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Calculation of Reorder Point Level under Stochastic Parameters: A Case Study in Healthcare Area

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***Abstract*—**We consider a single-echelon, single-item inventory system where both demand and lead-time are stochastic. Continuous review policy is used to control the inventory system. The objective is to calculate the reorder point level under stochastic parameters. A case study is presented in Neonatal Intensive Care Unit.

***Keywords*—**Inventory control system, reorder point level, stochastic demand, stochastic lead time

I. Introduction

THERE are a growing number of studies on inventory control systems. The majority of these studies relate to production applications, and backordering and shortages are allowed. Şatır and Cengiz [1] presented a stochastic, periodic-review model used to control pharmaceutical inventories in a university health centre. The objective of the inventory model formulated in that study has been taken to be the minimization of stockouts. It has been assumed that the lead-time demand is normally distributed and the lead-time is taken to be constant. Çakanyıldırım et al. [2] modeled (Q, r) policy where the lead-time depends on lot size. Salameh et al. [3] considered a continuous inventory model under permissible delays in payments. In this model, it has been assumed that expected demand is constant over time and the order lead-time is random. Seifbarghy and Jokar [4] developed an approximate cost function for a two-echelon inventory system with one warehouse and several identical retailers. The model considered that demand is independent Poisson and the control policy is continuous review. Chen and Levi [5] examined a continuous review model with infinite horizon and single product; pricing and inventory decisions have been made simultaneously and ordering cost has included a fixed cost. Mohebbi and Hao [6] investigated a problem of random supply interruptions in a continuous review inventory system with compound Poisson demand, Erlang-distributed lead-times and lost sales. Axsäter [7] developed a single-echelon inventory model controlled by continuous review (R, Q) policy in which it has been assumed that the lead-time demand has normally distributed and in which the aim is to minimize holding and ordering cost under fill rate constraint. Hill [8] investigated continuous review lost-sales inventory models with no fixed order cost and a Poisson demand process. In addition, Hill et al. [9] modeled a single-item, two-echelon, continuous review inventory model. In their model, demands made on the retailers follow a Poisson process and warehouse lead-time cannot exceed retailer transportation time. Kopytov et al. [10] considered two single product inventory control models with random parameters. Darwish [11] examined a continuous review model to determine the effects of transportation and purchasing issues on inventory decisions. Babai, Jemai and Dallery [12] developed a simple method to calculate the optimal order-up-to-level in a single echelon, single item inventory system under a compound Poisson process and stochastic lead-time. In this inventory system, unfilled demands are backordered.

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In most papers lead-time is assumed to be constant. But; when we take into account the real world, it is not true to expect constant lead-time. Delays in lead-time can occur because of unexpected situations (carrying problems, second class product, problems based on production system, problems based on supplier, etc.). As seen in Table 1, very few studies have been conducted based on stochastic demand and stochastic lead-time in literature.

TABLE I
SOME PUBLICATIONS ABOUT INVENTORY CONTROL

	Inventory Control			
	Demand		*Lead Time*	
Authors (Year)	***Deterministic***	***Stochastic***	***Deterministic***	***Stochastic***
Satır and Cengiz (1987)		Normal	+	
Cakanyildirim et al. (2000)	+			Exponential
Salameh et al. (2003)	+			Normal
Seifbarghy and Jokar (2006)		Poisson	+	
Axsater (2006)		Normal	+	
Chen and Levi (2006)		Poisson	Equal to zero	
Mohebbi and Hao (2006)		Poisson		Erlang
Hill et al. (2007)		Poisson	+	
Hill (2007)		Poisson	+	
Kopytov et al. (2007)		Poisson		Normal
Darwish (2008)		1.Uniform 2.Normal	+	
Babai, Jemai and Dallery (2010)		Poisson		General distribution

II. Case Study

A. System Description and Data Collecting

This study is conducted in a Neonatal Intensive Care Unit (NICU) in a university hospital in order to calculate reorder point level under stochastic demand and lead-time. The unit accepts patients both inpatients and outpatients and categorizes their illnesses according to the degree of severity. In the unit in which this study is conducted, there are three levels of care units. The patients in these care units may stabilize or worsen and subsequently be re-categorized into one of the other levels. The admitted patients are placed in one of three levels; Level-1 patients are non-critical, Level-2 patients are relatively critical and Level-3 patients are highly critical.

When patients arrive at the unit, if there is an unoccupied bed, they are accepted; if there is no bed available within the appropriate care level, they are rejected or transferred elsewhere. Inpatients that are transferred from a level one care unit to another have higher priority than outpatients. Upon being transferred from one level to another, if there is no unoccupied bed available the transferred patient stays in the same level until a bed becomes available. Patient flow between the levels can be seen in Figure 1.

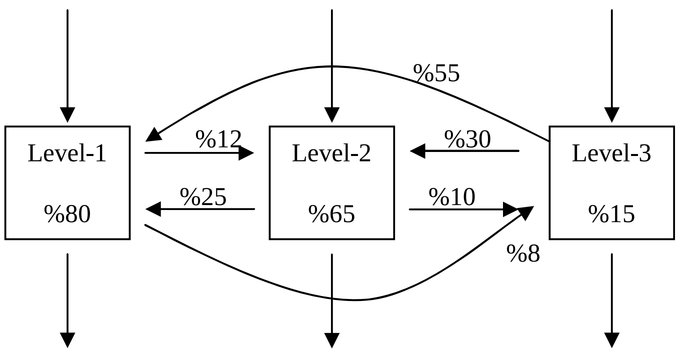

Fig. 1 Patient flow between levels in NICU

The number of arrivals, transferal rates between levels and length of stays (LOS) are random, which makes the variations in the number of patients in each level behave as a stochastic process.

As seen in Figure 1, eighty percent of the accepted patients in Level-1 are treated in Level-1 and then discharged from the same level. %12 of the accepted patients in Level-1 is transferred to Level-2 and %8 of the accepted patients in Level-1 is transferred to Level-3. %65 of the accepted patients in Level-2 are treated in Level-2 and then discharged from the same level. %25 of the accepted patients in Level-2 is transferred to Level-1 and %10 of the accepted patients in Level-2 is transferred to Level-3. %15 of the accepted patients in Level-3 are treated in Level-3 and then discharged from the same level. %55 of the accepted patients in Level-3 is transferred to Level-1 and %30 of the accepted patients in Level-3 is transferred to Level-2.

Each patient's requirement for medical supplies varies daily according to their condition; hence, the amount required of each inventory item depends on the number of patients in the NICU. The daily requirement depends on such random parameters as patient arrival rate, LOS and the probability of patient transferal to another level. To estimate these random parameters, data from 3330 patients treated in the NICU between the years 2000-2004 were collected. The patient arrival rates are calculated as the sum of the accepted patient arrivals and rejected patient arrivals for each level. The accepted patient arrivals, rejected patient arrivals, and daily demand per patient for material showed a Poisson distribution and LOS showed a lognormal distribution [13].

In order to determine the material requirement for any level in the NICU, data on the use of medical supplies within the NICU has been monitored for one year. Data have been collected on approximately 600 types of medical supplies. These materials are classified according to the ABC Analysis method. Among the A group medical equipment, a filter-pump-set (FPS) has been considered as an example.

The existing purchasing policy for FPS is given below:

1. Prepare technical specifications of FPS (max 29 days)
2. Approve the prepared technical specifications (max 60 days)
3. Call for tenders and put out an open tender (max 45 days)
4. Period for objection (max 15 days)
5. Make a contract (max 1 day)
6. Delivery period (max 60 days)

The maximum setup time (150 days) for an order is the total duration of the first five steps. The maximum lead-time (210 days) is the sum of the maximum setup time and the maximum delivery time.

B. Analytical Approach for Calculating Demand with Stochastic Parameters

In this study, the required demand during the stochastic lead time is assumed that the reorder point level. Therefore, we need to calculate the demand with stochastic parameters. Analyzing with collecting data, it seen that daily demand per patient for material has a Poisson distribution and lead time has a uniform distribution.

For a system with Poisson demand, distribution of demand (D_t) can be defined as follow (t represents the fixed time period) [10];

$$P(D_t = i) = \frac{(\lambda t)^i}{i!} e^{-\lambda t},\ i = 0,1,2,...... \tag{1}$$

$f_{LT}(t)$ is the uniform density function for lead-time (LT) and the demand distribution D_{LT} can be calculated as follow;

$$P(D_{LT} = i) = \int_a^b P(D_t = i)\, f_{LT}(t)\, dt \tag{2}$$

$$P\left(D_{LT}=i\right)=\int_{a}^{b}\frac{\left(\lambda t\right)^{i}}{i!}e^{-\lambda t}\frac{1}{b-a}dt=\frac{\lambda^{i}}{i!\left(b-a\right)}\int_{a}^{b}t^{i}e^{-\lambda t}dt \quad (3)$$

$$P\left(D_{LT}=i\right)=\frac{1}{\lambda(b-a)i!}(R_i(\lambda a)-R_i(\lambda b)) \quad (4)$$

$$R_i(z)=\sum_{m=0}^{i}\frac{(z)^m}{m!}e^{-z},\ i=0,1,2,\ldots;\ z=\lambda a;\ z=\lambda b \quad (5)$$

If Eq.5 is put into the Eq.4, demand within lead-time can be calculated using with Erlang distribution table.

But; as seen in Eq.4, to calculate demand within lead-time is considerably difficult. For this reason, in this study the simulation model is constructed for calculating the demand within stochastic lead-time.

C. Simulation Model

The required number of FPS depends on the number of patients in the NICU over the time. Therefore, a simulation model is constructed to determine the required number of FPS over time. The distributions obtained for arrivals, LOS and daily usage rate of FPS are used in the simulation model. It is assumed that all the distributions are constant during the simulation and the total required number of FPS is considered the reorder point level.

The simulation model is developed using the student version of the Arena simulation package [14]. To verify a simulation model, it is debugged interactively and the results are checked manually with animating the system. In this study, the simulation model is verified with bottom-up testing (bottom-up testing means testing the sub-models first and then the overall model [15]). First, the patients' flow system, including the three levels, is modeled and checked. Then, the inventory system for calculating the total required number of FPS is modeled and checked. Finally, the overall model is examined to determine whether or not it is working as intended. The model is validated by estimating the confidence interval for yearly demand of FPS needed from patients.

D.Numerical example using with simulation model

It is proposed determining the reorder point level under stochastic parameters for a variety of service level ($\alpha = 0.90$, $\alpha = 0.95$ and $\alpha = 0.99$). In the constructed simulation model, daily demand per patient for material has a Poisson distribution with $\lambda = 0.5$ parameter and lead time has a uniform distribution. For uniform distribution, it is assumed in this numerical example that $a = 30$ (minimum value parameter) and $b = 45$ (maximum value parameter). 4694 data are obtained from the simulation model (the simulation length is 1800 days and the replication number is 100). Using with statistical package, it is investigated whether the data are normally distributed or not. As seen in Figure 2, the data are normally distributed with $\mu = 487.16$ and $\sigma = 66.077$ parameters.

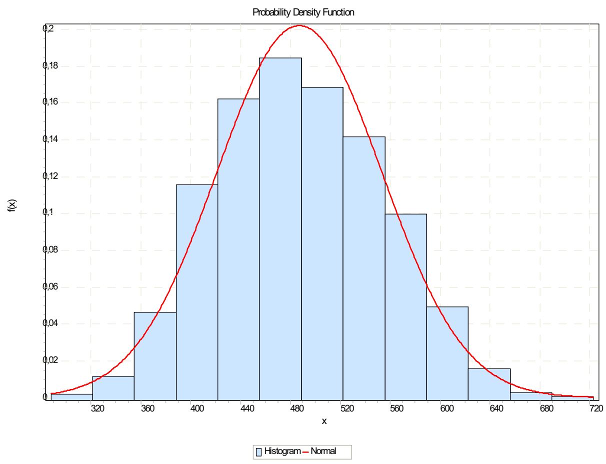

Fig. 2 Distribution of demand during the lead-time

Using the formula $r = \mu + \sigma Z_\alpha$, the reorder point level is calculated as seen in Table 2. Normal probability distribution table is used for Z_α values.

TABLE II
REORDER POINT LEVEL FOR DIFFERENT SERVICE LEVELS

Mean (μ)	Standard deviation (σ)	Service level (α)	Reorder point level (r)
$\mu = 487.16$	$\sigma = 66.077$	0.90	571.87
$\mu = 487.16$	$\sigma = 66.077$	0.95	595.85
$\mu = 487.16$	$\sigma = 66.077$	0.99	640.89

III. Conclusion

In real life; patient arrivals, transferal rates between levels, length of stays (LOS) and the required number of material are random, which makes the variations in the number of patients in each level behave as a stochastic process. In this study, all these random parameters are considered and a case study is presented using with real data. Therefore, this study is useful for hospital managers to determine the reorder point level in different service levels.

The simulation model developed in this study can easily be adapted to any changes in patient arrivals, LOS, probability of patient transferal, daily demand per patient for inventory and lead-time period. Therefore, this simulation model can be used in any inventory system.

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