A Novel FFT-Based Frequency Offset Estimator for OFDM Systems

Mahdi Masoumi, Mehrdad Ardebilipoor, and Seyed Aidin Bassam

***Abstract*—**This paper proposes a novel frequency offset (FO) estimator for orthogonal frequency division multiplexing. Simplicity is most significant feature of this algorithm and can be repeated to achieve acceptable accuracy. Also fractional and integer part of FO is estimated jointly with use of the same algorithm. To do so, instead of using conventional algorithms that usually use correlation function, we use DFT of received signal. Therefore, complexity will be reduced and we can do synchronization procedure by the same hardware that is used to demodulate OFDM symbol. Finally, computer simulation shows that the accuracy of this method is better than other conventional methods.

***Keywords*—**DFT, Estimator, Frequency Offset, IEEE802.11a, OFDM.

I. Introduction

RECENTLY, orthogonal frequency division multiplexing (OFDM) has received increasing attention as an effective multicarrier modulation scheme for high data rate transmissions over frequency-selective fading channels [1]. Channel equalization in OFDM can be accomplished through scalar division, which simplifies the receiver considerably [2]. So it has been chosen for several broadband wireless local area networks (WLANs) standards such as IEEE802.11a, HIPERLAN/2 and MMAC due to its high data rate transmission capability and its robustness to multi-path delay spread[3][4][5].

Despite its advantages, OFDM systems are very sensitive to synchronization errors caused by Doppler shift and/or oscillator instabilities. Orthogonality among subcarriers may easily be lost in the receiver due to frequency offset (FO). Timing offset can also cause the received symbol phase to be rotated. Both offsets lead to symbol decision error in the receiver. Various approaches have been proposed for estimating the time and frequency offsets either jointly or individually [6][7][8].

FO is normalized to the subcarrier spacing and the result is divided into fractional and integer part. Various algorithms have been proposed in the literatures for the estimation of the fractional part [2][9] and the integer part [2][7][10] of FO. This paper focuses on the estimation of the integer and fractional part jointly.

When the FO is an integer multiple of the subcarrier spacing, or normalized FO is an integer, the mutual orthogonality among subcarriers is still preserved. However, the demodulated data symbols are shifted in the frequency domain to wrong positions, yielding a large symbol error probability. When the FO is not an integer multiple of the subcarrier spacing, or normalized FO is fractional, different subcarriers are no longer orthogonal to one another, which introduces intercarrier interference (ICI) and results in significant performance degradation [11]. In the case of effect both fractional and integer part, we have been countered with two mentioned effect together. Thus, accurate FO estimation and compensation are very crucial in maintaining the effectiveness of OFDM.

Usually the correction of integer and fractional part of FO is performed by different algorithms, but in this paper the correction of these two parts is performed by the same algorithm. To do this idea, instead of using the previous methods, based on finding the maximum value of correlation function, we compute the DFT of received sequence and there is no more need to additional hardware to carry out FO correction. Also, the efficient algorithms for computation of DFT such as FFT increase the speed of system. Another significant feature of this algorithm is ability of correction FO iteratively that means we can repeat the algorithm until to reach acceptable accuracy. It is assumed that the timing and sampling clock synchronization tasks have already been accomplished.

We introduce this new method under AWGN channel, but simulation results show that in fading channel, it works as properly as it does in AWGN channel.

II. Definition of the Problem

Suppose that the preamble or training sequence transmitted at the start of every frame, has the structure of fig 1 in which $\mathbf{t}_i$, $1 \le i < M$ are sequences with length of L and for establishing a periodic property, suppose that $\mathbf{t}_1 = \mathbf{t}_2 = ... = \mathbf{t}_M = \mathbf{t}$.

S(n)	t_1	t_2	. . . .	t_{M-1}	t_M
	$a_1 a_2$ . . . . . . a_L				

Fig. 1 Structure of preamble

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$\mathbf{S}$ can be created with DFT of the upsampled version of a PN sequence with length of L [12]. This means that at first insert $M-1$ zero samples between every two consecutive nonzero samples, the DFT of this upsampeled sequence is the preamble.

If we suppose that timing offset is zero, the received signal $R(n)$ under AWGN channel can be presented as (1) [13]

$$R(n) = S(n)\,e^{j\frac{2\pi}{N}\varepsilon n} + V(n) \tag{1}$$

Where $S(n)$ is transmitted signal N is the total number of subchannels, $V(n)$ are the samples of additive white Gaussian noise and ε is the FO normalized to the subcarrier spacing which its effect is shown in (1) by $e^{j\frac{2\pi}{N}\varepsilon n}$ [14]. Our objective in this paper is finding ε more accurately. To do so, we define two vector A_i and B_i,

$$A_i = [a_i \quad a_i e^{j\frac{2\pi}{N}E} \quad \ldots \quad a_i e^{j\frac{2\pi}{N}(M-1)E}] \qquad i = 1 \ldots L$$

$$B_i = [1 \quad e^{j\frac{2\pi}{N}\hat{E}} \quad \ldots \quad e^{j\frac{2\pi}{N}(M-1)\hat{E}}] \qquad i = 1 \ldots L$$

Where a_i is the ith element of sequence $\mathbf{t}_1$, $E = L\varepsilon$, and $\hat{E}$ is the estimation of E.

We define a metric

$$C_i(\hat{E}) = \sum_{k=0}^{N-1}\left(a_i e^{j\frac{2\pi}{N}Ek}\right)\left(e^{-j\frac{2\pi}{N}\hat{E}k}\right) \tag{2}$$

In (2), $\left|C_i(\hat{E})\right|$ indicates similarity between A_i and B_i^* where B_i^* denotes the conjugate of B_i. It is clear that this similarity is maximized when $\hat{E} = E$. So

$$\hat{E}_0 = \arg_{\hat{E}}\left\{\max\left|C_i(\hat{E})\right|\right\} \tag{3}$$

where $\hat{E}_0$ is the closest estimation of E. It is clear that equation (2) is the DFT of sequence A_i. Therefore, finding ε or equally finding E is equal to find the point that the DFT of sequence A_i is maximized and we have $\hat{\varepsilon} = \frac{\hat{E}_0}{L}$ where $\hat{\varepsilon}$ is the estimation of ε. Our estimation is not accurate since $\hat{E}_0$ that is obtained from (3) is an integer, but E may have fractional part. Therefore, E is in the $\hat{E}_0 - 0.5 \le E < \hat{E}_0 + 0.5$ interval and our

By choosing larger L and using proceeding novel algorithm, the error, which is introduced in above, will be decreased to the acceptable value.

III. Proposed Algorithm

Suppose that $M = PQ$ where P and Q are integer and $2 \le Q \le N$. Our proposed estimation algorithm is performed in the following steps:

1- Form H_Q matrix that has Q row and LP column from received sequence $R(n)$

$$H_Q = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_P \\ \mathbf{r}_{P+1} & \mathbf{r}_{P+2} & \cdots & \mathbf{r}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{r}_{(Q-1)P+1} & \mathbf{r}_{(Q-1)P+2} & \cdots & \mathbf{r}_{QP} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,LP} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,LP} \\ \vdots & \vdots & & \vdots \\ h_{Q,1} & h_{Q,2} & \cdots & h_{Q,LP} \end{bmatrix}$$

where $h_{i,j}$, $2 \le i \le Q$, $1 \le j \le LP$, is the element of H_Q and $\mathbf{r}_i$ is received sequence corresponding to transmitted sequence $\mathbf{t}_i$ (see fig1) on the assumption of noiseless system ($V(n) = 0$). From above, it is clear that

$$\mathbf{r}_i = \mathbf{r}_1 e^{j\frac{2\pi}{N}(i-1)L\varepsilon} \qquad 1 \le i \le M$$
$$r(n) = t(n) e^{j\frac{2\pi}{N}\varepsilon n} \qquad 1 \le n \le L \tag{4}$$

2-Compute D-point DFT of each column of H_Q. Choosing of D depends on the estimation accuracy that is defined for system to work properly. If $Q < D$, we insert zero between consecutive samples until the number of samples become equal to D. Therefore,

$$\mathbf{F}_i(k) = DFT_D \begin{bmatrix} h_{1,i} \\ h_{2,i} \\ \vdots \\ h_{Q,i} \end{bmatrix}^T \qquad -\frac{D}{2} - 1 \le k \le \frac{D}{2} \tag{5}$$

where $\mathbf{F}_i(k)$ is the D-point DFT of the ith column of H_Q. Note that if D is odd then replace D with $D-1$ in boundary of (5).

3- Search for k that maximizes $\left|\mathbf{F}_i(k)\right|$

$$K_i = \arg_k \{\max|\mathbf{F}_i(k)|\} \quad (6)$$

4- Compute the average of K_i to cancel the destructive effect of the noise.

$$K = \frac{1}{LP}\sum_{i=1}^{LP} K_i \quad (7)$$

5- Then $\hat{\varepsilon}$, the estimation of ε, comes from

$$\hat{\varepsilon} = \frac{K}{LP} \quad (8)$$

It is clear from (8) that accuracy of estimation is proportional to LP. Therefore, we can choose P so that the accuracy becomes acceptable.

6- Compensate frequency offset by estimated $\hat{\varepsilon}$:

$$R'(n) = R(n).e^{-j\frac{2\pi}{N}\hat{\varepsilon}n} \quad (9)$$

Remained offset in $R'(n)$ is $\varepsilon_r = \varepsilon - \hat{\varepsilon}$, which can be removed in next stages.

To achieve better accuracy, we can repeat these six steps by choosing larger value of P. Referring to (8), it is clear that accuracy is proportional to P; therefore, choosing larger value for P leads to more accurate estimation. Nevertheless, we could not choose P very large, because εLP should have the same range as k in (5), otherwise ε will be out of estimation range. Since ε may be large in first stage, it is better to choose small P at first (for example $P = 1$), then in the next stages, increase the value of P to achieve better accuracy.

IV. Example

A practical example of an OFDM based system is IEEE802.11a [15]. The preamble structure of this system has been shown in fig 2. The packet preamble contains 10 identical short training symbols (each consist of 16 data samples) and 2 identical long training symbols (each contain 64 data samples). These symbols are used for packet detection, AGC, FO estimation, symbol timing synchronization and channel estimation. The short training symbols that exhibit periodic property are recommended to estimate symbol timing and FO. In this paper, we use short training symbols.

Our simulated system has $N = 64, L = 16$, and $M = 10 = 10 \times 1 = 5 \times 2$, therefore we can choose four different configuration for P and Q: ($P = 1, Q = 10$), ($P = 2, Q = 5$),($P = 5, Q = 2$), and ($P = 10, Q = 1$). These configurations are shown in figure 3. Frequency offset may be large in the first stage, so we choose minimum value for P to extend the range of estimation to the maximum value. Thus, in the first stage we choose configuration of fig 3a. By choosing this configuration, we have $Q = 10$, so each column of H_Q has ten samples. We insert adequate zero between samples until the number of them be equal to the number of subcarriers ($N = 64$). We can choose another value for upsampling, but this selected value leads to more simplicity to the proposed algorithm.

Performing proposed algorithm and computing the value of $\hat{K}$, $\hat{\varepsilon}$ can be calculated as

$$\hat{\varepsilon} = \frac{\hat{K}}{L} \quad (10)$$

We correct the FO in first stage by substitute $\hat{\varepsilon}$ in (9). Remaining error $\varepsilon_r = \varepsilon - \hat{\varepsilon}$ is smaller than ε, therefore we can use larger P to increase accuracy in the next stage.

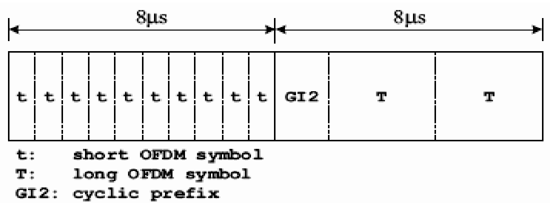

Fig. 2 Structure of preamble in IEEE802.11a standard

To compensate ε_r, we can use configuration of Fig. 3b or 3c. if we choose fig 3c and compute $\hat{\hat{K}}$, $\hat{\hat{\varepsilon}}$ will be:

$$\hat{\hat{\varepsilon}} = \frac{\hat{\hat{K}}}{5L} \quad (11)$$

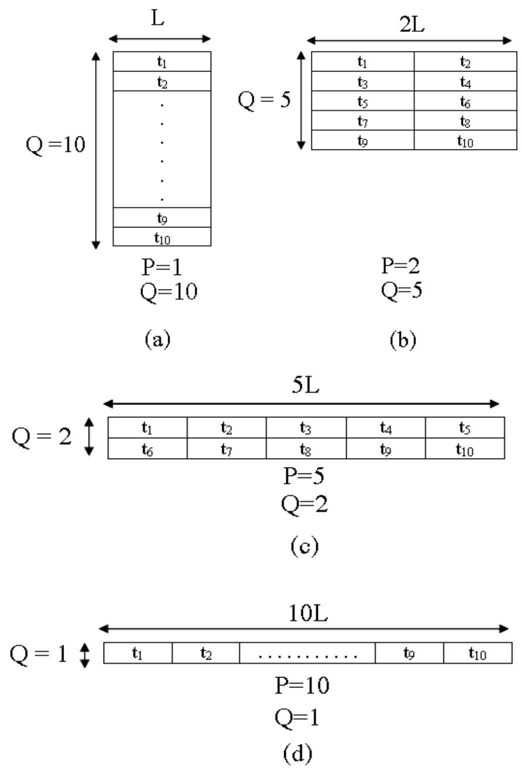

Fig. 3 Different configurations of P and Q

Note that we should insert 62 zero samples between samples in this stage.

FO estimation that comes from these two stages is $\varepsilon' = \hat{\varepsilon} + \hat{\hat{\varepsilon}}$.

It should be mentioned that by choosing proper value for L and M, we could select different values for P and Q dependent on frequency offset range, accuracy of estimation, and the number of iteration. For example, we can choose $M = 2^l$, so $P = 2^i$, and $Q = 2^{l-i}$ where $0 \le i \le l$. This selection will increase the different possible option for P and Q. In addition, this method can be used for the other packet-based OFDM systems such as IEEE802.16.

V. SIMULATON RESULTS

In order to verify the performance of the proposed algorithm, computer simulation is used under the multipath fading environments. Fig 4 shows the Mean-Square-Error (MSE) of the estimated frequency offset in IEEE802.11a system ($N = 64$, $L = 16$, $M = 10$) for the proposed frequency synchronization algorithm with two different number of FFT points, D=64,128. Other conventional methods (MNC [7], MMNC [8], and MML [17]) also are included for comparison. The HIPERLAN/2 channel model A [16] is used here. We observe that the proposed method with D=128 points DFT has considerable better performance than conventional methods. The proposed algorithm with D=64 has better performance than conventional methods for SNR<12dB.

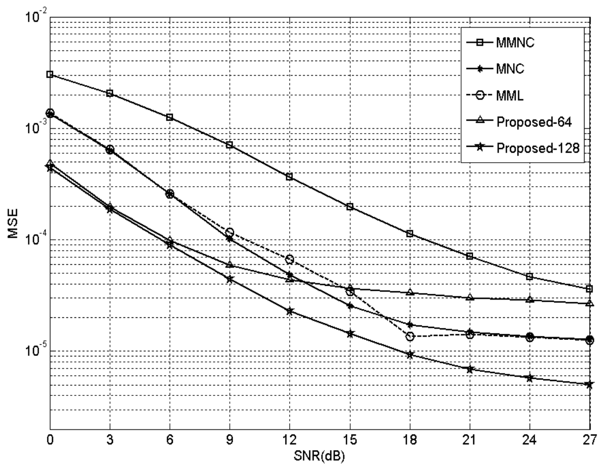

Fig. 4 MSE of FO estimation in fading channel

VI. CONCLUSION

In this paper, a new frequency estimation algorithm based on FFT is proposed. The proposed algorithm has low complexity to implement and can be repeated to obtain acceptable accuracy. Also the algorithm corrects the integer and fractional part of FO jointly. Simulation results show that the accuracy of the algorithm is better than other conventional methods and can be further improved by repeating it two or more times.

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