

Variable Structure Model Reference Adaptive Control for Vehicle Steering System

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Abstract—A variable structure model reference adaptive control (VS-MRAC) strategy for active steering assistance of a two wheel steering car is proposed. An ideal steering system with fixed properties and moving on an ideal road is used as the reference model, and the active steering assistance system is forced to attain the same behavior as the reference model. The proposed system can treat the nonlinear relationships between the side slip angles and lateral forces on tire, and the uncertainties on friction of the road surface, whose compensation are very important under critical situations. Simulation results show improvements on yaw rate and side slip.

Keywords—Variable Structure, Adaptive Control, Model reference, Active steering assistance.

I. INTRODUCTION

Motion control of autonomous vehicles has many problems to be considered according to the freedom of motion. Fig. 1 shows the dynamics model of a four wheel vehicle which is assumed to be a rigid body. It has six degrees of freedom. F1: up and down motion along to z-axis, F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, F4: rolling motion around an x-axis, F5: pitching motion around a y-axis and F6: yawing motion around a z-axis. Among them, in order to consider the automatic steering system, it will be derived the mathematical model with three degree of freedom i.e. F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, and F6: yawning motion around a z-axis, because it is the most important system for motion control of autonomous vehicle. Although the concept of a four-wheel steering (4WS) system has been introduced to enhance vehicle handling [1, 2], the need for 4WS is not so obvious. Some researchers have shown disadvantages on 4WS vehicles [3, 4]. In this paper, we will employ two–wheel steering (2WS) vehicle for cost and implementation issues in active car steering. However, it is generally fair to say that 4WS vehicle is easier to handle than the 2WS vehicle.

In this paper we investigate the Variable Structure Model Reference Adaptive Controller (VS-MRAC) [5], for a 2WS system.

The basic concept of the variable structure control is that of sliding mode control. Switching control functions are generally designed to generate sliding surface, or sliding modes, in the state space [6]. When this is attained the switching functions keep the trajectory on the sliding surfaces and the closed loop system becomes insensitive, to a certain extent, to parameter variations and disturbances.

A number of other control strategies for passenger vehicle are given in references [7, 8]. In addition, LMI approach to 4WS vehicle is investigated in reference [9].

This paper is organized as follows: in section II we introduce the model of steering system, section III subscribe the VS-MRAC controller and using steering system in this controller, section IV we modify our matrices that get in section III and show the simulation results in ICY and WET roads and in section V the conclusion is expressed.

II. DYNAMIC MODEL OF STEERING MOTION

The feature of car steering dynamics in horizontal plane is described by Fig. 2 [10, 11]. In the horizontal plane of Fig. 2 an inertial fixed coordinate system (X,Y) is shown together with a vehicle fixes coordinate system (x,y) that is rotated by a ψ. In the dynamic equations the r = ψ will appear as a state variable. Assuming that:

\[ \beta_f = \beta_{f1} = \beta_{f2} = \delta - \beta - \frac{l_f r}{V}, \]
\[ \beta_r = \beta_{r1} = \beta_{r2} = -\beta + \frac{l_r r}{V}, \]

and \(|\beta_f| << 1, |\beta_r| << 1, |\delta| << 1\), then the two wheel model [12]. Fig. 3 can be regarded as the equivalent model to the four wheel model (Fig. 2).
The equations of motion for three degrees of freedom in the horizontal plane are:

1. for longitudinal motion

   $$-mV(\beta + r)\sin \beta + mV\cos \beta = f_x$$  \hspace{1cm} (2)

2. for lateral motion

   $$mV(\beta + r)\cos \beta + mV\sin \beta = f_y$$  \hspace{1cm} (3)

3. for yaw motion

   $$\dot{r} = m_z$$  \hspace{1cm} (4)

It is obtained that from (2) to (4)

$$\begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} = \begin{bmatrix} -\sin \delta & 0 \\ \cos \delta & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_f \\ f_r \end{bmatrix}$$  \hspace{1cm} (1)

The side force $f_y$ is known to be the nonlinear function of the tire side slip angles $\beta_f$, $\beta_r$, i.e.

$$f_y = f_f(\beta_f) + f_r(\beta_r)$$  \hspace{1cm} (6)

Two wheel models (5) and (6) are nonlinear and we will introduce the additional assumptions [11] as follows.

(Assumption 1): the sideslip angle $\beta$ is assumed to be small. Then (5) become

$$\begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} = \begin{bmatrix} -\beta & 0 & f_f \\ 1 & 0 & f_y \\ 0 & 1 & m_z \end{bmatrix}$$  \hspace{1cm} (7)

The uncertain parameters in this model are $m$, $I$, $V$ and $\mu$. Solving (11) for $\beta$ and $r$ rearranging terms yields the nonlinear state space model:
Consider a linear time invariant plant with unknown parameters, which their bounds are known. Let the plant be of order with accessible states and described by the differential equation

\[ \dot{X} = AX + bu \]  

Where \( n \times n \) matrix \( A \) and vector \( b \) are unknown, and \((A,b)\) is controllable.

The reference model is characterized by the linear time invariant differential equation

\[ \dot{X}_m = A_m X_m + b_m r \]  

Where is \( A_m \) an \( n \times n \) asymptotically stable matrix, \( b_m \) is a known vector, and \( r \) is a bounded reference input. The purpose is to find control vector \( u \) such that the state error

\[ e = X - X_m \]  

Exponential tends to zero in finite time. In steering systems both of matrix \( A \) and vector \( b \) are unknown because of adhesion of road and cornering stiffness of front and rear tires. So the vector \( b_m \) of the reference model can be chosen as

\[ b_m = b q^* \]  

Where \( q^* \) is an unknown scalar and also it can change but the sign of it cannot be altered. It is further assumed that an unknown \( n \times n \) matrix \( \Theta^* \) exist such that

\[ A + b \Theta^* = A_m \]  

The control vector to the plant is generated introducing control law

\[ u = \sigma (\beta X + r) \]
\[ A = \begin{bmatrix} -0.4274 & -0.9957 \\ 2.8681 & -0.5427 \end{bmatrix}, \quad b = \begin{bmatrix} 0.2137 \\ 4.6368 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = [0]_{2 \times 1} \]

(28)

And also for vehicle case in wet road we have:
\[ m = 1170\text{Kg}, \quad l_f = 1.4m, \quad l_r = 1.8m, \]
\[ V = 30\text{m/s} = 108\text{Km/hr}, \quad i = 1.341m^2 \]
\[ c_f = 25\text{KN/m}\text{rad}, \quad c_r = 25\text{KN/m}\text{rad}, \quad \mu = 0.7 \]

(29)

And our matrices are as follow:
\[ A = \begin{bmatrix} -0.9972 & -0.99 \\ 6.6933 & -1.2663 \end{bmatrix}, \quad b = \begin{bmatrix} 0.4986 \\ 10.8192 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = [0]_{2 \times 1} \]

(30)

In this example our steering input shows in Fig. 4. Now we consider the results in icy and wet roads. In all figures green curve is model, blue curve is MRAC and red curve is Passive (open loop) control.

V. CONCLUSION

According to the Figs. 5 to 12 we see that the MRAC controller follow the model case in good condition. And we see that errors of this following are small and we can ignore them. On the other words, errors between model and MRAC are going to zero. Then we achieve our target and controller provide good situation for driver.

REFERENCES
