 1 shows the structure and components used by Cramer's rule.

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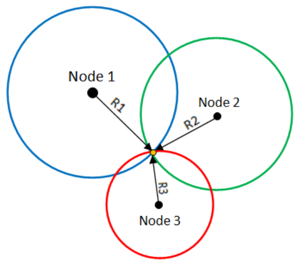

Fig. 1 Cramer's rule

The structure of positioning system using Cramer's rule consists of three referencing nodes, namely Node 1, Node 2 and Node 3 in fig.1. The object location is the intersection of the three circles centered at the referencing nodes. To derive formulas for computing the coordinate of the object, we define the following notations:

(x_i, y_i) = the coordinate of the referencing node i,
(x_u, y_u) = the coordinate of the object, and
R_i = the distance between the node i and the object.

We have the circle equation written

$$(x_i - x_u)^2 + (y_i - y_u)^2 = R_i^2 \quad for\ i = 1,2,3 \tag{1}$$

Then we have the relationship:

$$\left((x_1 - x_u)^2 - (x_3 - x_u)^2\right) + \left((y_1 - y_u)^2 - (y_3 - y_u)^2\right) = \left(R_1^2 - R_3^2\right) \tag{2}$$

$$\left((x_2 - x_u)^2 - (x_3 - x_u)^2\right) + \left((y_2 - y_u)^2 - (y_3 - y_u)^2\right) = \left(R_2^2 - R_3^2\right) \tag{3}$$

Arranging the equation (2) and (3), we obtain the following

$$2(x_3 - x_1)x_u - 2(y_3 - y_1)y_u = \left(R_1^2 - R_3^2\right) - \left(x_1^2 - x_3^2\right) - \left(y_1^2 - y_3^2\right) \tag{4}$$

$$2(x_3 - x_2)x_u - 2(y_3 - y_2)y_u = \left(R_2^2 - R_3^2\right) - \left(x_2^2 - x_3^2\right) - \left(y_2^2 - y_3^2\right) \tag{5}$$

Putting the equations (4) and (5) in the matrix form, we can write determinant matrix (7), (8) and (9).

$$2\begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix}\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} \left(R_1^2 - R_3^2\right) - \left(x_1^2 - x_3^2\right) - \left(y_1^2 - y_3^2\right) \\ \left(R_2^2 - R_3^2\right) - \left(x_2^2 - x_3^2\right) - \left(y_2^2 - y_3^2\right) \end{bmatrix} \tag{6}$$

$$\det(A) = \begin{vmatrix} (x_3 - x_1)*2 & (y_3 - y_1)*2 \\ (x_3 - x_2)*2 & (y_3 - y_2)*2 \end{vmatrix} \tag{7}$$

$$\det(A)_1 = \begin{vmatrix} \left(\left(R_1^2 - R_3^2\right) - \left(x_1^2 - x_3^2\right) - \left(y_1^2 - y_3^2\right)\right) & (y_3 - y_1)*2 \\ \left(\left(R_2^2 - R_3^2\right) - \left(x_2^2 - x_3^2\right) - \left(y_2^2 - y_3^2\right)\right) & (y_3 - y_2)*2 \end{vmatrix} \tag{8}$$

$$\det(A)_2 = \begin{vmatrix} (x_3 - x_1)*2 & \left(\left(R_1^2 - R_3^2\right) - \left(x_1^2 - x_3^2\right) - \left(y_1^2 - y_3^2\right)\right) \\ (x_3 - x_2)*2 & \left(\left(R_2^2 - R_3^2\right) - \left(x_2^2 - x_3^2\right) - \left(y_2^2 - y_3^2\right)\right) \end{vmatrix} \tag{9}$$

Finally, the coordinate of the object can be computed from (10)

$$x_u = \frac{\det[A]_1}{\det[A]} \quad \text{and} \quad y_u = \frac{\det[A]_2}{\det[A]} \tag{10}$$

III. Experimental Setup

This is paper aims to analyze of the localization accuracy of indoor positioning systems using Cramer's rule. The experiments were performed using XBee Pro mudule based on IEEE 8002.15.4 (ZigBee) standard. Structure of system consist reference nodes and target node. The reference nodes are configured as Coordinator and define on different place. The target node are configured as End device and connect to laptop can be mobile. The process of indoor positioning systems starting from, the target node request received signal strengths from each reference nodes. Next, we are convert the received signal strength into the distance for compute coordinates of the target node by Cramer's rule, as show in Fig. 2. This paper, we select the experimental area on the fourth floor of the C-building at Suranaree University of Technology. The dimension of floor is approximately 75 m x 75 m and we are install 4 reference nodes which are A, B, C, and D. We define 40 sample points. As show in Fig. 3 In the experimental, we propose 3 method of the convert received signal strength into the distance, the correlation of Line of Sight method, the correlation of Non Line of Sight method, and the correlation of path loss model method.

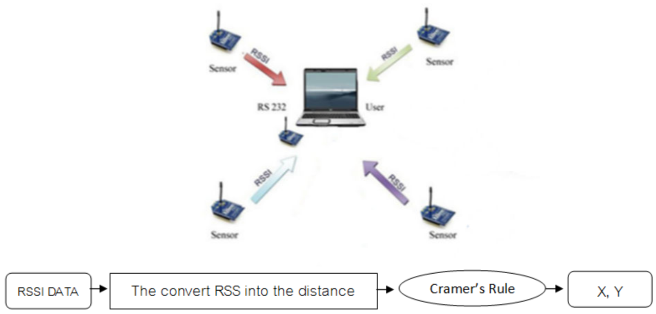

Fig. 2 Structure of the indoor positioning system

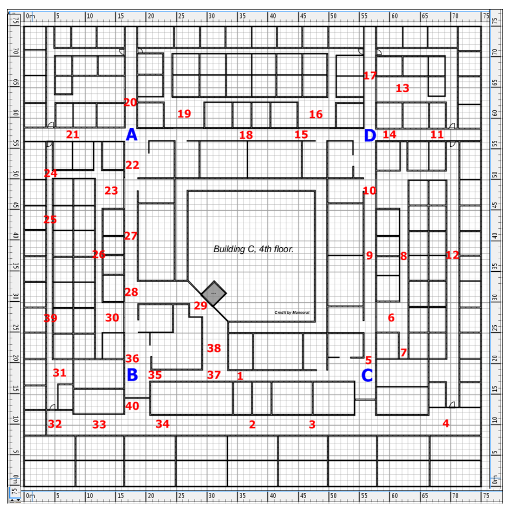

Fig. 3 Experimental setup: the indoor environment, locations of four referencing nodes and 40 test points

A. *Correlation of Line of Sight method*

The correlation of Line of Sight method there is characteristics of transmission is Line of Sight. We collecting RSS value form each test points, 10 times to find the average RSS value of the 62 test points. Next, we are using the average RSS value to build the relationship with the Matlab simulation by using the *cftool* command. We will have correlation equation between the received signal strength and the distance for the LOS case as equation (12) and the Fit Curve in Matlab is plotted in Fig. 5

Correlation equation of LOS

$$f(x) = p_1*x^4 + p_2*x^3 + p_3*x^2 + p_4*x + p_5 \tag{11}$$

then

$$f(x) = (-1.123\text{x}10^{-4}*x^4) + (-0.0239*x^3) + (-1.833*x^2) + (-61.58*x) + (-770.9) \tag{12}$$

When p_1 - p_5 refer the coefficient of linear model poly4, the distance function $f(x)$ is a distance between the reference node and target node (m) and the variable x represent the RSS value (dBm).

Fig. 4 Equipment used in the experiment: the referencing nodes and the object node

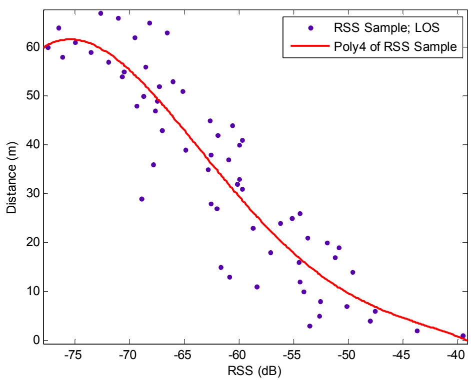

Fig. 5 LOS correlation

B. *Correlation of Non Line of Sight method*

The correlation of Non Line of Sight method there is no LOS path between transmitter and receiver. Because, there are effects of the indoor environment such as wall, door. We collecting RSS value form each test points, 10 times to find the average RSS value of the 57 test points. Then, we perform to build the relationship between the average RSS value and the distance for the NLOS case as follows (14)

Correlation equation of NLOS

$$f(x) = p_1*x^4 + p_2*x^3 + p_3*x^2 + p_4*x + p_5 \tag{13}$$

then

$$f(x) = (1.255\text{x}10^{-6}*x^4) + (4.863\text{x}10^{-4}*x^3) + (0.0515*x^2) + (1.133*x) + (-0.922) \tag{14}$$

When p_1 - p_5 represent the coefficient of linear model poly4, the distance function $f(x)$ is a distance between the reference node and target node (m) and the variable x is the RSS value (dBm).

C. *Correlation of path loss model method*

The correlation of path loss model method utilizes the property of signal radio propagation within a building [12]. The equation of the relationship for the path loss model case can be represented by equation (15)

$$Pr = Pt + K + Gt + Gr - 10\alpha\log_{10}(d/d_0) \tag{15}$$

$$K = 20\log_{10} (\lambda/4\pi\, d_0)^{2} \tag{16}$$

The variable Pr refers to the received signal strength value (dBm), the transmit power Pt assume is 18 dBm. K that depends on the average channel attenuation can be compute by equation (16). The λ is signal wavelength (m), d_0 is typically assumed to be 1 m. The variable α can be obtained to approximate either an analytical or empirical model, our experiment using path loss exponent is 3.45 [12]. Gt is the transmit antenna gain and Gr is the receive antenna gain, we determine these to parameters equal 1.5 dBi. The d refers to distance between the reference node and the target node (m).

Note that the using the 3 relationship above, we can be converting received signal strength value into the distance. By the condition, if the RSS value not in the range -80 dBm to -40 dbm was assume to be equal the boundary. For instance, measuring the received signal strength value is -95 dBm. We define a new received signal strength value are -80 dBm.

IV. Experimental Results and Analysis

In our experiments, we performed measurement of the 40 sample points. Each test points measuring 10 times in order to find the average RSS value of the 40 sample points. Use the 3 approach for convert the received signal strength into the distance as mentioned in Section III. Next, we are using the distance value to location estimate of the target node by

Cramer's rule. Examples of the position systems show as follows. We assume the method of the convert the received signal strength into the distance are LOS case. At sample point S1, starting that the target node measured RSS value form the 4 reference nodes are R_A = -97.9 dBm, R_B = -52.75 dBm, R_C = -65.9 dBm, and R_D = -89.65 dBm. Select the RSS value to the best RSS value of the 3 in 4 reference node (R_B=-52.75 dBm, R_C = -65.9 dBm, and $R_{D\ over\ bound}$ = -80 dBm). Next, converts the RSS value into the distance by the correlation equation of LOS method as equation (11). We obtained the distances are $f(R_B)$ = 12.62 m, $f(R_C)$ = 43.12 m, and $f(R_D)$ = 51.05 m. Then, using the distances value to compute the coordinate of the target node (X, Y) by the Cramer's rule. Final, we will be the position of the target node are (15.47, 28.30).Table I shows the error distance between the actual target location and the experiment target location when used the 3 method of the correlation equation. We can see that the correlation of path loss model method be provided the least accuracy positioning. The margin error of maximum error is 39.6306 m, average error is 22.418428 m, minimum error is 5.1764 m, and the standard deviation is approximately 8 m. And the method of the correlation equation give the most accuracy positioning is the correlation of NLOS method. A maximum error is 36.582 m, average error is 17.59365 m, minimum error is 2.7716 m, and the standard deviation is about 8.5 m.The plot in Figure 6, 7, and 8 shows the actual target location and the experiment target location all 40 sample points of the correlation method of the LOS, NLOS, path loss model, respectively. In Fig. 8 the correlation of path loss model method, it provides localization accuracy less than when comparing the localization accuracy of the correlation of LOS and NLOS method ostensibly.Histogram analysis for comparison the distance error of the 3 different methods as show in Figure 9. Notices, that the correlation of NLOS method there error distance distribution by spreading in the range 0 to 25 m that more than the correlation of LOS and path loss model method. Moreover, the Fig. 10 the cumulative distribution function (CDF) of the error distance of the 3 different methods. Shows that the correlation of NLOS method has a location precision of 80% within 25 m (the CDF of the error distance of 25 m is 0.8). Different from the correlation of LOS and path loss model method that a location precision of 80% within 30 m. Therefore, the estimation of the object within a building by Cramer's rule, using the correlation of NLOS method provided localization accuracy of the positioning more than the correlation of LOS and path loss model method.

TABLE I
ERROR DISTANCE OBTAINED BY DIFFERENT METHODS

Method	Error distance (m)			
	Min	Mean	Max	SD
LOS	3.26	20.46	36.99	9.48
NLOS	2.77	17.59	36.58	8.53
Path Loss model	5.17	22.41	39.63	8.21

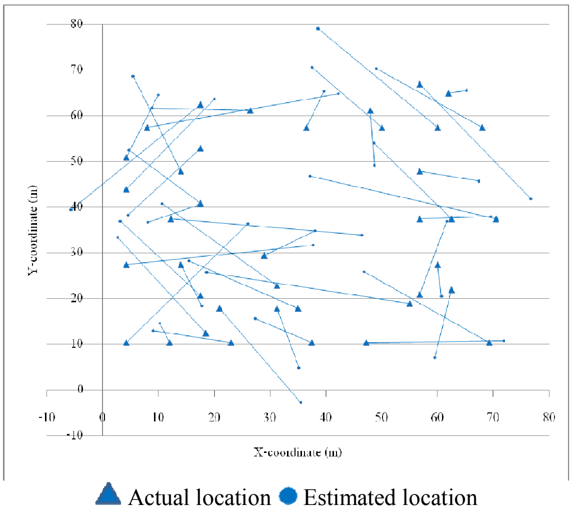

Fig. 6 the actual and estimated locations of 40 test points using LOS method

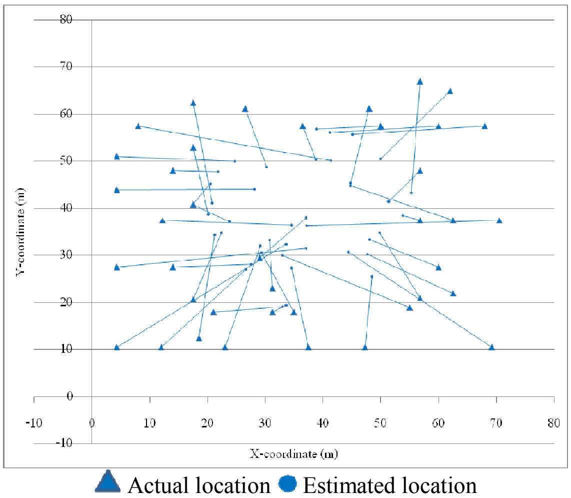

Fig. 7 the actual and estimated locations of 40 test points using NLOS method

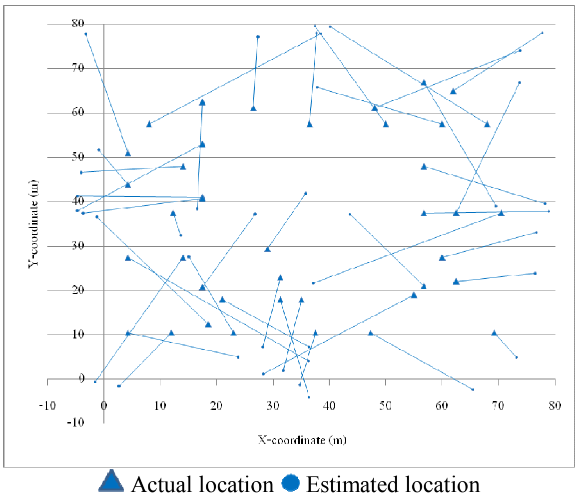

Fig. 8 the actual and estimated locations of 40 test points using path-loss model

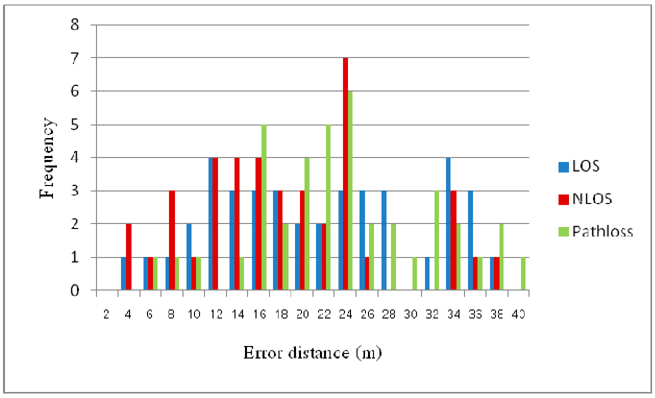

Fig. 9 Histogram of the error distances

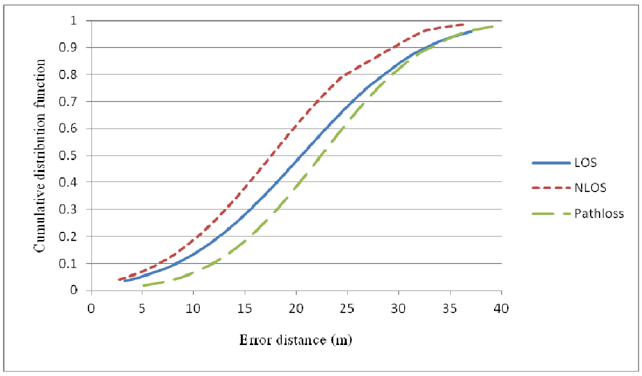

Fig. 10 Cumulative distribution function of the error distances

V. Conclusion

In this paper, we present the analysis of the localization accuracy of the wireless indoor positioning systems using Cramer's rule. We analyze the use of different methods to convert the received signal strength into the distance used in the Cramer's rule to compute the object location. Specifically, we compare three methods, including the method based on the empirical data measured in the line-of-sight (LOS) environment, the method based on the non line-of-sight (NLOS) empirical data and the method based on the path-loss model using the empirical path-loss exponent. The experimental study shows that the NLOS based method results in the highest localization accuracy, with the minimum error distance less than 3 m. Our ongoing work is to develop an efficient indoor positioning framework that can apply to various service environments including the single floor and multiple floor area.

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