

# Smith Predictor Design by CDM for Temperature Control System

Roengruen P., Tipsuwanporn V., Puawade P. and Numsomran A.

**Abstract**—Smith Predictor control is theoretically a good solution to the problem of controlling the time delay systems. However, it seldom gets use because it is almost impossible to find out a precise mathematical model of the practical system and very sensitive to uncertain system with variable time-delay. In this paper is concerned with a design method of smith predictor for temperature control system by Coefficient Diagram Method (CDM). The simulation results show that the control system with smith predictor design by CDM is stable and robust whilst giving the desired time domain system performance.

**Keywords**— CDM, Smith Predictor, temperature process

## I. INTRODUCTION

THE Smith Predictor [1] is a popular and very effective long dead-time compensator for stable processes. The main advantage of the Smith Predictor method is that the time delay is effectively taken outside the control loop in the transfer function relating the process output to setpoint. However, this method introduces extreme instability into the system for the uncertain system, unstable system and variable delay system. Furnkawa and Shimemura [2] augmented the scheme with an observer, Watanabe and Ito [3] deliberately replaced the known process by a mismatched process model, Gawthrop [4] used an adaptive least-square predictor. A simple algorithm of implementing self-tuning controller for first order system was present based on Smith Predictor control structure [5]. Huang [6] did the similar studies of self-tuning controller design, which used the infinite time integration of a quadratic function as the system performance index. These cited authors have thus solved the stabilization problem by involving greater complexity than the Smith Predictor. Also Hamamchi *et al* [7] proposed a Smith Predictor design by Coefficient Diagram Method (CDM) to achieve a better performance. [8]

Coefficient Diagram Method (CDM) [9] is a method where

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by the designer can obtain the characteristic polynomial of the closed loop system efficiently taking a good balance of stability, response and robustness. The strength of CDM lies in that the simplest and robust controller under practical limitations can be found for any plant [9]-[10]. Such, CDM is less sensitive to disturbances and bounded uncertainties resulted from parameter variations [11]-[12].

The rest of the paper organized as follows. In section II, gives overview of the traditional of Smith Predictor. Section III, explain concept of CDM design procedure. Section IV; introduce structure of Modified Smith Predictor design by CDM. Then simulation results are giving to illustrate the performance of method proposed for temperature control system in section V. Finally, conclusions are giving in section VI

## II. OVERVIEW OF SMITH PREDICTOR

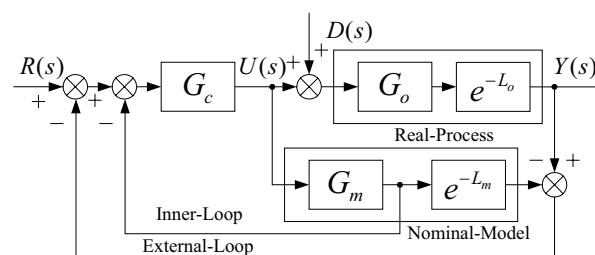


Fig.1 The conventional Smith Predictor structure

Smith Predictor control [1] is a feedback control scheme that has an inner loop as shown in Fig.1.  $G_o$  denotes a stable, strictly proper rational function characterizing the delay-free part of the plant.  $L_o$  denotes a positive constant standing for the time-delay.  $G_m$  and  $L_m$  are nominal model of  $G_o$  and  $L_o$ , respectively, obtained through modeling process.  $G_c$  denotes a rational function characterizing the compensator called primary controller. The inner loop works to eliminate the actual delayed output as well as to feed the predicted output to the primary controller. This makes it possible to design the primary controller assuming no time-delay in the control loop. From Fig.1, transfer function of conventional Smith Predictor can be writing as:

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_o(s)e^{-L_o s}}{1 + G_c(s)G_m(s) + G_c(s)[G_o(s)e^{-L_o s} - G_m(s)e^{-L_m s}]} \quad (1)$$

$$\frac{Y(s)}{D(s)} = \frac{G_o(s)e^{-L_o s} [1 + G_c(s)G_m(s)(1 - e^{-L_m s})]}{1 + G_c(s)G_m(s) + G_c(s)[G_o(s)e^{-L_o s} - G_m(s)e^{-L_m s}]} \quad (2)$$

In the case of  $G_o = G_m$  and  $L_o = L_m$  the transfer function can be writing as:

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_o(s)e^{-L_o s}}{1 + G_c(s)G_m(s)} \quad (3)$$

$$\frac{Y(s)}{D(s)} = \frac{G_o(s)e^{-L_o s} [1 + G_c(s)G_m(s)(1 - e^{-L_m s})]}{1 + G_c(s)G_m(s)} \quad (4)$$

### III. CONCEPT OF CDM

The CDM [9] is one of the methods of a controller design using polynomial approach. This method uses polynomials for system representation. By denominator and numerator of the transfer function are consider independently from each other. The CDM is a technique to arrange the poles of a closed loop transfer function, in order to get wanted response in the time domain. The arrangement of a suitable pole is get using to design parameter, stability index ( $\gamma_i$ ) and equivalent time constant ( $\tau$ ).

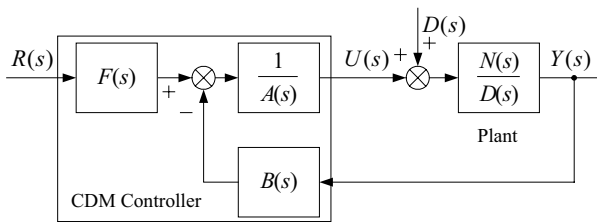


Fig.2 Standard block diagram of CDM control system

Fig. 2 represents the standard block diagram of control system designed by CDM. It is composed of plant and CDM controller. The polynomials form of the plant and the controller generally is write respectively in the form [9].

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_0 \quad (5)$$

$$N(s) = b_m s^m + b_{m-1} s^{m-1} + \dots + b_0$$

and

$$A(s) = l_i s^i + l_{i-1} s^{i-1} + \dots + l_0 \quad (6)$$

$$B(s) = k_i s^i + k_{i-1} s^{i-1} + \dots + k_0$$

$$F(s) = k_0$$

where  $m \leq n$  and  $i \leq n$

From Fig. 2 the characteristic polynomial define as

$$P(s) = A(s)D(s) + B(s)N(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad (7)$$

where  $a_0, a_1, \dots, a_n$  are the real coefficients.

And the stability index ( $\gamma_i$ ), the equivalent time constant ( $\tau$ ) and the stability limit ( $\gamma_i^*$ ) are defined as

$$\gamma_i = \frac{a_i^2}{(a_{i-1} a_{i+1})} ; i = 1, \dots, n-1 \quad (8)$$

$$\tau = \frac{a_1}{a_0} \quad (9)$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} ; \gamma_o = \gamma_n = \infty \quad (10)$$

The equivalent time constant determines the time response speed. The relation between the settling time and the equivalent time constant is consider according to the standard Manabe form [9]. If  $t_s$  denotes the desired settling time, this relation expresses the choice of  $\tau$  by  $\tau = t_s / (2.5 \sim 3)$ . The stability index specifies the stability and the waveform of the time response. The variation of the stability index due to plant parameter variation designates the robustness property [9]. According to the Manabe form, the stability index are chosen as

$$\gamma_1 = 2.5, \gamma_i = 2 ; i = 2 \sim (n-1), \gamma_o = \gamma_n = \infty \quad (11)$$

The standard values of stability index according Manabe form in Eq. (11) can be use to design the controller if the following condition is satisfied.

$$p_k / p_{k-1} > \tau / (\gamma_{n-1} \gamma_{n-2} \dots \gamma_1) \quad (12)$$

Where  $p_k$  and  $p_{k-1}$  are the coefficients of the plant at order  $k^{th}$  and  $(k-1)^{th}$ . If the above condition is not satisfied, we can first increase  $\gamma_{n-1}$  then  $\gamma_{n-2}$  and so on, until Eq. (12) is satisfied. From Eqs. (8) ~ (10), the characteristic polynomial to be used to design the parameters of a controller is

$$P_t(s) = a_0 \left\{ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}} \right) (\tau s)^i + \tau s + 1 \right\} \quad (13)$$

By equating the characteristic polynomial (7) with a controller included to the characteristic polynomial (13) resulting from the known equivalent time constant ( $\tau$ ) and stability index ( $\gamma_i$ ), the parameters of a controller are then obtained.

From the CDM standard block diagram, it can be rearranged in structure of two degree of freedom [13] as shown in Fig. 3 where the controller  $G_c(s)$  and the pre-filter  $G_f(s)$  are

$$G_c(s) = \frac{B(s)}{A(s)} \quad (14)$$

$$G_f(s) = \frac{F(s)}{B(s)} \quad (15)$$

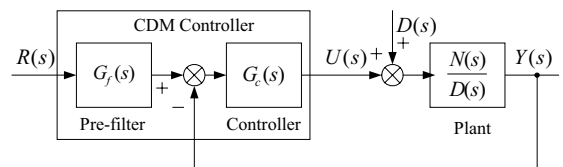


Fig. 3 Rearranged block diagram of CDM control systems.

#### IV. MODIFIED SMITH PREDICTOR USING CDM

From Fig.4 represents the Modified Smith Predictor using CDM. It is composed pre-filter  $G_{c1}(s)$ , controller  $G_{c2}(s)$  in forward-loop and controller  $G_{c3}(s)$  in inner-loop. Assuming the real-process and nominal-model match exactly, the closed loop transfer function of the Modified Smith Predictor using CDM as per the Fig.4 is given by

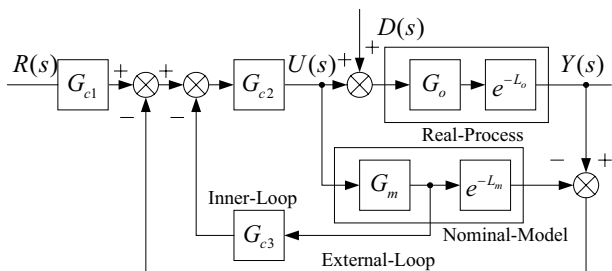


Fig.4 Modified Smith Predictor using CDM

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}(s)G_o(s)e^{-L_o s}}{1 + G_{c2}(s)G_{c3}(s)G_m(s)} \quad (16)$$

$$\frac{Y(s)}{D(s)} = \frac{G_o(s)e^{-L_o s} [1 + G_{c2}(s)G_m(s)(1 - e^{-L_m s})]}{1 + G_{c2}(s)G_{c3}(s)G_m(s)} \quad (17)$$

Where  $G_{c1}(s) = F(s)$ ,  $G_{c2}(s) = 1/A(s)$  and  $G_{c3}(s) = B(s)$ . As it is seen from Eqs. (16) ~ (17), the characteristic polynomial of closed loop system is time delay free. Hence, the controller parameters of  $G_{c1}(s)$ ,  $G_{c2}(s)$  and  $G_{c3}(s)$  can be found using only the time delay free part of plant transfer function. According structure of two degree of freedom in Fig.3 and Eqs. (14) ~ (15), the closed loop transfer function can be writing as :

$$\frac{Y(s)}{R(s)} = \frac{G_f(s)G_c(s)G_o(s)e^{-L_o s}}{1 + G_c(s)G_m(s)} \quad (18)$$

$$\frac{Y(s)}{D(s)} = \frac{G_o(s)e^{-L_o s} [1 + G_c(s)G_m(s)(1 - e^{-L_m s})]}{1 + G_{c2}(s)G_m(s)} \quad (19)$$

#### V. SIMULATION RESULTS

The In this section, the simulation of the Modified Smith Predictor design by CDM use to control the temperature of oven process in laboratory show in Fig.5. By changing the control signal 10% from operating point at 53 °C. The open-loop response of the temperature process obtained from the experiment is show in Fig.6 and its transfer function found that

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{K}{Ts + 1} e^{-L_o s} = \frac{62.78}{1020s + 1} e^{-30s} \quad (20)$$

where  $T$ ,  $L_o$  and  $K$  are the time constant, dead time and gain of the process respectively.



Fig. 5 Temperature process

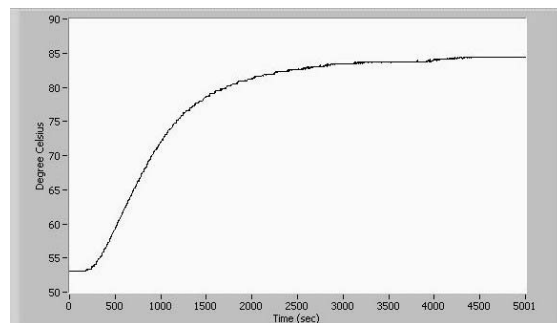


Fig. 6 Open-Loop response

In the following examples, the temperature of oven process to be controlled is 63 °C which change 10 °C from operating point and the results will be show in three parts. First is the system performance of the temperature process for various stability index ( $\gamma_i$ ). Next, the step response due to comparison between proposed system and internal model control (IMC). Finally, the illustration of robustness property due to the variation of  $T$ ,  $L_o$  and  $K$ .

##### A. System Performance

According to design procedure in section III and IV, the model in Eq. (20) is use to design CDM controller by neglect dead time. Therefore, parameter of CDM controller for this case is equivalent with PI controller. Then design CDM controller by define desired settling time  $t_s = 1,000$  sec, so as the equivalent time constant  $\tau = 200$ . Next, define stability index corresponding stability limit condition in Eq. (10). In this case, the comparison performance for various values of stability index is show that by define  $\gamma_i = 2.5, 5, 7.5, 10$  respectively.

TABLE I  
 Coefficients of CDM-PI Controller for Various Values of Stability Index

Stability index	$K_p$	$K_i$
$\gamma_i = 2.5$	0.1869	0.001
$\gamma_i = 5$	0.3898	0.002
$\gamma_i = 7.5$	0.5926	0.003
$\gamma_i = 10$	0.7954	0.004

The step responses of Modified Smith Predictor using CDM-PI for various values of stability index are show in Fig.7, and the control signals are show in Fig. 8. It is apparent that parameter assignment according to the standard Manabe form satisfied for case of settling time while overshoot more

than 0 %. The coefficient of CDM-PI controller and performances values of time response for various stability index are also summarized in Table I and II respectively.

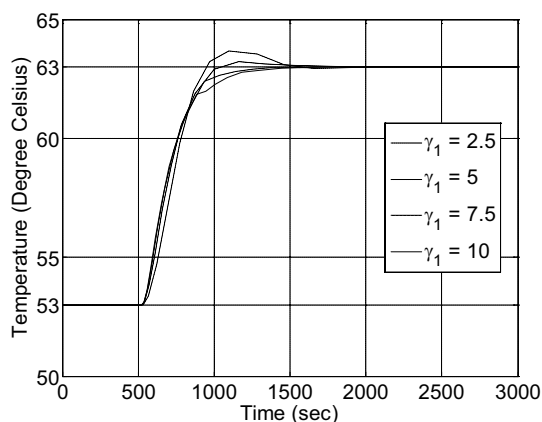


Fig. 7 Step response of system with various stability index

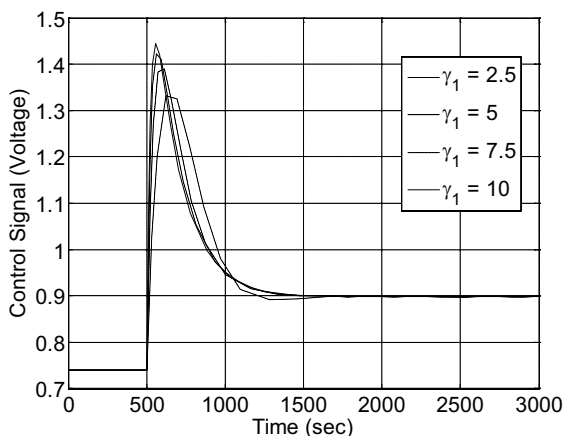


Fig. 8 Control signal of system with various stability index

TABLE II

Comparison Performance Values of The Time Response Curves Shown in Fig. 7 and 8

Stability index	settling time	max overshoot	max u
$\gamma_I = 2.5$	920 s	6.8 %	1.3315 V
$\gamma_I = 5$	680 s	2.1 %	1.39 V
$\gamma_I = 7.5$	640 s	0 %	1.423 V
$\gamma_I = 10$	715 s	0 %	1.445 V

### B. Comparison Between CDM-PI and IMC-PI

In this part, the performance comparison between CDM-PI and IMC-PI is shown. In this case, IMC-PI controller is designed by conventional method that approximate  $e^{-L_0s} \approx 1 - L_0s$  and define filter parameter ( $\tau_f$ ) to be five as fast as the open loop response. Hence, from Eq. (20)  $\tau_f = 204$  and parameter of PI controller is  $K_p = 0.0693$   $K_i = 6.7984 \times 10^{-5}$ . From previous part, settling time is satisfied for all case of definition stability index but the response without overshoot is satisfied for case of  $\gamma_I = 7.5$  and  $\gamma_I = 10$  only. According to concept of CDM in section III, stability index specifies the robustness property. Therefore, choose parameter design of CDM-PI controller  $\gamma_I =$

10 and  $\tau = 200$  which obtained  $K_p = 0.7954$   $K_i = 0.004$  for comparison with IMC-PI controller.

The step responses of Modified Smith Predictor using CDM-PI compare with IMC-PI are shown in Fig.9, and the control signals are shown in Fig. 10. It is obvious that CDM-PI controller produces a step response without an overshoot, fast settling time and this is achieved by a control signal having a smaller magnitude when compared with IMC-PI. Table III represents the performance values of the control system in Fig. 9 and 10.

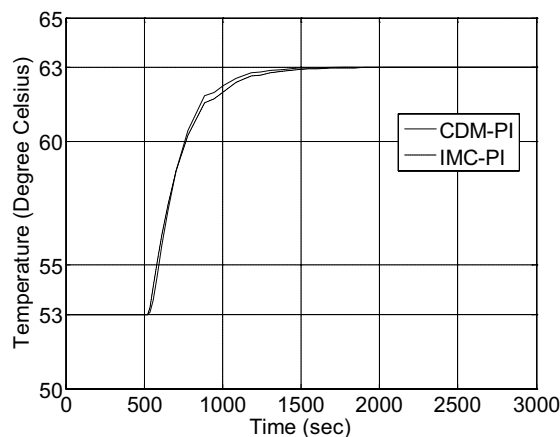


Fig.9 Step response for comparison between CDM-PI and IMC-PI

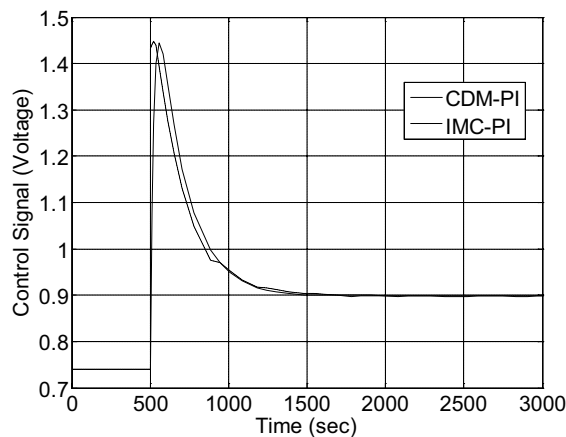


Fig.10 Control Signal for comparison between CDM-PI and IMC-PI

TABLE III

Performance Values of the Time Response Curves Shown in Fig. 9 and 10.

Methods	settling time	max overshoot	max u
CDM-PI	715 s	0 %	1.445 V
IMC-PI	835 s	0 %	1.448 V

### C. Robustness Property

This part the robustness property comparison between CDM-PI and IMC-PI is shown by using parameter of controller from previous part. Which the robustness of the control systems to the  $\pm 20\%$  changes in the parameter  $K$ ,  $T$  and  $L_0$  appearing in Eq. (20) is shown in Fig.11, 12 and 13 respectively. The parameters are changed at steps of 5% and the resulting step responses are plotted for each control

system. Investigation of these figures reveals that Smith Predictor design by CDM-PI control system is much more robust than IMC-PI

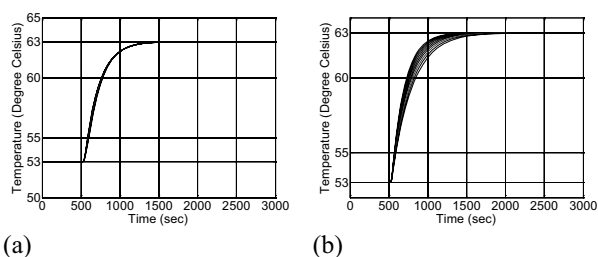


Fig.11 Time response with parameter variation in  $K$   
(a) CDM-PI (b) IMC-PI

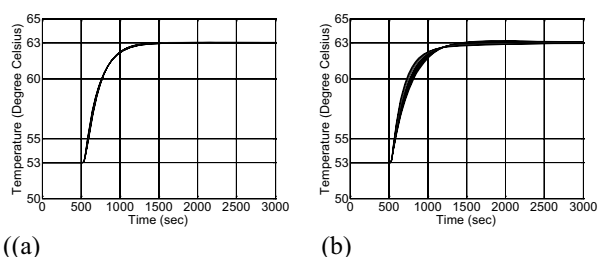


Fig. 12 Time response with parameter variation in  $T$   
(a) CDM-PI (b) IMC-PI

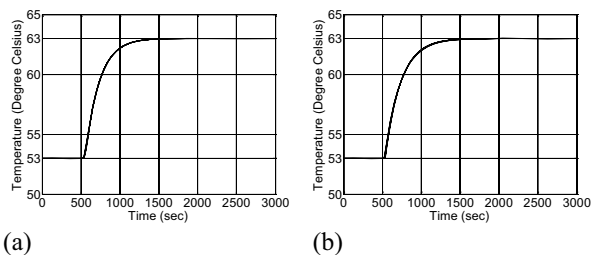


Fig.13 Time response with parameter variation in  $L_0$   
(a) CDM-PI (b) IMC-PI

## VI. CONCLUSION

In this study, the smith predictor design by CDM for the temperature control system has been proposed in this paper. The control system is tested with Matlab/Simulink program and the time domain characteristics are compared with IMC method based on PI controller. It has been show that the proposed method is successful in the controller design than the IMC method for the temperature control to overcome the effects of plant perturbations. Moreover, the proposed method can be neglect dead time which cause of easily and flexible for controller design.

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