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Variable Structure Model Reference Adaptive Control for Vehicle Steering System

Ardeshir Karami Mohammadi, and Mohammadreza Saee

***Abstract*—**A variable structure model reference adaptive control (VS-MRAC) strategy for active steering assistance of a two wheel steering car is proposed. An ideal steering system with fixed properties and moving on an ideal road is used as the reference model, and the active steering assistance system is forced to attain the same behavior as the reference model. The proposed system can treat the nonlinear relationships between the side slip angles and lateral forces on tire, and the uncertainties on friction of the road surface, whose compensation are very important under critical situations. Simulation results show improvements on yaw rate and side slip.

***Keywords*—**Variable Structure, Adaptive Control, Model reference, Active steering assistance.

I. Introduction

MOTION control of autonomous vehicles has many problems to be considered according to the freedom of motion. Fig. 1 shows the dynamics model of a four wheel vehicle which is assumed to be a rigid body. It has six degrees of freedom. F1: up and down motion along to z-axis, F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, F4: rolling motion around an x-axis, F5: pitching motion around a y-axis and F6: yawing motion around a z-axis. Among them, in order to consider the automatic steering system, it will be derived the mathematical model with three degree of freedom i.e. F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, and F6: yawing motion around a z-axis, because it is the most important system for motion control of autonomous vehicle. Although the concept of a four-wheel steering (4WS) system has been introduced to enhance vehicle handling [1, 2], the need for 4WS is not so obvious. Some researchers have shown disadvantages on 4WS vehicles [3, 4]. In this paper, we will employ two–wheel steering (2WS) vehicle for cost and implementation issues in active car steering. However, it is generally fair to say that 4WS vehicle is easier to handle than the 2WS vehicle.

In this paper we investigate the Variable Structure Model Reference Adaptive Controller (VS-MRAC) [5], for a 2WS system.

A. Karami mohammadi is with the Department of Mechanical Engineering of Islamic Azad University of Iran, Karaj Branch (corresponding author to provide e-mail: akaramim@yahoo.com).

M. Saee, is with the Department of Mechanical Engineering of Islamic Azad University of Iran, Karaj Branch (e-mail: akaramim@yahoo.com).

The basic concept of the variable structure control is that of sliding mode control. Switching control functions are generally designed to generate sliding surface, or sliding modes, in the state space [6]. When this is attained the switching functions keep the trajectory on the sliding surfaces and the closed loop system becomes insensitive, to a certain extent, to parameter variations and disturbances.

A number of other control strategies for passenger vehicle are given in references [7, 8]. In addition, LMI approach to 4WS vehicle is investigated in reference [9].

This paper is organized as follows: in section II we introduce the model of steering system, section III subscribe the VS-MRAC controller and using steering system in this controller, section IV we modify our matrices that get in section III and show the simulation results in ICY and WET roads and in section V the conclusion is expressed.

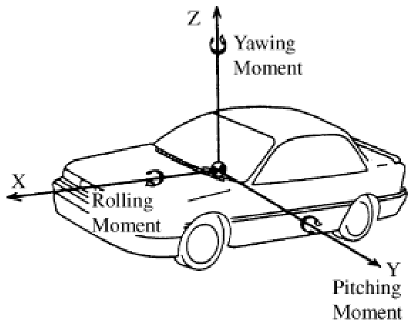

Fig. 1 Rigid-body model of four wheel vehicles

II. Dynamic Model of Steering Motion

The feature of car steering dynamics in horizontal plane is described by Fig. 2 [10, 11]. In the horizontal plane of Fig. 2 an inertial fixed coordinate system (X,Y) is shown together with a vehicle fixes coordinate system (x,y) that is rotated by a ψ. In the dynamic equations the $r=\dot{\psi}$ will appear as a state variable. Assuming that:

$$\beta_f = \beta_{f1} = \beta_{f2} = \delta - \beta - {l_f r}/{V} \ ,$$

$$\beta_r = \beta_{r1} = \beta_{r2} = -\beta + {l_f r}/{V} \ ,$$

and $|\beta_f| << 1, |\beta_r| << 1, |\delta| << 1$, then the two wheel model [12]. Fig. 3 can be regarded as the equivalent model to the four wheel model (Fig. 2).

The side forces $f_f = 2Y_f$, $f_r = 2Y_r$ are projected through the steering angle into chassis coordinate (x, y), where they appear as forces f_x , f_y and the torque m_z around a z-axis which is pointing upward from the center of gravity (P).

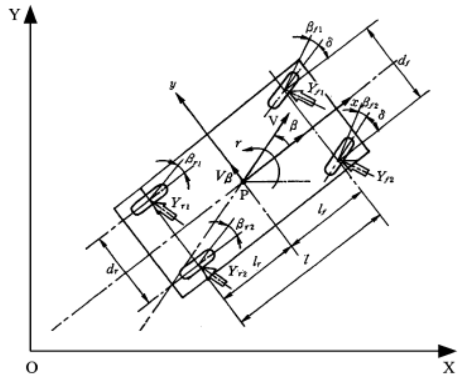

Fig. 2 Four wheel model for car steering

$$\begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} = \begin{bmatrix} -\sin\delta & 0 \\ \cos\delta & 1 \\ l_f\cos\delta & -l_r \end{bmatrix} \cdot \begin{bmatrix} f_f \\ f_r \end{bmatrix} \tag{1}$$

Via the dynamics model the forces cause state variables β ,V ,r . The equations of motions for three degrees of freedom in the horizontal plane are:

1. for longitudinal motion

$$-mV(\dot{\beta}+r)\sin\beta + m\dot{V}\cos\beta = f_x \tag{2}$$

2. for lateral motion

$$mV(\dot{\beta}+r)\cos\beta + m\dot{V}\sin\beta = f_y \tag{3}$$

3. for yaw motion

$$I\dot{r} = m_z \tag{4}$$

It is obtained that from (2) to (4)

$$\begin{bmatrix} mV(\dot{\beta}+r) \\ m\dot{V} \\ I\dot{r} \end{bmatrix} = \begin{bmatrix} -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} \tag{5}$$

The side force f_y is known to be the nonlinear function of the tire side slip angles β_f , β_r i.e.

$$f_y = f_f(\beta_f) + f_r(\beta_r) \tag{6}$$

Two wheel models (5) and (6) are nonlinear and we will introduce the additional assumptions [11] as follows.

(Assumption 1): the sideslip angle β is assumed to be small. Then (5) become

$$\begin{bmatrix} mV(\dot{\beta}+r) \\ m\dot{V} \\ I\dot{r} \end{bmatrix} = \begin{bmatrix} -\beta & 1 & 0 \\ 1 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} \tag{7}$$

(Assumption 2): the velocity is constant, $\dot{V} = 0$. Then, the second row of (7) is eliminated.

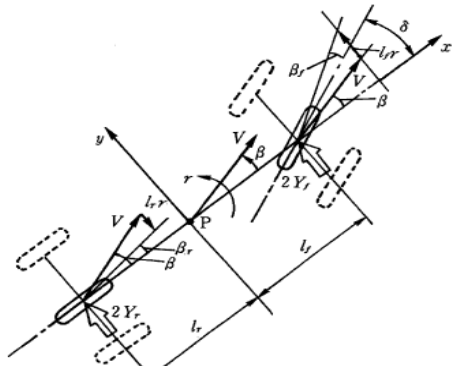

Fig. 3 Two wheel model for car steering

$$\begin{bmatrix} mV(\dot{\beta}+r) \\ I\dot{r} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_y \\ m_z \end{bmatrix} \tag{8}$$

The velocity V is treated as an uncertain constant parameter.

(Assumption 3): the nonlinear characteristic of (6) is approximated by nominal value of the tangent at $\beta_f = \beta_r = 0$ and small nonlinear functions i.e.

$$f_f(\beta_f) = c_f\mu(\beta_f + \Delta_1(\beta, r)) \quad , \qquad f_r(\beta_r) = c_r\mu(\beta_r + \Delta_2(\beta, r)), \tag{9}$$

Where $\Delta_i(\beta, r)$, $(i = 1,2)$ is C^∞ nonlinear function and $\|\Delta_i(\beta, r)\| << 1$ [13,14]. Typical experimental values of μ [11,15,16] are

$$\begin{array}{ll} \mu = 1 & dryroad \\ \mu = 0.7 & wetroad \\ \mu = 0.3 & icyroad \end{array} \tag{10}$$

The steering model follows from (7) to (9) and using (1) as

$$\begin{bmatrix} mV(\dot{\beta}+r) \\ I\dot{r} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ l_f & -l_r \end{bmatrix} \begin{bmatrix} c_f\mu(\overbrace{\delta - \beta - \frac{l_f r}{V}}^{\beta_f} + \Delta_1(\beta, r)) \\ c_r\mu(\overbrace{-\beta - \frac{l_r r}{V}}^{\beta_r} + \Delta_2(\beta, r)) \end{bmatrix} \tag{11}$$

The uncertain parameters in this model are m, I, V and μ. Solving (11) for $\dot{\beta}$ and $\dot{r}$ rearranging terms yields the nonlinear state space model:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \delta + \begin{bmatrix} g_1(\beta, r) \\ g_2(\beta, r) \end{bmatrix} \mu \quad (12)$$

Where

$$a_{11} = -\frac{c_f + c_r}{\tilde{m}V}, \quad a_{12} = -1 + \frac{c_r l_r - c_f l_f}{\tilde{m}V^2}$$

$$a_{21} = \frac{c_r l_r - c_f l_f}{\tilde{I}}, \quad a_{22} = -\frac{c_f l_f^2 + c_r l_r^2}{\tilde{I}V}$$

$$b_1 = \frac{c_f}{\tilde{m}V}, \quad b_2 = \frac{c_f l_f}{\tilde{I}}$$

$$g_1(\beta, r) = c_f \Delta_1(\beta, r) + c_r \Delta_2(\beta, r)$$
$$g_2(\beta, r) = l_f c_f \Delta_1(\beta, r) - l_r c_r \Delta_2(\beta, r) \quad (13)$$
$$\tilde{m} = \frac{m}{\mu}, \quad \tilde{I} = \frac{I}{\mu}$$

III. Control Strategy – Variable Structure Adaptive Controller

Consider a linear time invariant plant with unknown parameters, which their bounds are known. Let the plant be of $n-th$ order with accessible states and described by the differential equation

$$\dot{X} = AX + bu \quad (14)$$

Where $n \times n$ matrix A and vector b are unknown, and (A, b) is controllable.
The reference model is characterized by the linear time invariant differential equation

$$\dot{X}_m = A_m X_m + b_m r \quad (15)$$

Where is A_m an $n \times n$ asymptotical stable matrix, b_m is a known vector, and r is a bounded reference input. The purpose is to find control u such that the state error

$$e = X - X_m \quad (16)$$

Exponential tends to zero in finite time. In steering systems both of matrix A and vector b are unknown because of adhesion of road and cornering stiffness of front and rear tires. So the vector b_m of the reference model can be chosen as

$$b_m = bq^* \quad (17)$$

Where q^* is a unknown scalar and also it can change but the sign of it cannot be altered. It is further assumed that an unknown $m \times n$ matrix θ^* exist such that

$$A + b\theta^* = A_m \quad (18)$$

The control u to the plant, is generated introducing control law

$$u = \sigma(\beta X + r) \quad (19)$$

Where n dimensional raw feedback vector β, with the elements β_i are adjust using VS approach by designing switching functions β_i as described in following.

$$\beta_i = -\bar{\theta}_i \operatorname{sgn}\left(b^T.P.e.x_i\right), \quad \bar{\theta}_i > \left|\theta_i^*\right| \quad (20)$$

Where

$$A_m^T P + P A_m = -Q_0 \quad (21)$$

Is a Lyaponov equation with Lyaponov function $V = e^T.P.e$ .And Q_0 is a positive definite symmetric matrix. Also switching function σ is define as follow

$$\sigma = -\bar{q} \operatorname{sgn}\left(b_m^T P e u^0\right) \operatorname{sgn}\left(q^*\right), \quad \bar{q} > \left|q^*\right| \quad (22)$$

You can refer to [5] for the proof of equations.

IV. VS-MRAC for Steering System - Simulation Results

In steering system we have three uncertainties. μ as adhesion of road and c_f, c_r as cornering stiffness of front and rear tires respectively. Although these are our uncertainties, we know the range of variation of them. For adhesion of road we have

$$0 \le \mu \le 1 \quad (23)$$

And also for cornering stiffness of front and rear tires we have

$$25\,{}^{KN}\!/_{rad} \le c_f \le 150\,{}^{KN}\!/_{rad}$$
$$25\,{}^{KN}\!/_{rad} \le c_r \le 150\,{}^{KN}\!/_{rad} \quad (24)$$

For showing our results we let these parameters as follow for model and vehicle cases. In model case all uncertainties are imagine. So for model case we have:

$$m = 1170Kg,\ l_f = 1.4m,\ l_r = 1.8m, \quad (25\text{-}1)$$
$$V = 30\,{}^{m}\!/_{s} = 108\,{}^{Km}\!/_{hr},\ i = 1.341m^2 \quad (25\text{-}2)$$
$$c_f = 6\,{}^{KN}\!/_{rad},\ c_r = 10\,{}^{KN}\!/_{rad},\ \mu = 1 \quad (25\text{-}3)$$

So our model matrix according to equations (12) and (13) is:

$$A_m = \begin{bmatrix} -0.4558 & -0.9906 \\ 6.2971 & -0.6436 \end{bmatrix}, b_m = \begin{bmatrix} 0.1709 \\ 3.7094 \end{bmatrix},$$
$$C_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_m = [0]_{2\times 1} \quad (26)$$

For vehicle case in icy road we have

$$m = 1170Kg,\ l_f = 1.4m,\ l_r = 1.8m,$$
$$V = 30\,{}^{m}\!/_{s} = 108\,{}^{Km}\!/_{hr},\ i = 1.341m^2 \quad (27)$$
$$c_f = 25\,{}^{KN}\!/_{rad},\ c_r = 25\,{}^{KN}\!/_{rad},\ \mu = 0.3$$

So we have

$$A = \begin{bmatrix} -0.4274 & -0.9957 \\ 2.8681 & -0.5427 \end{bmatrix}, b = \begin{bmatrix} 0.2137 \\ 4.6368 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = [0]_{2\times 1} \quad (28)$$

And also for vehicle case in wet road we have:

$$m = 1170Kg, l_f = 1.4m, l_r = 1.8m,$$

$$V = 30 \, m/s = 108 \, Km/hr, i = 1.341m^2 \quad (29)$$

$$c_f = 25 \, KN/rad, c_r = 25 \, KN/rad, \mu = 0.7$$

And our matrices are as follow:

$$A = \begin{bmatrix} -0.9972 & -0.99 \\ 6.6933 & -1.2663 \end{bmatrix}, b = \begin{bmatrix} 0.4986 \\ 10.8192 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = [0]_{2\times 1} \quad (30)$$

In this example our steering input shows in Fig. 4. Now we consider the results in icy and wet roads. In all figures green curve is model, blue curve is MRAC and red curve is Passive (open loop) control.

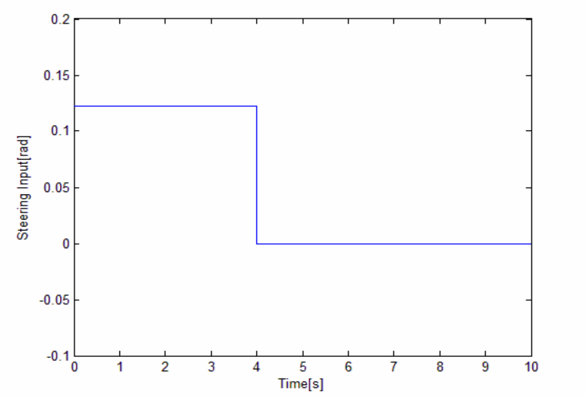

Fig. 4 Steering input

V. Conclusion

According to the Figs. 5 to 12 we see that the MRAC controller follow the model case in good condition. And we see that errors of this following are small and we can ignore them. On the other words, errors between model and MRAC are going to zero. Then we achieve our target and controller provide good situation for driver.

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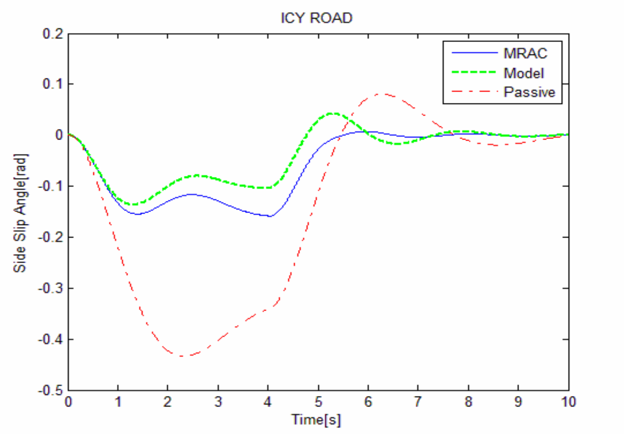

Fig. 5 Side slip angle for icy road

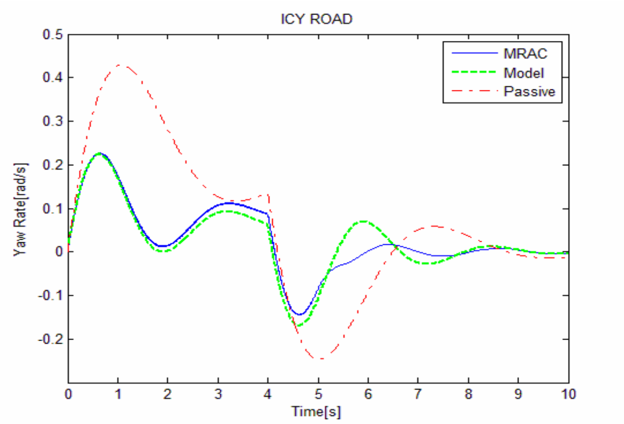

Fig. 6 Yaw rate for icy road

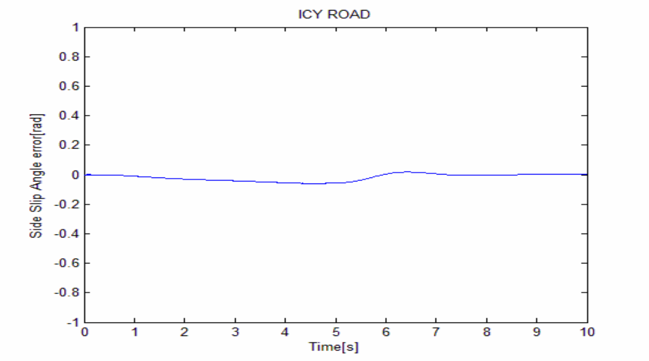

Fig. 7 Error of side slip angle for icy road

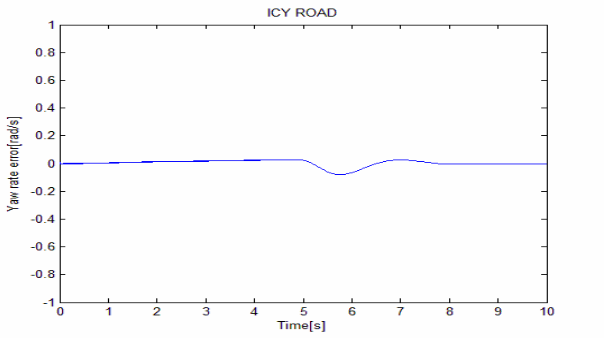

Fig. 8 Error of yaw rate for icy road

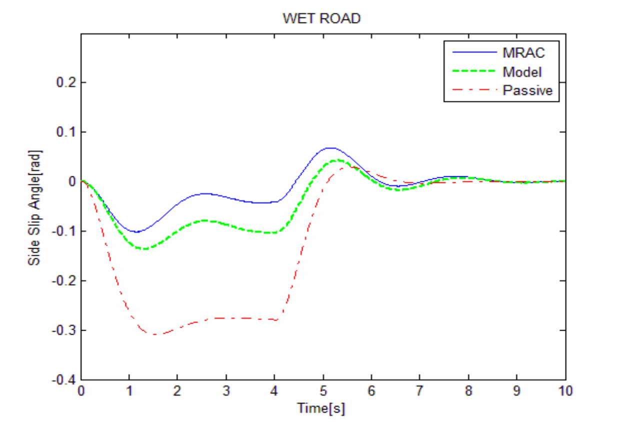

Fig. 9 Side slip angle for wet road

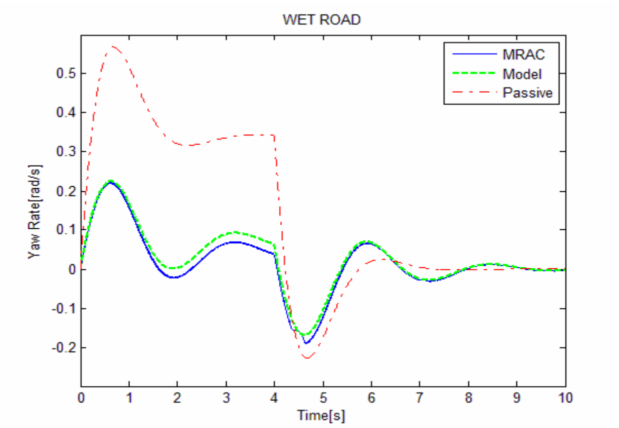

Fig. 10 Yaw rate for wet road

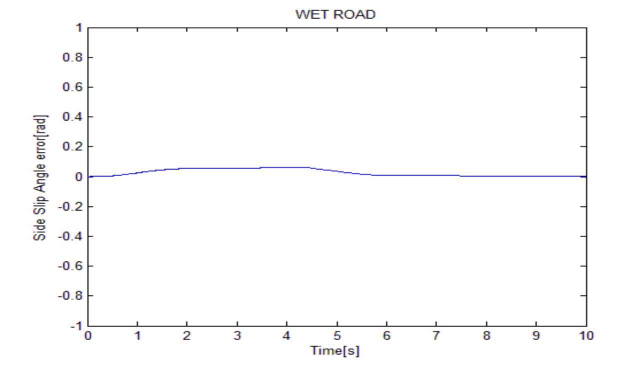

Fig. 11 Error of side slip angle for wet road

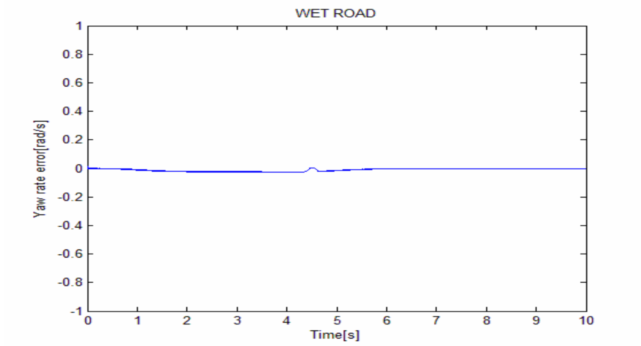

Fig. 12 Error of yaw rate for wet road