

Preliminary Chaos Analyses of Explosion Earthquakes Followed by Harmonic Tremors at Semeru volcano, East Java, Indonesia

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Abstract—Successive event of explosion earthquake and harmonic tremor recorded at Semeru volcano were analyzed to investigate the dynamical system regarding to their eruptive mechanism. The eruptive activity at Semeru volcano East Java, Indonesia is intermittent emission of ash and bombs with Strombolian style which occurred at interval of 15 to 45 minutes. The explosive eruptions accompanied by explosion earthquakes and followed by volcanic tremor which generated by continuous emission of volcanic ash. The spectral and Lyapunov exponent of successive event of explosion and harmonic tremor were analyzed. Peak frequencies of explosion earthquakes range 1.2 to 1.9 Hz and those of the harmonic tremor have peak frequency range 1.5 – 2.2 Hz. The phase space is reconstructed and evaluated based on the Lyapunov exponents. Harmonic tremors have smaller Lyapunov exponent than explosion earthquakes. It can be considerably as correlated complexity of the mechanism from the variance of spectral and fractal dimension and can be concluded that the successive event of harmonic tremor and explosions are chaotic.

Keywords—Semeru volcano, explosion earthquakes, harmonic tremor, lyapunov exponent, chaotic.

I. INTRODUCTION

SEMERU volcano is an andesitic stratovolcano located in East Java (Fig. 1). The peak's summit rises 3676 m above sea level and it is the highest, active volcano on the island of Java. Frequent explosions at the summit crater of Jonggring Seloko have been occurring continuously since 1941. [1,2] During this period, small to moderate strombolian and vulcanian type explosions have occurred, producing explosion plumes rising 400–1000 m above the summit. During active periods, lava flow, lava dome extrusion and pyroclastic flows have also been observed. In 2005 the frequency of the

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explosions averaged 3453 times per month and 115 per day, respectively.[1,2]

In previous study, Maryanto et al., 2005, classified harmonic tremor at Sakurajima volcano into 2 groups: Harmonic Tremors occurred after B-type earthquake swarms (HTB) and Harmonic Tremor which occurred immediately after Eruption (HTE). There is no study focus on harmonic tremor after an eruption at Semeru volcano previously. We have analyzed Harmonic Tremors after an Eruption (HTE) that occurred at Semeru volcano on March 2005. HTEs that occurred at Sakurajima volcano were characterized to understand more detail volcanic activity as supporting information for prediction of volcanic eruption .[5]

Most of the previous study are mainly based on the characteristics of spectra which mostly triggered by linear process, however little intent has been given on nonlinear process. In contrary, theoretically volcanic tremors can be generated by some kind of nonlinear processes.¹⁵⁾ Methods based on the discipline of nonlinear dynamics have been rarely applied to the volcanic tremor and explosion earthquake recorded at some volcanoes.

In the present paper, another approach of chaos analyses of the volcanic earthquakes and tremor analysis has been applied. We applied the method to successive event of explosion earthquakes and harmonic tremors recorded at Semeru volcano, East Java, Indonesia. First, we briefly describe the data obtained from Center for Volcanology and Geological Hazard Mitigation, Indonesia. We then present a method to estimate the Lyapunov exponent of the explosion earthquakes and harmonic tremor. We follow with an application of the method to our data and terminate with a discussion of the implications of these results for driving mechanism of harmonic tremor.

II. OBSERVATION

Semeru volcano has been monitored continuously by CVGHM using 5 seismometers at stations PCK, KPL, TRS, LEK and BES, distributed 0.76-8.9 km apart from the active crater. Seismic sensor installed at the summit and northwest-south slope (Fig. 1). The 5 stations are equipped with short-period (1 Hz) vertical seismometer. The temporary stations (PCK, KPL, and TRS) installed during September – December 2005. The signals from seismometers transmitted to the Sawur Volcano Observatory by FM radio telemetry. The

recorded seismic signals transmitted to the G. Sawur Volcano Observatory by FM radio telemetry. [6]

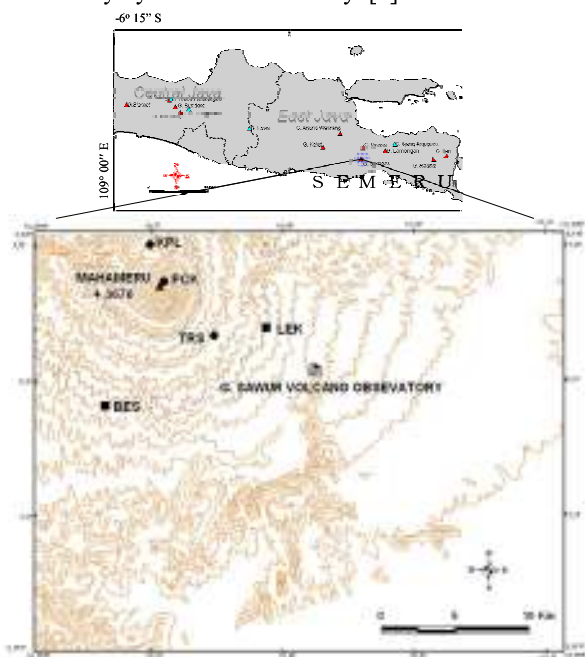


Fig. 1. Map of Semeru volcano and locations of seismic stations at this volcano operated by Center of Volcanology and Geological Hazard Mitigation. Triangle, circles and squares represent summit crater, temporary and permanent stations, respectively. In this study, we used permanent stations (BES and LEK)[6].

The seismic signals are recorded by analog drum recorders; Kinematics PS-2 and digitally sampled 100 Hz by data loggers (Datamark LS-7000) with GPS time calibration. Characteristic signals such as volcanic earthquakes, tectonic earthquakes, and eruption earthquakes also harmonic tremor are recognized on the seismograms. The selected successive events of explosion earthquakes and harmonic tremor are represented in Table I. The harmonic tremor occurred about 2-3 minutes after explosion.

A typical of seismogram HTE recorded at Semeru volcano shown in Fig. 2. The explosive eruption occurred on 00h05m42.93s on March 15, 2005 and waveform became harmonic tremor at 00h09m49.73s, three minutes after the beginning of the eruption. The harmonic tremor continued for 3.3 minutes. The duration and time interval of harmonic tremors after the beginning explosion earthquake on March, 2005 can be detected at the range of 30s - 1020s.

TABLE I EXPLOSION EARTHQUAKE AND HARMONIC TREMOR EVENTS [6]

Time of eruption	Estimated time interval between eruption and harmonic tremor (min)	Estimated duration (min)
2005:03:12 23:36	2	6
2005:03:13 00:39	2	10
2005:03:14 05:34	3	21
2005:03:15 00:05	3	23

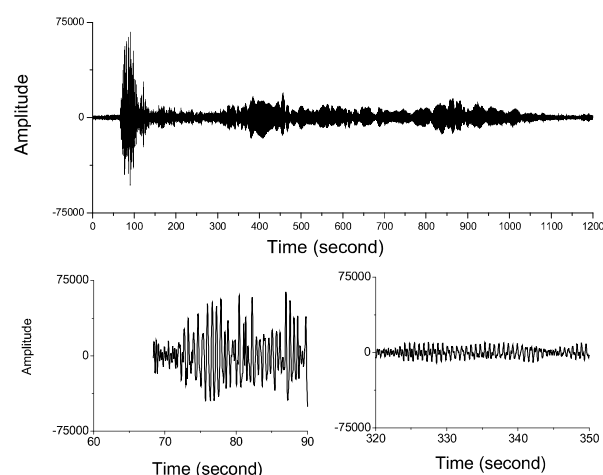


Fig. 2. Vertical component seismogram of small eruption followed by harmonic tremor (upper part). Left bottom shown a typical of the explosion earthquake and right bottom is harmonic tremor.[6]

III. SPECTRA OF HTE

In order to investigate the frequency contents of harmonic tremor, the spectra of harmonic tremors at Semeru volcano were examined. Fast Fourier Transform (FFT) algorithm was applied to digitized seismograms. The time windows used for FFT analysis were 10.24 second long starting from the onset of explosion earthquakes. The time localization can be obtained by windowing the data at different times with sliding window functions. The Fourier transform of this procedure applied on a signal is called short-time Fourier transform (*STFT*), or spectrogram. Figure 3 represent a spectrogram of HTE recorded at station LEK which occurred on March 13, 2005. Broader frequency bands at the beginning of the spectrogram are caused by explosion earthquakes. A few minutes later, sharp peaks are recognized and their frequencies spaced regularly.

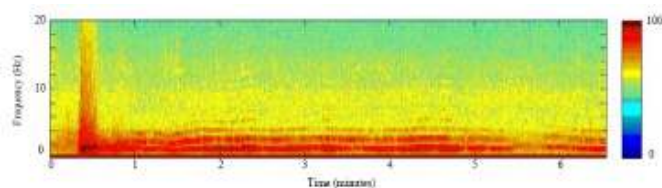


Fig. 3. Spectrogram of vertical component of HTE recorded at station LEK March 13, 2005 which calculated using time windows 10.24 s overlap 50%.

To grasp temporal changes of peak frequencies more clearly, we calculated the running spectra of HTE without overlap of time windows, by shifting the time windows of 10.24 s. Temporal changes of dominant frequency of harmonic tremor at BES and LEK station are shown in Fig. 4. Waveforms with duration 240 seconds are divided into 24 time windows. In the 1st time windows, dominant frequency has 2.1 Hz and then 2nd to 12th dominant frequency gradually decreased, it is shifted to 1.8 Hz. In 12th to 24th, the frequency increased from 1.8 to 2.3 Hz on

BES station. The temporal change at LEK station has similar pattern. 1st to 5th dominant frequency has 2 Hz and then 6th to 14th the frequency gradually decreased it is shifted to 1.8 Hz. In 14th to 23rd the frequency become increased from 1.8 to 2.3 Hz. Frequency of second mode also changes similarly.

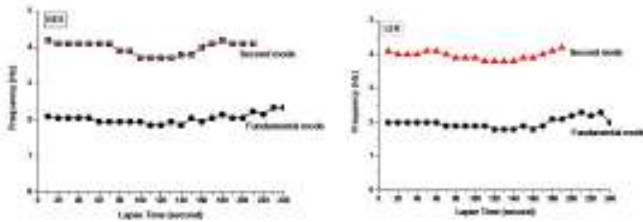


Fig. 4. Temporal change of peak frequency of harmonic tremor recorded at LEK and BES station on March 14, 2005.[6]

IV. QUANTIFYING CHAOS

One of the easiest ways in obtaining the Lyapunov exponent can be done by observing the separation of two close initial trajectories on the attractor and taking the logarithm of the separation. This method cannot be applied to the observation data that have no dealing with two or more sets data closed to initial conditions. In this case, we have to reconstruct the phase space from the time series data.

3.1 Reconstruction of phase space

Reconstructing the phase space from the time series with appropriate time delay and embedding dimension makes it possible to obtain an attractor whose Lyapunov spectrum is identical to that of the original attractor. Mathematically, a reconstructed phase space can be described as follows:

$$y(k) = S(k), S(k + \tau), S(k + 2\tau) \dots, S(k + (m - 1)\tau) \quad (1)$$

where $S(k)$ is the time series from single observation, τ represents appropriate delay time and for a digitized time-series is a multiple of the sampling interval used. The dimension m of the reconstructed space is considered as the sufficient dimension for recovering the object without distorting any of its topological properties, thus it may be different from the true dimension of the space where this object lies. This procedure of phase space reconstruction is termed embedding and the formulation of Takens is called the delay embedding theorem, with m being the embedding dimension. In practical applications both the delay time and the embedding dimension have to be determined from the time-series itself.

3.1.1. Selection of delay time

In a phase space reconstruction procedure, we must ensure that the points in each dimension (coordinate) are independent of each other. Therefore, time delay τ must be chosen so as to result in points that are not correlated to previously generated points. One proposed way of choosing the delay time τ for phase space reconstruction is by calculating the Mutual Information of the time series data and choosing τ as the time of the first local minimum [7]. The Mutual Information of time series $x_i \{i=1, 2, \dots, M\}$ is defined:

$$MI(X, Y) = \sum_i \sum_j p_{xy}(i, j) \log \frac{p_{xy}(i, j)}{p_x(i)p_y(j)} \quad (2)$$

Where in the first step, a sequence $X_i \{i=1, 2, \dots, M\}$ of length M is considered in the time series X_i . Let the probability distribution in X_i be $p_x(i)$ and the probability distribution in the sequence $Y_i \{j=\alpha, \alpha+1, \dots, \alpha+M\}$ obtained by shifting X_i by α be $p_y(j)$. By varying the time shift α , let the time delay, for which first takes the local minimum, be τ . Figure 5 shows the mutual information of the HTB. The first minimum value of the mutual information is approximately 0.05 sec.

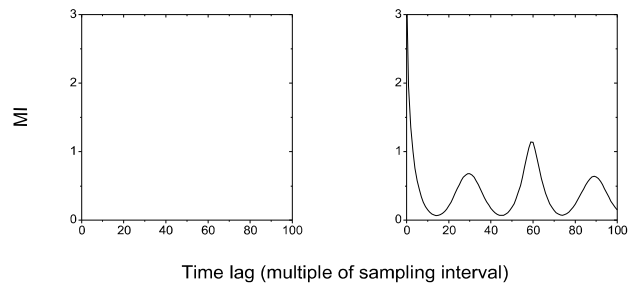


Fig. 5. Example of calculated Mutual information of explosion and harmonic tremor at station LEK calculated for delay times 1–100s.

3.1.2. Minimum embedding dimension Cao's method (1997)[8]

The next step in reconstructing phase space is to recover the appropriate number of coordinates m of the phase space. The idea of a number of coordinates m is a dimension in which the geometrical structure of the phase space is completely unfolded.

The basic method in determining the embedding dimension in phase-space reconstruction is the False Nearest Neighbor method.[9] Suppose the vector y^{NN} is a false neighbor of y , having arrived in its neighborhood by projection from a higher dimension, because the present dimension m does not unfold the attractor, then by going to the next dimension $m+1$, we may move this false neighbor out of the neighborhood of y . Thus, if the additional distance is large compared to the distance in dimension d between nearest neighbors, we have a false neighbor. Otherwise, we have a true neighbor. In order to have a straightforward representation of the minimum embedding dimension, Cao [8] defined the mean value of E_1 , which generally represents the relative Euclidean distance between y^{NN} and y in two consecutive dimensions. Cao's number E_1 consequently will stop changing when the dimension m is greater than the minimum embedding dimension m_0 .

Figure 6 depicts the Cao number as a function of embedding dimension. It can be observed that E_1 approaches a constant value for a dimension higher than four. Thus, we can conclude that the minimum dimension that will totally unfold the phase space is 5.

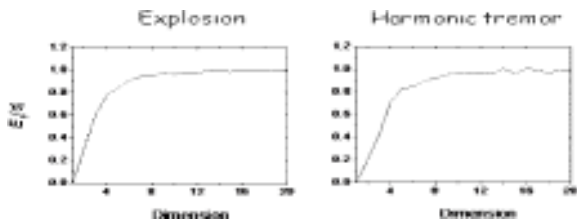


Fig. 6. Example of calculated Minimum embedding dimension for Explosion and harmonic tremor at station LEK calculated using Cao's method.

Examples of an attractor of explosion earthquake and harmonic tremor are shown in Figs. 7 and 8. The attractor obtained from the seismogram of HTE recorded at station LEK.

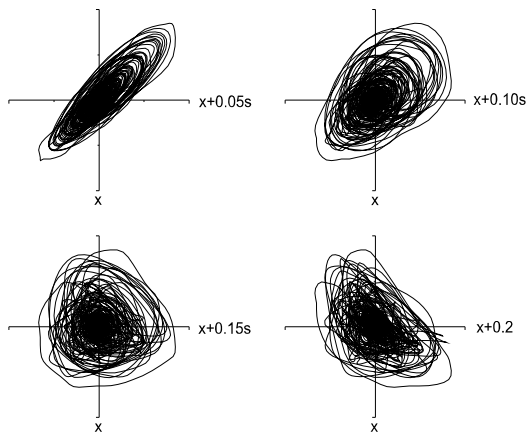


Fig. 7. Four projections of explosion earthquake for delay time 0.05 s, 0.10 s, 0.15 and 0.20, respectively.

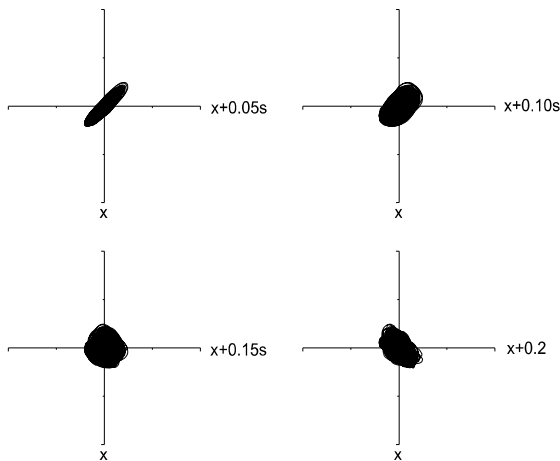


Fig. 8. Four projections of harmonic tremor event for delay time 0.05 s, 0.10 s, 0.15 and 0.20, respectively.

3.2. Estimation of Lyapunov exponent

The current method is principally based on the work of Sano and Sawada.[10] Let us consider a general m -dimensional discrete time dynamical system

$$\mathbf{X}_{i+1} = \mathbf{F}(\mathbf{X}_i), \mathbf{X}_i \in \mathbf{R}^m \quad (3)$$

Where \mathbf{X}_i is a state of the system and $\mathbf{F} : \mathbf{R}^m \rightarrow \mathbf{R}^m$ is a m -dimensional mapping. Considering the infinitesimal small displacement at \mathbf{X}_i be $\delta\mathbf{X}_i$. Then, there holds:

$$\mathbf{X}_{i+1} + \delta\mathbf{X}_{i+1} = \mathbf{F}(\mathbf{X}_i + \delta\mathbf{X}_i) \quad (4)$$

By forming the Taylor expansion and applying the linear approximation, the following mapping is obtained for the small displacement $\delta\mathbf{X}_i$ at \mathbf{X}_i

$$\delta\mathbf{X}_{i+1} = \mathbf{DF}(\mathbf{X}_i) \delta\mathbf{X}_i \quad (5)$$

where $\mathbf{DF}(\mathbf{X}_i)$ is the Jacobian matrix of \mathbf{F} at point \mathbf{X}_i . The mapping is a time dependent linear mapping. Defining $\delta\mathbf{X}_0$ as the initial value and executing N time mappings by \mathbf{DF} , there results:

$$\delta\mathbf{X}_N = \mathbf{DF}(\mathbf{X}_{N-1})\mathbf{DF}(\mathbf{X}_{N-2})\dots\mathbf{DF}(\mathbf{X}_0)\delta\mathbf{X}_0 \quad (6)$$

The Jacobian matrix \mathbf{DF}_N at the N -th mapping is defined as:

$$\mathbf{DF}_N = \mathbf{DF}(\mathbf{X}_{N-1})\mathbf{DF}(\mathbf{X}_{N-2})\dots\mathbf{DF}(\mathbf{X}_0) \quad (7)$$

Using the eigenvalues $\sigma_i(N)$, the Lyapunov spectrum λ_i ($i = 1, 2 \dots m$) is derived as followed:[10, 11]

$$\lambda_i = \lim_{N \rightarrow \infty} \frac{1}{N} \log |\sigma_i(N)| \quad (8)$$

The Lyapunov exponent, λ_i is defined as a divergence rate in bits per unit time (bits/sec) i.e. logarithm of base two. In the case of experimental time series, the Jacobian matrix of Eq. (6) cannot directly be used since the dynamical system of the object of measurement is unknown. The Jacobian matrix of Eq. (6) in the tangent space is estimated from the experimental data according to the method by Sano and Sawada.[10]

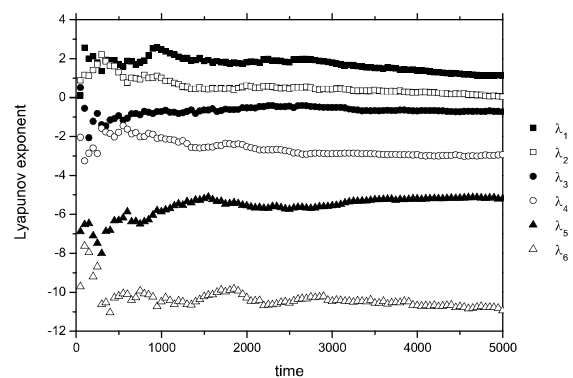


Fig. 9. Lyapunov exponent spectrum of explosion earthquakes recorded at station BES (event20050313 00:39).

Fig. 9 shows the Lyapunov exponent, λ_i of explosion earthquakes, part of HTE recorded at station BES. The largest Lyapunov exponent, λ_1 is positive ($\lambda_1 \sim 2$) for the explosion

earthquakes. The harmonic tremor part also has a positive largest Lyapunov exponent ($\lambda_1 \sim 1$) as shown in Fig. 10.

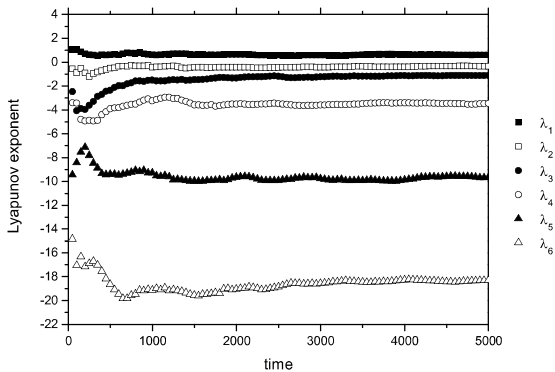


Fig. 10. Lyapunov exponent spectrum of harmonic tremor recorded at station BES (event20050313 00:39).

Furthermore, to check the spatial change, we also calculate Lyapunov exponent of HTE recorded at station LEK. Fig.11 shows the largest Lyapunov exponent of explosion earthquakes at station LEK. The largest Lyapunov exponent, λ_1 is also positive ($\lambda_1 \sim 2.5$). At this station, the harmonic tremor part also has positive largest Lyapunov exponent ($\lambda_1 \sim 1.5$) as shown in Fig. 12.

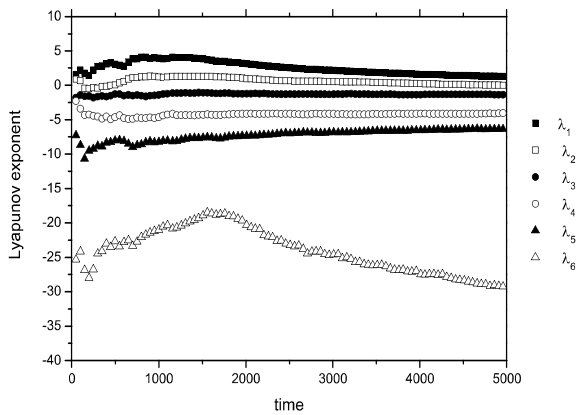


Fig. 11. Lyapunov exponent spectrum of explosion earthquakes recorded at station LEK (event20050313 00:39).

V. DISCUSSIONS AND SOME REMARKS

In this paper, the Lyapunov exponent of Harmonic Tremors after an Eruption (HTE) estimated by using the algorithm proposed by Sano and Sawada method [10]. We applied the method to both of explosion earthquakes and harmonic tremor parts of HTE recorded at station BES and LEK. The largest Lyapunov exponent, λ_1 of harmonic tremor are smaller than the explosion earthquakes at both stations. These indicate that explosion earthquakes have more complicated structure of signals or the quantity information embodied in a pattern.

Similar results appear from some previous studies in the frame fractal dimension as shown in Fig. 13.[6] Various studies have suggested that the dominant frequency of tremor is related to the dimension of magma body and coupling between magma and gas. Change of spectral structure with time is associated by the change of dimension and/or physical boundary conditions of the origin (Kamo, 1977).[12] The changes dominant frequency harmonic tremor at Semeru volcano may be existence changes the conduit size and dimension. It was interpreted as the implication of gas accumulation release that occurred sporadically. The source of gas is magma chamber convection that accumulates gas near the surface and form gas pocket. This gas pocket is covered by semi-solid lava plug. After the gas release process, the pocket will be empty and then refill continuously by another gas that produced from magma chamber. In addition, the variation of fractal dimension along the seismogram can be explained in terms of stationary or non-stationary behavior of source. Maryanto and Iyan (2009) suggest that explosion earthquakes show non-stationary behavior indicated by higher fractal dimension, $\approx 2.2-2.5$, while harmonic tremors present stationary behavior characterized by lower fractal dimension $\approx 1.6-1.9$.[6]

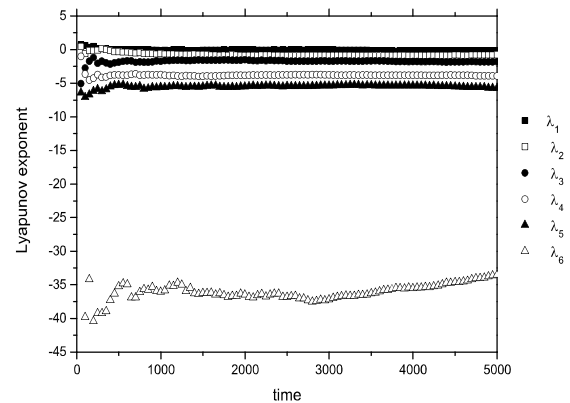


Fig. 12. Lyapunov exponent spectrum of harmonic tremor recorded at station LEK (event20050313 00:39).

Compared with harmonic tremor after explosion at Sakurajima, the change of frequency is smaller at Semeru. The harmonic tremor at Sakurajima increased from 0.9 to 3.7 Hz (Maryanto et al. [13]. This suggests that condition of conduit is more stable and the size is smaller at Semeru. This associated by the change dimension and/or physical boundary conditions of conduit. Furthermore, Maryanto et al.[6,13]and Tameguri et al.[14,15] suggest the following facts should be considered as constraints for modeling of their generation mechanism; spectrum peaks appear at the frequencies of multiple integers of the fundamental frequencies, peak frequencies of harmonic tremor are rather stable, source depths coincide with the gas pocket at uppermost part the conduit.

Because of successive events contain of two different types of volcanic seismic signals, we estimated that the source of these events should be changed from unstable to more stable source as detected by temporal change of fractal dimension and higher

Lyapunov exponent of explosion earthquake than harmonic tremor. In this case, when an explosion earthquake occurred a resonance may occur immediately at the upper part of the conduit.

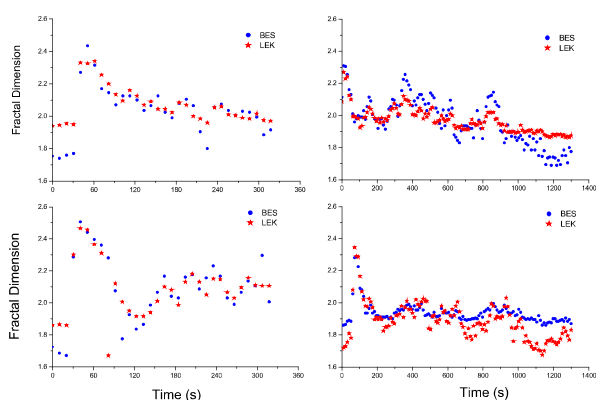


Fig. 13. Time dependent of fractal dimension of four events. Data are from vertical component of ground velocity recorded at station BES (blue dots) and LEK (red stars).[6]

Due to the signals contain of fluctuations concerned with frequency, amplitude, self-affine property, and chaotic behavior, these volcanic tremor may caused by non-linear source processes involving one or several different kinds of magmatic activity that not only supported by theoretical considerations [16, 17], but also by certain characteristics observed in volcanic seismic signals.

Even though the results of spectra, fractal and chaos analyses can tell us about change of conduit and/or source of harmonic tremor, they cannot provide a clue as to what the physical mechanism of this source is. Other methods, that can extract information concerning the physical properties and geometrical configuration of the rock-fluid system from seismic and acoustic data combined with visual observations, are needed in order to accomplish this task. Future volcano monitoring and prediction efforts should, therefore, rely on a multidisciplinary approach on experimental, observational and theoretical when trying to study the nature of harmonic tremor sources whether independent or associated with other volcanic earthquakes .

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