Model Reduction of Linear Systems by Conventional and Evolutionary Techniques

S. Panda, S. K. Tomar, R. Prasad, C. Ardil

Abstract—Reduction of Single Input Single Output (SISO) continuous systems into Reduced Order Model (ROM), using a conventional and an evolutionary technique is presented in this paper. In the conventional technique, the mixed advantages of Mihailov stability criterion and continued fraction expansions (CFE) technique is employed where the reduced denominator polynomial is derived using Mihailov stability criterion and the numerator is obtained by matching the quotients of the Cauer second form of Continued fraction expansions. In the evolutionary technique method Particle Swarm Optimization (PSO) is employed to reduce the higher order model. PSO method is based on the minimization of the Integral Squared Error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. Both the methods are illustrated through numerical example.

Keywords—Reduced Order Modeling, Stability, Continued Fraction Expansions, Mihailov Stability Criterion, Particle Swarm Optimization, Integral Squared Error.

I. INTRODUCTION

REDUCTION of high order systems to lower order models has been an important subject area in control engineering for many years. The mathematical procedure of system modeling often leads to detailed description of a process in the form of high order differential equations. These equations in the frequency domain lead to a high order transfer function. Therefore, it is desirable to reduce higher order transfer functions to lower order systems for analysis and design purposes.

Bosley and Lees [1] and others have proposed a method of reduction based on the fitting of the time moments of the system and its reduced model, but these methods have a serious disadvantage that the reduced order model may be unstable even though the original high order system is stable. To overcome the stability problem, Hutton and Friedland [2],

Appiah [3] and Chen *et. al.* [4] gave different methods, called stability based reduction methods which make use of some stability criterion. Other approaches in this direction include the methods such as Shamash [5] and Gutman *et. al.* [6]. These methods do not make use of any stability criterion but always lead to the stable reduced order models for stable systems. Some combined methods are also given for example Shamash [7], Chen *et. al.* [8] and Wan [9]. In these methods the denominator of the reduced order model is derived by some stability criterion method while the numerator of the reduced model is obtained by some other methods [6, 8, 10].

In recent years, one of the most promising research fields has been "Evolutionary Techniques", an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Recently, the particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population-based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [11]. PSO shares many similarities with the genetic algorithm (GA), such as initialization of population of random solutions and search for the optimal by updating generations. However, unlike GA, PSO has no evolution operators, such as crossover and mutation. One of the most promising advantages of PSO over the GA is its algorithmic simplicity: it uses a few parameters and is easy to implement [12].

In the present paper, two methods for order reduction of Single Input Single Output (SISO) continuous systems are presented. In the first method, the mixed advantages of Mihailov stability criterion [13] and continued fraction expansions (CFE) technique [14] is employed where the reduced denominator polynomial is derived using Mihailov stability criterion and the numerator is obtained by matching the quotients of the Cauer second form of Continued fraction expansions. The Mihailov stability criterion is to improve the Pade approximation method, to the general case. In this method, several reduced models can be obtained depending upon the different values of the constant λ_2 in the model and bring the Mihailov frequency characteristic of the reduced model to approximate that of the original system at the low frequency region. In the second method, PSO is employed for the order reduction where both the numerator and denominator coefficients of LOS are determined by minimizing an ISE error criterion.

S. Panda is working as a Professor in the Department of Electrical and Electronics Engineering, NIST, Berhampur, Orissa, India, Pin: 761008. (e-mail: panda_sidhartha@rediffmail.com).

S. K. Tomar is a research scholar in the department of Electrical engineering, Indian Institute of Technology, Roorkee 247667, Uttarakhand, India (e-mail: shivktomar@gmail.com).

R. Prasad is working as an Associate Professor in the Department of Electrical Engineering, Indian Institute of Technology, Roorkee 247 667 Uttarakhand, India (e-mail: rpdeefee@ernet.in)

C. Ardil is with National Academy of Aviation, AZ1045, Baku, Azerbaijan, Bina, 25th km, NAA (e-mail: cemalardil@gmail.com).

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The reminder of the paper is organized in five major sections. In Section II statement of the problem is given. Order reduction by Mihailov stability criterion and CFE technique is presented in Section III. In Section IV, order reduction by PSO has been presented. In Section V, a numerical example is taken and both the proposed methods are applied to obtain the reduced order models for higher order models and results are shown. A comparison of both the proposed method with other well known order reduction techniques is presented in Section VI. Finally, in Section VII conclusions are given.

II. STATEMENT OF THE PROBLEM

Given an original system of order 'n' that is described by the transfer function $G(s)$ and its reduced model $R(s)$ of order 'r' be represented as:

$$G(s) = \sum_{j=1}^{n} b_{1,j} s^{j-1} \Big/ \Delta(s) \tag{1}$$

where

$$\Delta(s) = \sum_{j=1}^{n+1} a_{1,j} s^{j-1}$$

$$R(s) = \sum_{j=1}^{r} b_{2,j} s^{j-1} \Big/ D_r(s) \tag{1}$$

$$D_r(s) = \sum_{j=1}^{r+1} a_{2,j} s^{j-1} \tag{4}$$

Where a_1, b_1, a_2 and b_2 are constants. $D_r(s)$ is the reduced degree polynomial of order 'r', with ($r < n$).

The objective is to find a reduced r^{th} order reduced model $R(s)$ such that it retains the important properties of $G(s)$ for the same types of inputs.

III. REDUCTION BY CONVENTIONAL METHOD

The reduction procedure by conventional method (Mihailov stability criterion and continued fraction expansions) may be described in the following steps:

Step-1

Expand $G(s)$ into Cauer second form of continued fraction expansion:

$$G(s) = \cfrac{1}{h_1 + \cfrac{1}{h_2/s + \cfrac{1}{h_3 + \cfrac{1}{h_4/s + \cfrac{1}{\dots}}}}} \tag{5}$$

Where the quotients h_i for $i = 1,2,3,...,r$ are determined using Routh algorithm [14] as:

$$a_{i,j} = a_{i-2,j+1} - h_{i-2} a_{i-1,j+1} \tag{6}$$

Where, $i = 3,4,.....,$ $j = 1,2,.....,$ and $h_i = a_{i,1}/a_{i+1,1}$

provided $a_{i+1,1} \neq 0$

Step-2

Determine the reduced denominator $D_r(s)$ using Mihailov stability criterion as follows:

Substituting $s = j\omega$ in $\Delta(s)$, expanding and separating it into real and imaginary parts, gives:

$$\Delta(j\omega) = a_{11} + a_{12}(j\omega) + a_{13}(j\omega)^2 + ... + a_{1n+1}(j\omega)_n \tag{7}$$

$$= (a_{11} - a_{13}\omega^2 + ...) + j(a_{12}\omega - a_{14}\omega_3 + ...) \tag{8}$$

$$= \phi(\omega) + j\varphi(\omega) \tag{9}$$

Where 's' is angular frequency in rad/sec.

Setting $\phi(\omega) = 0$ and $\varphi(\omega) = 0$, the intersecting frequencies $\omega = 0, \pm\omega_1, \pm\omega_3, ... \pm\omega_{n-1}$ are obtained where $|\omega_1| < |\omega_2| < ... < |\omega_{n-1}|$.

Similarly substituting $s = j\omega$ in $D_r(s)$ gives

$$D_r(j\omega) = \xi(\omega) + j\eta(\omega) \tag{10}$$

Where

$$\xi(\omega) = e_0 - e_2\omega^2 + e_4\omega^4 - ..... \tag{11}$$

and

$$\eta(\omega) = e_1\omega - e_3\omega^3 + e_5\omega^5 - ..... \tag{12}$$

If the reduced model is stable, its Mihailov frequency characteristic must intersect 'r' times with abscissa and ordinate alternately in the same manner as that of the original system.

In other words, roots of $\xi(\omega) = 0$ and $\eta(\omega) = 0$ must be real and positive and alternately distributed along the ω-axis. So, the first 'r' intersecting frequencies $0, \omega_1, \omega_2, ... \omega_{r-1}$ are kept unchanged and are set to be the roots $\xi(\omega) = 0$ and $\eta(\omega) = 0$.

Therefore:

$$\xi(\omega) = \lambda_1(\omega^2 - \omega_1^2)(\omega^2 - \omega_3^2)(\omega^2 - \omega_5^2)... \tag{13}$$

$$\eta(\omega) = \lambda_2\omega(\omega^2 - \omega_2^2)(\omega^2 - \omega_4^2)... \tag{14}$$

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Where the values of the coefficients 'λ_1' and 'λ_2' are computed from $\phi(0) = \xi(0)$ and $\psi(\omega_1) = \eta(\omega_1)$ respectively, putting these values of 'λ_1' and 'λ_2' in (13) and (14) respectively, $\xi(\omega)$ and $\eta(\omega)$ are obtained and $D_r(j\omega)$ is found as given in equation (10).

Now replacing $j\omega$ by 's', the 'r^{th}' order reduced denominator $D_r(s)$ is obtained as given by equation (2).

Two other sets of 'λ_1' and 'λ_2' are also obtained resulting in reducing the denominator $\Delta(s)$ to different values of $D_r(s)$ to provide a range of different solutions. This is achieved as follows:

In the first criterion, 'λ_1' is determined by $\phi(0) = \xi(0)$ and 'λ_2' is determined by $(d\psi/d\phi)_{\omega_0} = (d\eta/d\xi)_{\omega_0}$ in the reduced model to keep the initial slope of the Mihailov frequency characteristic unchanged.

In the second criterion, 'λ_1' is again unchanged but 'λ_2' is determined by keeping the ratio of the first two coefficients (a_{11} and a_{12}) of the characteristic equation (2) unchanged in the reduced model [13].

Step-3

Match the coefficients $a_{2,j}$ in (16) and h_i in (6) to determine reduced numerator polynomial $N_r(s)$ by applying the following reverse Routh algorithm:

$$a_{i+1} = a_{i,1}/h_i \qquad (15)$$

For $i = 1,2,...r$ with $r < n$

$$a_{i+1,j+1} = (a_{i,j+1} - a_{i+2,j}) \qquad (16)$$

With $j = 1,2,...(r-1)$ and $a_{1,r+1} = 1$

The reduced order model (ROM), $R(s)$ is obtained as:

$$R(s) = \frac{N_r(s)}{D_r(s)} \qquad (17)$$

Step-4

There is a steady state error between the outputs of original and reduced systems. To avoid steady state error we match the steady state responses by following relationship, to obtain correction factor 'k' a constant as follows:

$$\frac{b_{11}}{a_{11}} = k\frac{b_{21}}{a_{21}} \qquad (18)$$

The final reduced order model is obtained by multiplying 'k' with numerator of the reduced model obtained in step 3.

IV. PARTICLE SWARM OPTIMIZATION METHOD

In conventional mathematical optimization techniques, problem formulation must satisfy mathematical restrictions with advanced computer algorithm requirement, and may suffer from numerical problems. Further, in a complex system consisting of number of controllers, the optimization of several controller parameters using the conventional optimization is very complicated process and sometimes gets struck at local minima resulting in sub-optimal controller parameters. In recent years, one of the most promising research field has been "Heuristics from Nature", an area utilizing analogies with nature or social systems. Application of these heuristic optimization methods a) may find a global optimum, b) can produce a number of alternative solutions, c) no mathematical restrictions on the problem formulation, d) relatively easy to implement and e) numerically robust. Several modern heuristic tools have evolved in the last two decades that facilitates solving optimization problems that were previously difficult or impossible to solve. These tools include evolutionary computation, simulated annealing, tabu search, genetic algorithm, particle swarm optimization, etc. Among these heuristic techniques, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) techniques appeared as promising algorithms for handling the optimization problems. These techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions.

The PSO method is a member of wide category of swarm intelligence methods for solving the optimization problems. It is a population based search algorithm where each individual is referred to as particle and represents a candidate solution. Each particle in PSO flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also to the flying experience of the other particles. In PSO each particles strive to improve themselves by imitating traits from their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as *pbest* and the overall best out of all the particles in the population is called *gbest* [11].

The modified velocity and position of each particle can be calculated using the current velocity and the distances from the *pbest*$_{j,g}$ to *gbest*$_g$ as shown in the following formulas [12,15, 16]:

$$v_{j,g}^{(t+1)} = w*v_{j,g}^{(t)} + c_1 * r_1(\ )*(pbest_{j,g} - x_{j,g}^{(t)})$$
$$+ c_2 * r_2(\ )*(gbest_g - x_{j,g}^{(t)}) \qquad (19)$$

$$x_{j,g}^{(t+1)} = x_{j,g}^{(t)} + v_{j,g}^{(t+1)} \qquad (20)$$

With $j = 1,2,...,n$ and $g = 1,2,...,m$

Where,

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n = number of particles in the swarm

m = number of components for the vectors v_j and x_j

t = number of iterations (generations)

$v_{j,g}^{(t)}$ = the g-th component of the velocity of particle j at iteration t , $v_g^{min} \leq v_{j,g}^{(t)} \leq v_g^{max}$;

w = inertia weight factor

c_1, c_2 = cognitive and social acceleration factors respectively

r_1, r_2 = random numbers uniformly distributed in the range (0, 1)

$x_{j,g}^{(t)}$ = the g-th component of the position of particle j at iteration t

$pbest_j$ = $pbest$ of particle j

$gbest$ = $gbest$ of the group

The j-th particle in the swarm is represented by a d-dimensional vector $x_j = (x_{j,1}, x_{j,2}, \dots, x_{j,d})$ and its rate of position change (velocity) is denoted by another d-dimensional vector $v_j = (v_{j,1}, v_{j,2}, \dots, v_{j,d})$. The best previous position of the j-th particle is represented as $pbest_j = (pbest_{j,1}, pbest_{j,2}, \dots, pbest_{j,d})$. The index of best particle among all of the particles in the swarm is represented by the $gbest_g$. In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group's previous best solution. The velocity update in a PSO consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm. The parameters c_1 and c_2 determine the relative pull of $pbest$ and $gbest$ and the parameters r_1 and r_2 help in stochastically varying these pulls. In the above equations, superscripts denote the iteration number.
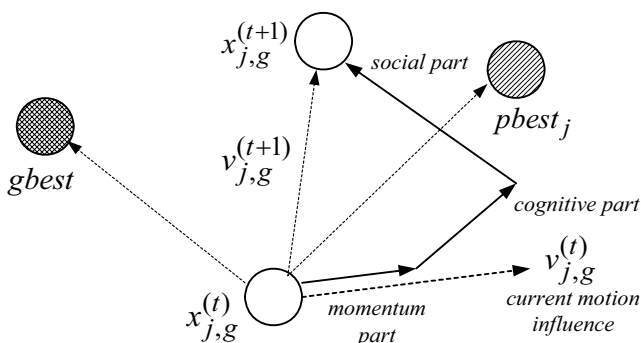

Fig. 1. Description of velocity and position updates in particle swarm optimization for a two dimensional parameter space
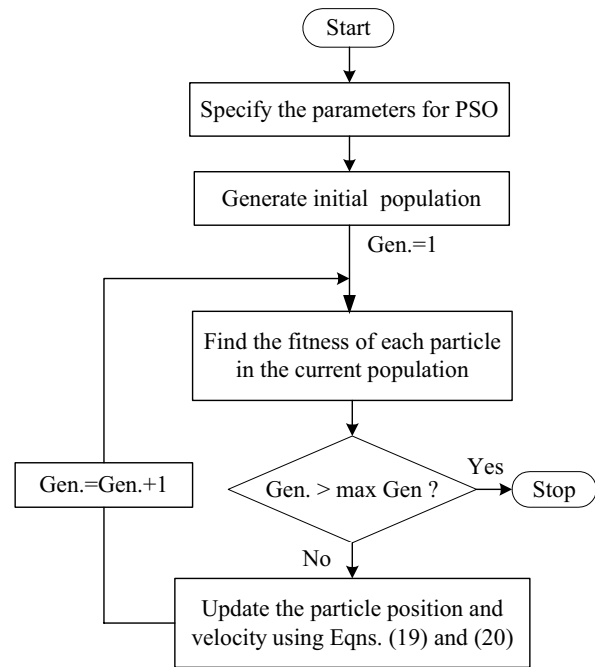

Fig. 2. Flowchart of PSO for order reduction

Fig.1. shows the velocity and position updates of a particle for a two-dimensional parameter space. The computational flow chart of PSO algorithm employed in the present study for the model reduction is shown in Fig. 2.

V. NUMERICAL EXAMPLES

Let us consider the system described by the transfer function [15]:

$$G(s) = \frac{10s^4 + 82s^3 + 264s^2 + 396s + 156}{2s^5 + 21s^4 + 84s^3 + 173s^2 + 148s + 40} \quad (21)$$

For which a second order reduced model $R_2(s)$ is desired.

A. Conventional Method

Step-1

The quotients h_i h$_i$ for $i = 1,2,3,\dots,r$ are determined using Routh algorithm as:

$$h_1 = 0.256, \; h_2 = 2.92$$
$$h_3 = 0.872, \; h_4 = 1.72 \quad (22)$$

Step-2

Determine the reduced denominator $D_r(s)$ using Mihailov stability criterion as follows:

$$\Delta(s) = 2s^5 + 21s^4 + 84s^3 + 173s^2 + 148s + 40 \quad (23)$$

Expanding and separating it into real and imaginary parts, gives:

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$$\phi(\omega) = 40 - 173\omega^2 + 21\omega^4 \tag{24}$$

The roots are:

$$\omega^2 = 0.238, 8$$

and $\psi(\omega) = 148\omega - 84\omega^3 + 2\omega^4 \tag{25}$

The roots are:

$$\omega^2 = 0, 0.167, 41.83$$

Now, reduced denominator polynomial is derived using the second criterion of Mihailov stability criterion method by calculating the values of 'λ_1' and 'λ_2' as given in (13) and (14) which come out to be 239.5 and 148 respectively, after putting $j\omega = s$ results into reduced denominator polynomial of a second order ROM as:

$$D_r(s) = 40 + 148 + 239.5s^2 \tag{26}$$

Step-3

The numerator is obtained by matching the quotients h_i of the Cauer second form of Continued fraction expansions with the coefficients of reduced denominator and using the reverse Routh algorithm as:

$$N_r(s) = 369s + 156 \tag{27}$$

The transfer function for the reduced order model (ROM) of second order can therefore be expressed as:

$$R_2(s) = \frac{369s + 156}{239.5s^2 + 148s + 40} \tag{28}$$

Step-4

In this particular example there is no steady state error between the step responses of the original system and the ROM, hence $k = 1$, and the final reduced model remains unchanged.

B. Particle Swarm Optimization Method

For the implementation of PSO, several parameters are required to be specified, such as c_1 and c_2 (cognitive and social acceleration factors, respectively), initial inertia weights, swarm size, and stopping criteria. These parameters should be selected carefully for efficient performance of PSO.

The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward *pbest* and *gbest* positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, target regions. Hence, the acceleration constants were often set to be 2.0 according to past experiences. Suitable selection of inertia weight, w, provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. As originally developed, w often decreases linearly from about 0.9 to 0.4 during a run [16, 17]. One more important point that more or less affects the optimal solution is the range for unknowns. For the very first execution of the program, wider solution space can be given, and after getting the solution, one can shorten the solution space nearer to the values obtained in the previous iterations.

The objective function J is defined as an integral squared error of difference between the responses given by the expression:

$$J = \int_0^{t_\infty} [y(t) - y_r(t)]^2 \, dt \tag{29}$$

Where

$y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems.

The reduced 2nd order model employing PSO technique is obtained as follows:

$$R_2(s) = \frac{347.0245s + 225.6039}{135.6805s^2 + 166.3810s + 57.8472} \tag{30}$$
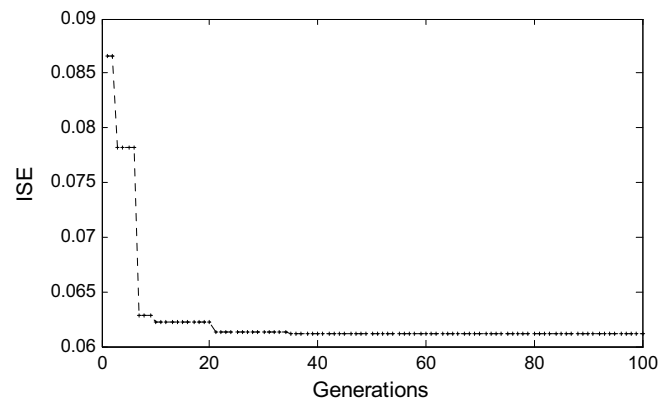

Fig. 3. Convergence of objective function for example-1

The convergence of objective function with the number of generations is shown in Fig. 3. The unit step responses of original and reduced systems by both the methods are shown in Fig. 4. It can be seen that the steady state responses of both the proposed reduced order models are exactly matching with that of the original model. However, compared to conventional method of reduced models, the transient response of evolutionary reduced model by PSO is very close to that of original model.

VI. COMPARISON OF METHODS

The performance comparison of both the proposed algorithm for order reduction techniques is given in Table I. The comparison is made by computing the error index known as integral square error ISE [16] in between the transient parts of the original and reduced order model, is calculated to

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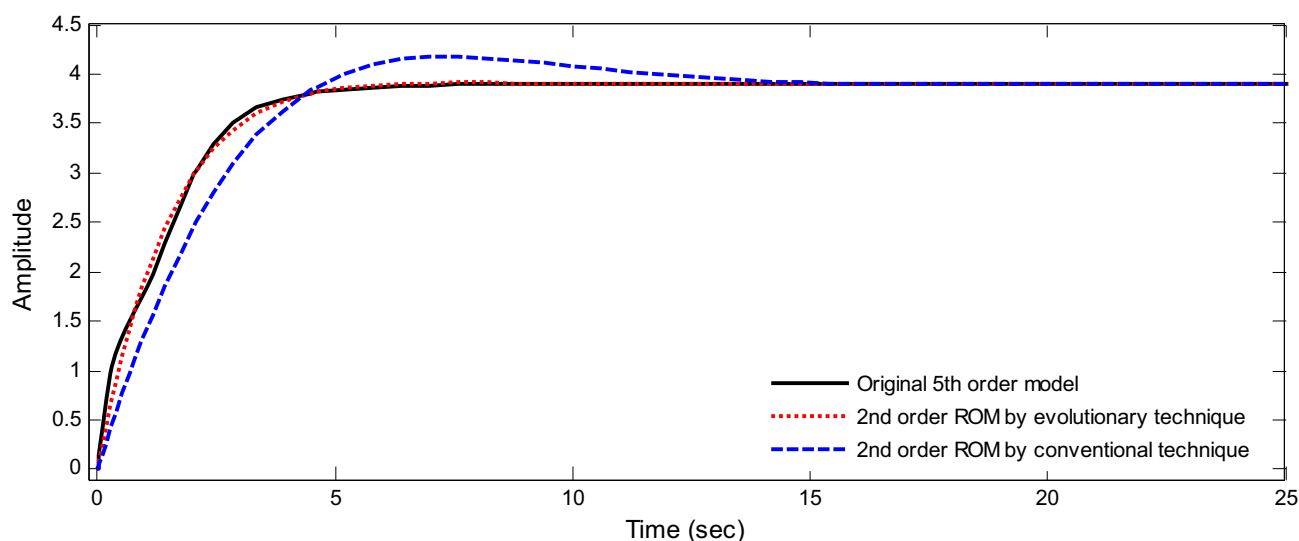International Journal of Electrical and Computer Engineering
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Fig. 4. Step Responses of original system and reduced model

measure the goodness/quality of the [i.e. the smaller the ISE, the closer is $R(s)$ to $G(s)$, which is given by:

$$ISE = \int_0^{t_\infty}[y(t) - y_r(t)]^2\, dt \qquad (31)$$

Where $y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems for a second- order reduced respectively. This error index is calculated for various reduced order models which are obtained by us and compared with the other well known order reduction methods available in the literature.

TABLE I COMPARISON OF METHODS

Method	Reduced model	ISE
Proposed evolutionary method	$\dfrac{347.0245s + 225.6039}{135.6805s^2 + 166.3810s + 57.8472}$	0.0613
Proposed conventional method	$\dfrac{369s + 156}{239.5s^2 + 148s + 40}$	1.0806

VI. CONCLUSION

In this paper, two methods for reducing a high order system into a lower order system have been proposed. In the first method, a conventional technique has been proposed which uses the advantages of both the Mihailov stability criterion to deduce the denominator polynomial and the Cauer second form of continued fraction expansions to obtain the numerator polynomial, by matching the denominator polynomial. In the second method, an evolutionary swarm intelligence based method known as Particle Swarm Optimization (PSO) is employed to reduce the higher order model. PSO method is based on the minimization of the Integral Squared Error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. Both the methods are illustrated through a numerical example.

Also, a comparison of both the proposed methods has been presented. It is observed that both the proposed methods preserve steady state value and stability in the reduced models and the error between the initial or final values of the responses of original and reduced order models is very less. However, PSO method seems to achieve better results in view of its simplicity, easy implementation and better response.

REFERENCES

[1] M. J. Bosley and F. P. Lees, "A survey of simple transfer function derivations from high order state variable models", *Automatica*, Vol. 8, pp. 765-775, !978.
[2] M. F. Hutton and B. Fried land, "Routh approximations for reducing order of linear time- invariant systems", *IEEE Trans. Auto. Control*, Vol. 20, pp 329-337, 1975.
[3] R. K. Appiah, "Linear model reduction using Hurwitz polynomial approximation", *Int. J. Control*, Vol. 28, no. 3, pp 477-488, 1978.
[4] T. C. Chen, C. Y. Chang and K. W. Han, "Reduction of transfer functions by the stability equation method", *Journal of Franklin Institute*, Vol. 308, pp 389-404, 1979.
[5] Y. Shamash, "Truncation method of reduction: a viable alternative", *Electronics Letters*, Vol. 17, pp 97-99, 1981.
[6] P. O. Gutman, C. F. Mannerfelt and P. Molander, "Contributions to the model reduction problem", *IEEE Trans. Auto. Control*, Vol. 27, pp 454-455, 1982.
[7] Y. Shamash, "Model reduction using the Routh stability criterion and the Pade approximation technique", *Int. J. Control*, Vol. 21, pp 475-484, 1975.
[8] T. C. Chen, C. Y. Chang and K. W. Han, "Model Reduction using the stability-equation method and the Pade approximation method", *Journal of Franklin Institute*, Vol. 309, pp 473-490, 1980.
[9] Bai-Wu Wan, "Linear model reduction using Mihailov criterion and Pade approximation technique", *Int. J. Control*, Vol. 33, pp 1073-1089, 1981.
[10] V. Singh, D. Chandra and H. Kar, "Improved Routh-Pade Approximants: A Computer-Aided Approach", *IEEE Trans. Auto. Control*, Vol. 49. No. 2, pp292-296, 2004.
[11] J. Kennedy and R. C. Eberhart, "Particle swarm optimization", *IEEE Int.Conf. on Neural Networks,* IV, 1942-1948, Piscataway, NJ, 1995.
[12] S. Panda, and N. P. Padhy "Comparison of Particle Swarm Optimization and Genetic Algorithm for FACTS-based Controller Design", *Applied Soft Computing.* Vol. 8, pp. 1418-1427, 2008.
[13] Wan Bai-Wu, "Linear model reduction using Mihailov criterion and Pade approximation technique." Int. J. Control, 1981, No 33, pp 1073-1089.

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:3, No:11, 2009

[14] Chen, C.F. and Sheih, L.S "A Novel approach to linear model simplification." Int. J. Control, 1968, No 8, pp 561-570.
[15] T. N. Lukas. "Linear system reduction by the modified factor division method" IEEE Proceedings Vol. 133 Part D No. 6, nov.-1986, pp-293-295.
[16] M. Lal and H. Singh, "On the determination of a transfer function matrix from the given state equations." *Int. J. Control*, Vol. 15, pp 333-335, 1972.
[17] Sidhartha Panda, N.P.Padhy, R.N.Patel, "Power System Stability Improvement by PSO Optimized SSSC-based Damping Controller", *Electric Power Components & Systems*, Vol. 36, No. 5, pp. 468-490, 2008.
[18] Sidhartha Panda and N.P.Padhy, "Optimal location and controller design of STATCOM using particle swarm optimization", *Journal of the Franklin Institute*, Vol.345, pp. 166-181, 2008.

Sidhartha Panda is a Professor at National Institute of Science and Technology (NIST), Berhampur, Orissa, India. He received the Ph.D. degree from Indian Institute of Technology, Roorkee, India in 2008, M.E. degree in Power Systems Engineering in 2001 and B.E. degree in Electrical Engineering in 1991. His areas of research include power system transient stability, power system dynamic stability, FACTS, optimisation techniques, distributed generation and wind energy.

Shiv Kumar Tomar is a Research Scholar in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). He is the Head of the Department, Electronics Dept. I.E.T. M.J.P. Rohilkhand University, Bareilly and at present on study leave on Quality Improvement Programme (QIP) for doing his Ph.D. at IIT, Roorkee. His field of interest includes Control, Optimization, Model Order Reduction and application of evolutionary techniques for model order reduction.

Dr. Rajendra Prasad received B.Sc. (Hons.) degree from Meerut University, India, in 1973. He received B.E., M.E. and Ph.D. degrees in ElectricalEngineering from University of Roorkee, India, in 1977, 1979, and 1990 respectively. Presently, he is an Associate Professor in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). His research interests include Control, Optimization, SystemEngineering and Model Order Reduction of large scale systems.

C. Ardil is with National Academy of Aviation, AZ1045, Baku, Azerbaijan, Bina, 25th km, NAA